Computer algebra independent integration tests

1-Algebraic-functions/1.2-Trinomial-products/1.2.3-General/1.2.3.4-f-x-^m-d+e-x^n-^q-a+b-x^n+c-x^-2-n-^p

Nasser M. Abbasi

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [156]. This is test number [48].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (156)	% 0. (0)
Mathematica	% 94.23 (147)	% 5.77 (9)
Maple	% 87.82 (137)	% 12.18 (19)
Maxima	% 38.46 (60)	% 61.54 (96)
Fricas	% 67.95 (106)	% 32.05 (50)
Sympy	% 44.23 (69)	% 55.77 (87)
Giac	% 69.87 (109)	% 30.13 (47)

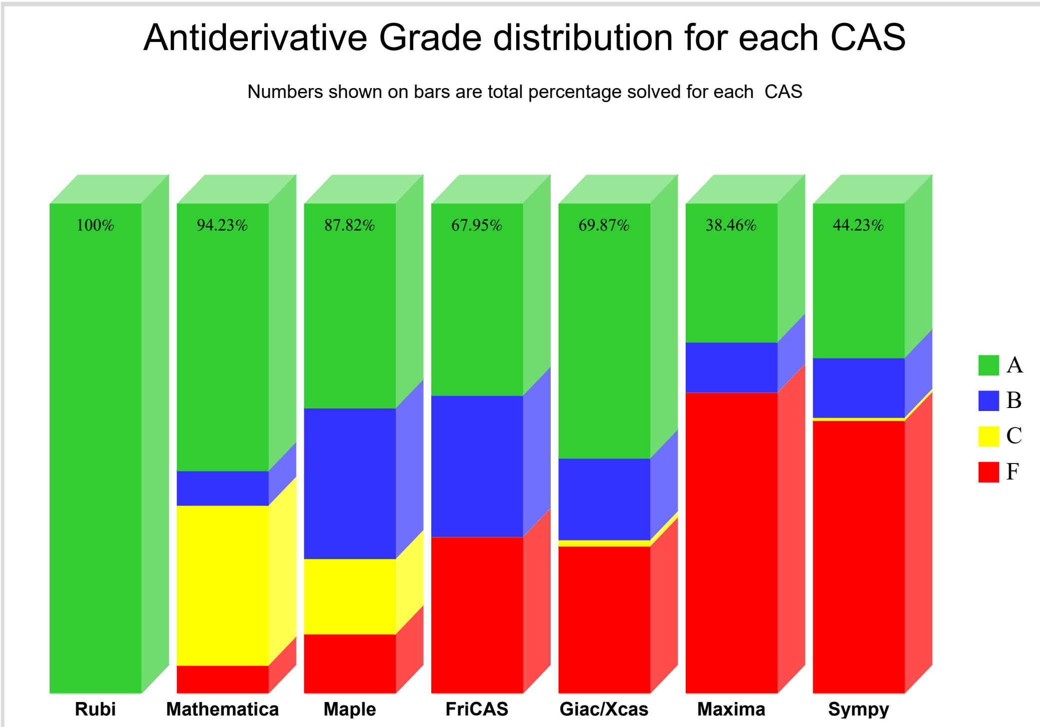
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

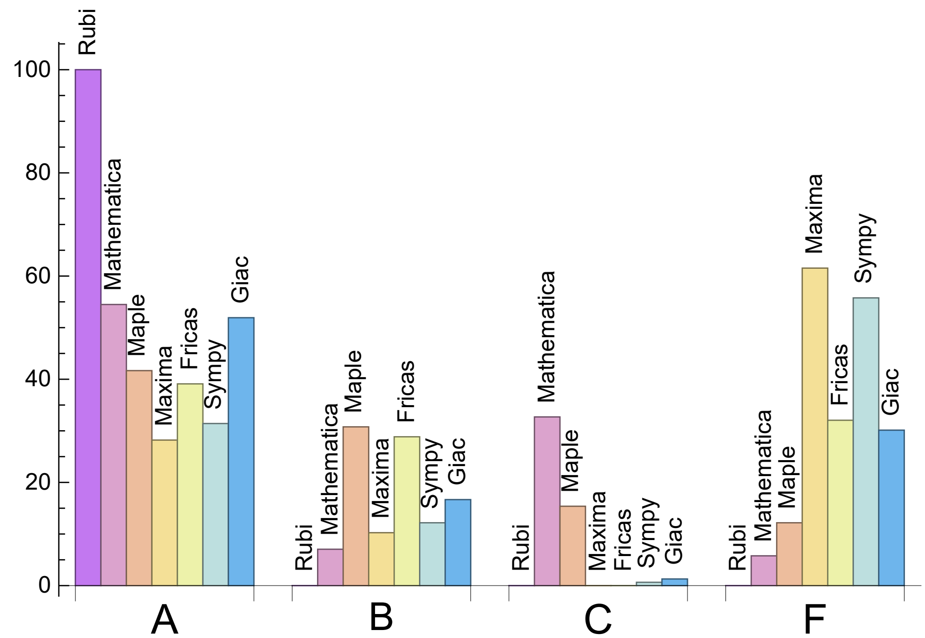
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	54.49	7.05	32.69	5.77
Maple	41.67	30.77	15.38	12.18
Maxima	28.21	10.26	0.	61.54
Fricas	39.1	28.85	0.	32.05
Sympy	31.41	12.18	0.64	55.77
Giac	51.92	16.67	1.28	30.13

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.4	208.71	0.98	153.5	1.
Mathematica	0.78	386.67	1.84	80.	0.96
Maple	0.02	2589.11	116.42	70.	1.24
Maxima	1.2	220.17	10.71	48.	1.39
Fricas	7.77	1867.	23.41	481.	6.52
Sympy	8.86	232.78	8.77	76.	1.
Giac	2.28	429.07	9.99	105.	1.4

1.4 list of integrals that has no closed form antiderivative

{86, 155, 156}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {79, 80, 81, 143, 152, 153, 154}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

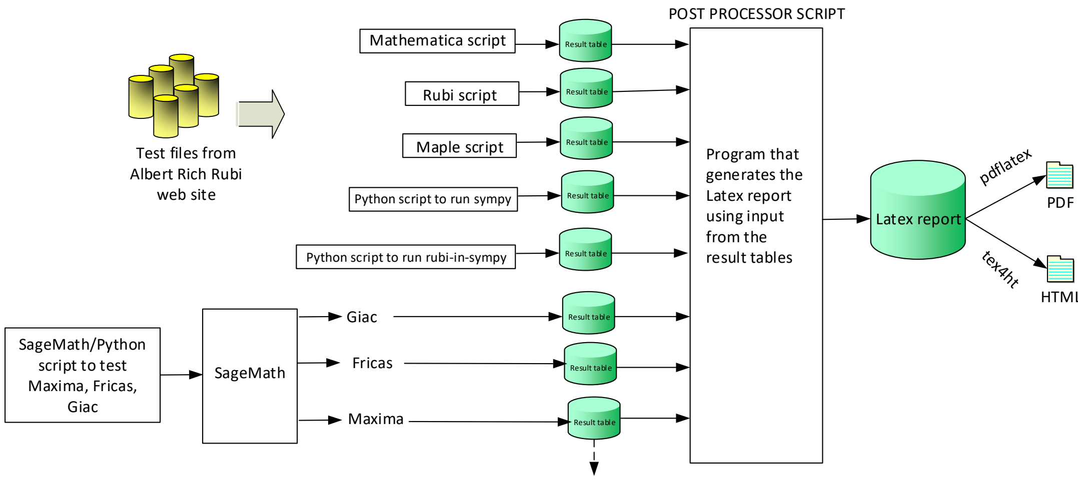
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

Nasser M. Abbasi
June 22, 2018

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 20, 21, 22, 32, 44, 46, 53, 55, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 86, 87, 88, 89, 96, 100, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 141, 144, 149, 152, 153, 154, 155, 156 }

B grade: { 93, 94, 95, 97, 98, 99, 101, 102, 103, 142, 143 }

C grade: { 12, 13, 14, 15, 16, 17, 18, 19, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 45, 47, 48, 49, 50, 51, 52, 54, 56, 57, 58, 59, 60, 79, 80, 81, 82, 83, 84, 85, 138, 139, 140

}

F grade: { 90, 91, 92, 145, 146, 147, 148, 150, 151}

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 7, 8, 10, 11, 12, 13, 20, 21, 22, 23, 24, 32, 33, 34, 44, 48, 53, 55, 57, 59, 64, 65, 66, 67, 74, 86, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 144, 155, 156 }

B grade: { 6, 9, 35, 36, 37, 38, 39, 40, 41, 42, 46, 50, 61, 62, 63, 68, 69, 70, 71, 72, 73, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 125, 126, 127, 128 }

C grade: { 14, 15, 16, 17, 18, 19, 25, 26, 27, 28, 29, 30, 31, 43, 45, 47, 49, 51, 52, 54, 56, 58, 60, 140 }

F grade: { 87, 88, 89, 90, 91, 92, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 20, 21, 22, 23, 24, 32, 33, 53, 57, 86, 93, 97, 101, 105, 106, 107, 108, 109, 113, 114, 115, 116, 117, 121, 122, 123, 125, 130, 131, 132, 134, 135, 136, 138, 139, 140, 144, 155, 156

B grade: { 94, 95, 98, 99, 102, 103, 110, 111, 112, 118, 119, 120, 124, 126, 127, 128 }

C grade: { }

F grade: { 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 25, 26, 27, 28, 29, 30, 31, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 96, 100, 104, 129, 133, 137, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 20, 21, 22, 23, 24, 32, 33, 34, 44, 48, 52, 53, 55, 57, 58, 61, 62, 63, 64, 65, 66, 86, 105, 106, 107, 108, 113, 114, 115, 116, 121, 122, 123, 124, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 144, 155, 156 }

B grade: { 8, 17, 25, 26, 27, 28, 29, 30, 31, 45, 46, 47, 50, 54, 56, 59, 60, 72, 73, 74, 75, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 109, 110, 111, 112, 117, 118, 119, 120, 125, 126, 127, 128 }

C grade: { }

F grade: { 14, 15, 16, 18, 19, 35, 36, 37, 38, 39, 40, 41, 42, 43, 49, 51, 67, 68, 69, 70, 71, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 16, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 52, 53, 54, 55, 56, 57, 58, 59, 60, 105, 106, 107, 113, 114, 115, 121, 122, 123, 124, 137 }

B grade: { 9, 10, 11, 44, 93, 94, 95, 97, 98, 99, 101, 102, 103, 109, 117, 125, 129, 133, 138 }

C grade: { 144 }

F grade: { 12, 13, 14, 15, 17, 18, 19, 40, 41, 42, 43, 45, 46, 47, 48, 49, 50, 51, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 96, 100, 104, 108, 110, 111, 112, 116, 118, 119, 120, 126, 127, 128, 130, 131, 132, 134, 135, 136, 139, 140, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 20, 21, 22, 23, 24, 32, 33, 34, 44, 48, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 86, 105, 106, 107, 108, 109, 110, 112, 113, 114, 115, 116, 117, 118, 120, 121, 122, 123, 124, 125, 126, 127, 128, 132, 136, 137, 138, 139, 140, 144, 155, 156 }

B grade: { 25, 26, 27, 28, 29, 30, 31, 59, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 129, 130, 131, 133, 134, 135 }

C grade: { 46, 50 }

F grade: { 14, 15, 16, 17, 18, 19, 35, 36, 37, 38, 39, 40, 41, 42, 43, 45, 47, 49, 51, 79, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 111, 119, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	164	169	224	448	187	234
normalized size	1	1.	1.01	1.04	1.37	2.75	1.15	1.44
time (sec)	N/A	0.185	0.048	0.001	1.006	1.139	0.096	1.133

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	135	136	182	360	151	190
normalized size	1	1.	1.	1.01	1.35	2.67	1.12	1.41
time (sec)	N/A	0.125	0.036	0.001	0.983	1.164	0.089	1.114

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	104	103	138	282	117	147
normalized size	1	1.	1.01	1.	1.34	2.74	1.14	1.43
time (sec)	N/A	0.097	0.029	0.002	1.053	1.157	0.082	1.077

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	73	70	93	192	75	103
normalized size	1	1.	1.	0.96	1.27	2.63	1.03	1.41
time (sec)	N/A	0.062	0.022	0.	0.962	1.097	0.075	1.162

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	37	49	107	39	58
normalized size	1	1.	1.	0.88	1.17	2.55	0.93	1.38
time (sec)	N/A	0.028	0.009	0.	1.058	1.128	0.063	1.099

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	176	313	0	1096	175	279
normalized size	1	1.	0.94	1.66	0.	5.83	0.93	1.48
time (sec)	N/A	0.211	0.155	0.004	0.	1.433	1.07	1.103

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	199	345	0	1548	206	306
normalized size	1	1.	0.93	1.62	0.	7.27	0.97	1.44
time (sec)	N/A	0.226	0.199	0.008	0.	1.389	1.984	1.122

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	209	362	0	2067	246	340
normalized size	1	1.	0.86	1.5	0.	8.54	1.02	1.4
time (sec)	N/A	0.262	0.271	0.01	0.	1.398	7.551	1.145

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	126	260	0	902	619	177
normalized size	1	1.	0.95	1.97	0.	6.83	4.69	1.34
time (sec)	N/A	0.218	0.065	0.004	0.	3.254	15.002	1.361

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	93	175	0	664	434	128
normalized size	1	1.	0.96	1.8	0.	6.85	4.47	1.32
time (sec)	N/A	0.12	0.07	0.003	0.	1.928	9.096	1.35

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	71	99	0	481	287	95
normalized size	1	1.	0.99	1.38	0.	6.68	3.99	1.32
time (sec)	N/A	0.073	0.05	0.002	0.	1.445	4.389	1.375

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	80	106	0	556	0	103
normalized size	1	1.	1.03	1.36	0.	7.13	0.	1.32
time (sec)	N/A	0.128	0.034	0.006	0.	2.198	0.	1.384

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	130	191	0	845	0	173
normalized size	1	1.	1.16	1.71	0.	7.54	0.	1.54
time (sec)	N/A	0.197	0.052	0.009	0.	4.111	0.	1.366

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	723	723	88	70	0	0	0	0
normalized size	1	1.	0.12	0.1	0.	0.	0.	0.
time (sec)	N/A	1.813	0.049	0.007	0.	0.	0.	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	718	718	88	67	0	0	0	0
normalized size	1	1.	0.12	0.09	0.	0.	0.	0.
time (sec)	N/A	1.457	0.052	0.006	0.	0.	0.	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	634	634	59	49	0	0	920	0
normalized size	1	1.	0.09	0.08	0.	0.	1.45	0.
time (sec)	N/A	0.728	0.031	0.002	0.	0.	133.311	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	634	634	61	47	0	28045	0	0
normalized size	1	1.	0.1	0.07	0.	44.24	0.	0.
time (sec)	N/A	0.654	0.031	0.003	0.	108.462	0.	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	653	653	85	70	0	0	0	0
normalized size	1	1.	0.13	0.11	0.	0.	0.	0.
time (sec)	N/A	1.175	0.049	0.006	0.	0.	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	655	655	89	68	0	0	0	0
normalized size	1	1.	0.14	0.1	0.	0.	0.	0.
time (sec)	N/A	1.11	0.048	0.006	0.	0.	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	38	50	109	42	50
normalized size	1	1.	1.	0.83	1.09	2.37	0.91	1.09
time (sec)	N/A	0.058	0.016	0.003	1.491	1.778	0.15	1.11

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	25	32	76	32	32
normalized size	1	1.	1.	0.81	1.03	2.45	1.03	1.03
time (sec)	N/A	0.035	0.008	0.003	1.457	1.8	0.126	1.167

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	33	43	95	37	43
normalized size	1	1.	1.	0.85	1.1	2.44	0.95	1.1
time (sec)	N/A	0.04	0.009	0.002	1.507	1.681	0.14	1.146

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	44	35	51	108	41	47
normalized size	1	1.	1.07	0.85	1.24	2.63	1.	1.15
time (sec)	N/A	0.055	0.013	0.005	1.458	1.766	0.154	1.11

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	45	25	32	84	36	32
normalized size	1	1.	1.45	0.81	1.03	2.71	1.16	1.03
time (sec)	N/A	0.045	0.013	0.004	1.526	1.352	0.158	1.106

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	418	418	47	46	0	3918	31	867
normalized size	1	1.	0.11	0.11	0.	9.37	0.07	2.07
time (sec)	N/A	0.538	0.011	0.006	0.	1.607	0.177	1.168

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	382	382	48	44	0	5949	32	1103
normalized size	1	1.	0.13	0.12	0.	15.57	0.08	2.89
time (sec)	N/A	0.328	0.014	0.004	0.	1.942	0.193	1.16

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	378	378	46	41	0	3906	24	853
normalized size	1	1.	0.12	0.11	0.	10.33	0.06	2.26
time (sec)	N/A	0.259	0.013	0.004	0.	1.673	0.179	1.178

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	411	411	55	44	0	5936	22	1108
normalized size	1	1.	0.13	0.11	0.	14.44	0.05	2.7
time (sec)	N/A	0.276	0.012	0.006	0.	2.013	0.183	1.196

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	411	411	57	44	0	3903	26	860
normalized size	1	1.	0.14	0.11	0.	9.5	0.06	2.09
time (sec)	N/A	0.277	0.012	0.004	0.	1.639	0.185	1.195

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	416	416	47	46	0	5952	31	1119
normalized size	1	1.	0.11	0.11	0.	14.31	0.07	2.69
time (sec)	N/A	0.275	0.014	0.006	0.	1.937	0.19	1.157

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	418	418	47	46	0	3945	32	867
normalized size	1	1.	0.11	0.11	0.	9.44	0.08	2.07
time (sec)	N/A	0.359	0.012	0.006	0.	1.626	0.2	1.164

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	37	33	43	96	37	43
normalized size	1	1.	1.03	0.92	1.19	2.67	1.03	1.19
time (sec)	N/A	0.039	0.01	0.003	1.5	1.276	0.136	1.171

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	55	35	51	107	41	47
normalized size	1	1.	1.41	0.9	1.31	2.74	1.05	1.21
time (sec)	N/A	0.056	0.014	0.006	1.486	1.476	0.143	1.128

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	55	35	0	107	41	47
normalized size	1	1.	1.41	0.9	0.	2.74	1.05	1.21
time (sec)	N/A	0.063	0.01	0.004	0.	1.498	0.148	1.094

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	396	396	103	1070	0	0	400	0
normalized size	1	1.	0.26	2.7	0.	0.	1.01	0.
time (sec)	N/A	0.416	0.179	0.161	0.	0.	10.083	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	356	101	1010	0	0	257	0
normalized size	1	1.	0.28	2.84	0.	0.	0.72	0.
time (sec)	N/A	0.311	0.153	0.029	0.	0.	5.848	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	316	98	956	0	0	124	0
normalized size	1	1.	0.31	3.03	0.	0.	0.39	0.
time (sec)	N/A	0.246	0.134	0.027	0.	0.	3.115	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	98	907	0	0	119	0
normalized size	1	1.	0.35	3.26	0.	0.	0.43	0.
time (sec)	N/A	0.182	0.091	0.028	0.	0.	2.629	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	289	102	934	0	0	119	0
normalized size	1	1.	0.35	3.23	0.	0.	0.41	0.
time (sec)	N/A	0.189	0.106	0.038	0.	0.	18.698	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	309	309	129	1005	0	0	0	0
normalized size	1	1.	0.42	3.25	0.	0.	0.	0.
time (sec)	N/A	0.211	0.139	0.042	0.	0.	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	349	349	166	1095	0	0	0	0
normalized size	1	1.	0.48	3.14	0.	0.	0.	0.
time (sec)	N/A	0.324	0.191	0.043	0.	0.	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	389	389	200	1182	0	0	0	0
normalized size	1	1.	0.51	3.04	0.	0.	0.	0.
time (sec)	N/A	0.395	0.243	0.048	0.	0.	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	433	433	88	67	0	0	0	0
normalized size	1	1.	0.2	0.15	0.	0.	0.	0.
time (sec)	N/A	1.132	0.075	0.004	0.	0.	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	71	99	0	481	287	95
normalized size	1	1.	0.99	1.38	0.	6.68	3.99	1.32
time (sec)	N/A	0.072	0.056	0.003	0.	2.224	7.519	6.167

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	375	375	59	51	0	26996	0	0
normalized size	1	1.	0.16	0.14	0.	71.99	0.	0.
time (sec)	N/A	0.456	0.047	0.003	0.	97.806	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	179	340	0	3077	0	6484
normalized size	1	1.	0.97	1.85	0.	16.72	0.	35.24
time (sec)	N/A	0.213	0.156	0.022	0.	2.278	0.	7.939

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	375	375	61	47	0	26437	0	0
normalized size	1	1.	0.16	0.13	0.	70.5	0.	0.
time (sec)	N/A	0.351	0.048	0.003	0.	20.939	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	80	106	0	556	0	105
normalized size	1	1.	1.03	1.36	0.	7.13	0.	1.35
time (sec)	N/A	0.126	0.034	0.007	0.	4.899	0.	6.592

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	392	392	85	72	0	0	0	0
normalized size	1	1.	0.22	0.18	0.	0.	0.	0.
time (sec)	N/A	0.683	0.065	0.006	0.	0.	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	89	365	0	5485	0	2365
normalized size	1	1.	0.45	1.83	0.	27.56	0.	11.88
time (sec)	N/A	0.311	0.049	0.02	0.	5.412	0.	7.081

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	394	394	86	68	0	0	0	0
normalized size	1	1.	0.22	0.17	0.	0.	0.	0.
time (sec)	N/A	0.625	0.074	0.007	0.	0.	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	46	34	0	613	170	281
normalized size	1	1.	0.17	0.12	0.	2.21	0.61	1.01
time (sec)	N/A	0.3	0.016	0.006	0.	1.832	0.328	1.151

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	33	43	96	37	43
normalized size	1	1.	1.	0.85	1.1	2.46	0.95	1.1
time (sec)	N/A	0.042	0.013	0.004	1.487	1.752	0.182	1.119

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	355	355	55	46	0	2313	27	342
normalized size	1	1.	0.15	0.13	0.	6.52	0.08	0.96
time (sec)	N/A	0.289	0.016	0.007	0.	2.07	1.35	1.14

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	44	39	0	104	42	42
normalized size	1	1.	0.88	0.78	0.	2.08	0.84	0.84
time (sec)	N/A	0.04	0.016	0.012	0.	1.726	0.162	1.112

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	355	355	57	44	0	2313	26	342
normalized size	1	1.	0.16	0.12	0.	6.52	0.07	0.96
time (sec)	N/A	0.216	0.013	0.	0.	1.868	1.542	1.127

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	44	35	51	109	41	51
normalized size	1	1.	1.07	0.85	1.24	2.66	1.	1.24
time (sec)	N/A	0.053	0.013	0.006	1.545	1.466	0.201	1.108

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	280	47	38	0	617	168	284
normalized size	1	1.	0.17	0.14	0.	2.2	0.6	1.01
time (sec)	N/A	0.208	0.016	0.007	0.	1.629	0.306	1.135

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	49	70	0	566	76	348
normalized size	1	1.	0.55	0.79	0.	6.36	0.85	3.91
time (sec)	N/A	0.09	0.016	0.01	0.	1.569	0.263	1.263

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	370	370	47	46	0	1962	32	348
normalized size	1	1.	0.13	0.12	0.	5.3	0.09	0.94
time (sec)	N/A	0.269	0.015	0.01	0.	1.798	1.599	1.17

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	280	283	662	0	2079	0	398
normalized size	1	1.	1.01	2.36	0.	7.42	0.	1.42
time (sec)	N/A	0.597	0.241	0.014	0.	137.062	0.	1.108

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	218	218	218	512	0	1623	0	302
normalized size	1	1.	1.	2.35	0.	7.44	0.	1.39
time (sec)	N/A	0.395	0.181	0.006	0.	84.083	0.	1.115

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	178	388	0	1237	0	250
normalized size	1	1.	1.01	2.2	0.	7.03	0.	1.42
time (sec)	N/A	0.285	0.187	0.007	0.	30.694	0.	1.096

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	132	275	0	872	0	201
normalized size	1	1.	0.89	1.85	0.	5.85	0.	1.35
time (sec)	N/A	0.21	0.128	0.007	0.	9.367	0.	1.117

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	107	169	0	694	0	171
normalized size	1	1.	0.86	1.36	0.	5.6	0.	1.38
time (sec)	N/A	0.145	0.079	0.003	0.	3.256	0.	1.109

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	105	168	0	697	0	170
normalized size	1	1.	0.85	1.37	0.	5.67	0.	1.38
time (sec)	N/A	0.107	0.077	0.006	0.	3.293	0.	1.096

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	159	152	285	0	0	0	221
normalized size	1	1.01	0.96	1.8	0.	0.	0.	1.4
time (sec)	N/A	0.271	0.189	0.008	0.	0.	0.	1.104

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	194	412	0	0	0	284
normalized size	1	1.	1.01	2.13	0.	0.	0.	1.47
time (sec)	N/A	0.343	0.178	0.011	0.	0.	0.	1.108

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	252	562	0	0	0	377
normalized size	1	1.	1.	2.23	0.	0.	0.	1.5
time (sec)	N/A	0.428	0.234	0.012	0.	0.	0.	1.099

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	343	338	943	0	0	0	763
normalized size	1	1.	0.99	2.75	0.	0.	0.	2.22
time (sec)	N/A	0.907	0.38	0.012	0.	0.	0.	1.134

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	269	765	0	0	0	643
normalized size	1	1.	0.98	2.79	0.	0.	0.	2.35
time (sec)	N/A	0.563	0.316	0.012	0.	0.	0.	1.131

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	207	580	0	3005	0	556
normalized size	1	1.	0.84	2.36	0.	12.22	0.	2.26
time (sec)	N/A	0.395	0.248	0.007	0.	110.084	0.	1.117

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	159	389	0	2365	0	455
normalized size	1	1.	0.82	2.01	0.	12.19	0.	2.35
time (sec)	N/A	0.306	0.24	0.008	0.	36.977	0.	1.122

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	148	328	0	2217	0	441
normalized size	1	1.	0.81	1.79	0.	12.11	0.	2.41
time (sec)	N/A	0.236	0.262	0.007	0.	31.559	0.	1.136

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	151	386	0	2295	0	447
normalized size	1	1.	0.8	2.04	0.	12.14	0.	2.37
time (sec)	N/A	0.305	0.226	0.009	0.	17.499	0.	1.112

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	249	246	589	0	0	0	528
normalized size	1	1.	0.99	2.38	0.	0.	0.	2.13
time (sec)	N/A	0.409	0.285	0.013	0.	0.	0.	1.136

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	287	791	0	0	0	657
normalized size	1	1.	0.99	2.72	0.	0.	0.	2.26
time (sec)	N/A	0.563	0.381	0.016	0.	0.	0.	1.109

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	372	372	370	993	0	0	0	792
normalized size	1	1.	0.99	2.67	0.	0.	0.	2.13
time (sec)	N/A	0.851	0.468	0.019	0.	0.	0.	1.119

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	981	981	10904	11938	0	0	0	0
normalized size	1	1.	11.12	12.17	0.	0.	0.	0.
time (sec)	N/A	6.172	14.328	0.148	0.	0.	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	778	778	7531	9182	0	0	0	0
normalized size	1	1.	9.68	11.8	0.	0.	0.	0.
time (sec)	N/A	2.349	13.756	0.062	0.	0.	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	636	636	5350	6302	0	0	0	0
normalized size	1	1.	8.41	9.91	0.	0.	0.	0.
time (sec)	N/A	0.993	13.094	0.049	0.	0.	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	550	550	693	4361	0	0	0	0
normalized size	1	1.	1.26	7.93	0.	0.	0.	0.
time (sec)	N/A	0.655	12.161	0.045	0.	0.	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	955	955	1258	3023	0	0	0	0
normalized size	1	1.	1.32	3.17	0.	0.	0.	0.
time (sec)	N/A	3.313	10.681	0.048	0.	0.	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	929	929	1372	3553	0	0	0	0
normalized size	1	1.	1.48	3.82	0.	0.	0.	0.
time (sec)	N/A	2.725	11.486	0.045	0.	0.	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1287	1287	811	4957	0	0	0	0
normalized size	1	1.	0.63	3.85	0.	0.	0.	0.
time (sec)	N/A	5.302	12.653	0.051	0.	0.	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.019	0.191	0.424	0.	0.	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	358	358	249	0	0	0	0	0
normalized size	1	1.	0.7	0.	0.	0.	0.	0.
time (sec)	N/A	0.237	0.314	0.074	0.	0.	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	189	0	0	0	0	0
normalized size	1	1.	0.72	0.	0.	0.	0.	0.
time (sec)	N/A	0.16	0.168	0.075	0.	0.	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	136	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.095	0.076	0.073	0.	0.	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	194	194	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.224	0.116	0.094	0.	0.	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	302	302	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.333	0.163	0.094	0.	0.	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	412	412	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.449	0.607	0.092	0.	0.	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	201	46548	19	3474	1326	1952
normalized size	1	1.	12.56	2909.25	1.19	217.12	82.88	122.
time (sec)	N/A	0.06	0.169	0.004	1.036	0.936	0.305	1.113

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	233	46552	1674	3578	1384	1963
normalized size	1	1.	12.94	2586.22	93.	198.78	76.89	109.06
time (sec)	N/A	0.329	0.174	0.003	1.037	0.893	0.307	1.15

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	233	46552	1674	3594	1394	1963
normalized size	1	1.	12.94	2586.22	93.	199.67	77.44	109.06
time (sec)	N/A	0.302	0.179	0.003	1.07	0.927	0.305	1.147

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	22	2042	0	2969	0	2286
normalized size	1	1.	0.96	88.78	0.	129.09	0.	99.39
time (sec)	N/A	0.056	0.068	0.062	0.	1.298	0.	1.291

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	201	47685	22	3474	1326	1958
normalized size	1	1.	11.17	2649.17	1.22	193.	73.67	108.78
time (sec)	N/A	0.069	0.173	0.006	1.23	0.808	0.313	1.11

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	233	47688	1677	3578	1384	1963
normalized size	1	1.	11.65	2384.4	83.85	178.9	69.2	98.15
time (sec)	N/A	0.322	0.168	0.002	1.054	0.86	0.311	1.146

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	233	47688	1677	3594	1394	1963
normalized size	1	1.	11.65	2384.4	83.85	179.7	69.7	98.15
time (sec)	N/A	0.31	0.165	0.001	1.062	0.968	0.321	1.14

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	24	2046	0	2969	0	2286
normalized size	1	1.	0.96	81.84	0.	118.76	0.	91.44
time (sec)	N/A	0.06	0.056	0.06	0.	1.358	0.	1.287

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	172	155	18	387	175	208
normalized size	1	1.	11.47	10.33	1.2	25.8	11.67	13.87
time (sec)	N/A	0.014	0.005	0.003	1.002	0.937	0.118	1.088

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	182	157	211	402	182	211
normalized size	1	1.	11.38	9.81	13.19	25.12	11.38	13.19
time (sec)	N/A	0.054	0.006	0.004	1.134	0.897	0.157	1.089

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	186	157	211	408	185	211
normalized size	1	1.	11.62	9.81	13.19	25.5	11.56	13.19
time (sec)	N/A	0.056	0.006	0.004	1.17	0.818	0.121	1.094

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	230	0	467	0	255
normalized size	1	1.	1.	10.95	0.	22.24	0.	12.14
time (sec)	N/A	0.033	0.118	0.033	0.	1.103	0.	1.132

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	10	12	15	30	10	15
normalized size	1	1.	0.91	1.09	1.36	2.73	0.91	1.36
time (sec)	N/A	0.004	0.003	0.001	1.138	0.979	0.297	1.096

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	20	38	14	20
normalized size	1	1.	1.	0.94	1.18	2.24	0.82	1.18
time (sec)	N/A	0.019	0.006	0.	1.139	0.995	0.418	1.137

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	20	38	14	20
normalized size	1	1.	1.	0.94	1.18	2.24	0.82	1.18
time (sec)	N/A	0.024	0.007	0.002	1.03	0.988	0.53	1.368

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	24	31	41	0	26
normalized size	1	1.	1.	1.26	1.63	2.16	0.	1.37
time (sec)	N/A	0.027	0.109	0.019	1.161	1.109	0.	1.082

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	15	15	19	737	359	19
normalized size	1	1.	0.94	0.94	1.19	46.06	22.44	1.19
time (sec)	N/A	0.005	0.011	0.	1.031	1.236	51.451	1.141

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	475	748	0	22
normalized size	1	1.	1.	0.94	26.39	41.56	0.	1.22
time (sec)	N/A	0.02	0.014	0.002	1.673	1.244	0.	50.318

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	475	749	0	0
normalized size	1	1.	1.	0.94	26.39	41.61	0.	0.
time (sec)	N/A	0.023	0.013	0.001	1.705	1.268	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	22	22	562	851	0	28
normalized size	1	1.	0.96	0.96	24.43	37.	0.	1.22
time (sec)	N/A	0.027	0.062	0.059	3.203	1.356	0.	1.183

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	12	14	18	30	10	18
normalized size	1	1.	0.92	1.08	1.38	2.31	0.77	1.38
time (sec)	N/A	0.005	0.005	0.	1.186	0.95	0.393	1.121

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	23	38	14	23
normalized size	1	1.	1.	0.95	1.21	2.	0.74	1.21
time (sec)	N/A	0.019	0.007	0.003	1.014	1.038	0.418	1.158

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	23	38	14	23
normalized size	1	1.	1.	0.95	1.21	2.	0.74	1.21
time (sec)	N/A	0.024	0.006	0.	0.979	1.016	0.512	1.332

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	26	34	41	0	28
normalized size	1	1.	1.	1.24	1.62	1.95	0.	1.33
time (sec)	N/A	0.029	0.114	0.02	1.148	1.208	0.	1.095

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	16	17	22	737	359	22
normalized size	1	1.	0.89	0.94	1.22	40.94	19.94	1.22
time (sec)	N/A	0.004	0.013	0.002	1.006	1.318	55.204	1.109

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	481	748	0	24
normalized size	1	1.	1.	0.95	24.05	37.4	0.	1.2
time (sec)	N/A	0.02	0.017	0.001	1.481	1.324	0.	55.488

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	481	749	0	0
normalized size	1	1.	1.	0.95	24.05	37.45	0.	0.
time (sec)	N/A	0.024	0.017	0.	1.531	1.327	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	23	24	566	851	0	31
normalized size	1	1.	0.92	0.96	22.64	34.04	0.	1.24
time (sec)	N/A	0.029	0.068	0.059	2.891	1.513	0.	1.238

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	9	9	14	24	8	15
normalized size	1	1.	0.9	0.9	1.4	2.4	0.8	1.5
time (sec)	N/A	0.004	0.004	0.002	1.026	1.096	0.353	1.096

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	15	15	14	23	39	12	20
normalized size	1	0.94	0.94	0.88	1.44	2.44	0.75	1.25
time (sec)	N/A	0.024	0.006	0.004	0.997	1.021	0.324	1.097

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	15	15	14	23	39	12	20
normalized size	1	0.94	0.94	0.88	1.44	2.44	0.75	1.25
time (sec)	N/A	0.03	0.007	0.006	1.202	1.168	0.348	1.121

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	18	63	42	48	23
normalized size	1	1.	1.	1.2	4.2	2.8	3.2	1.53
time (sec)	N/A	0.035	0.012	0.018	1.028	1.086	134.943	1.091

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	14	177	18	171	87	18
normalized size	1	1.	0.93	11.8	1.2	11.4	5.8	1.2
time (sec)	N/A	0.004	0.02	0.017	0.981	1.189	4.082	1.112

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	197	109	177	0	20
normalized size	1	1.	1.	12.31	6.81	11.06	0.	1.25
time (sec)	N/A	0.021	0.028	0.019	1.038	1.05	0.	1.125

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	197	109	177	0	20
normalized size	1	1.	1.	12.31	6.81	11.06	0.	1.25
time (sec)	N/A	0.025	0.035	0.013	1.086	1.138	0.	1.121

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	203	826	236	0	27
normalized size	1	1.	1.	9.67	39.33	11.24	0.	1.29
time (sec)	N/A	0.032	0.181	0.05	1.215	1.315	0.	1.139

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	19	21	0	63	104	72
normalized size	1	1.	0.95	1.05	0.	3.15	5.2	3.6
time (sec)	N/A	0.005	0.008	0.004	0.	1.078	51.315	1.118

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	45	74	0	84
normalized size	1	1.	1.	0.96	1.8	2.96	0.	3.36
time (sec)	N/A	0.019	0.011	0.003	1.163	1.188	0.	1.143

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	45	74	0	84
normalized size	1	1.	1.	0.96	1.8	2.96	0.	3.36
time (sec)	N/A	0.024	0.012	0.006	1.191	1.089	0.	1.158

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	26	40	53	82	0	36
normalized size	1	1.	0.96	1.48	1.96	3.04	0.	1.33
time (sec)	N/A	0.028	0.034	0.055	1.29	1.18	0.	1.152

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	21	23	0	63	104	81
normalized size	1	1.	0.95	1.05	0.	2.86	4.73	3.68
time (sec)	N/A	0.005	0.011	0.003	0.	1.08	51.531	1.083

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	26	50	74	0	93
normalized size	1	1.	1.	0.96	1.85	2.74	0.	3.44
time (sec)	N/A	0.02	0.014	0.003	1.168	1.079	0.	1.151

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	26	50	74	0	93
normalized size	1	1.	1.	0.96	1.85	2.74	0.	3.44
time (sec)	N/A	0.025	0.016	0.004	1.215	1.062	0.	1.148

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	28	45	58	82	0	39
normalized size	1	1.	0.97	1.55	2.	2.83	0.	1.34
time (sec)	N/A	0.028	0.035	0.057	1.253	1.054	0.	1.189

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	17	24	0	53	46	50
normalized size	1	1.	0.89	1.26	0.	2.79	2.42	2.63
time (sec)	N/A	0.004	0.01	0.004	0.	1.005	0.577	1.112

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	97	31	47	63	85	59
normalized size	1	1.	4.04	1.29	1.96	2.62	3.54	2.46
time (sec)	N/A	0.014	0.074	0.005	1.156	1.084	20.152	1.146

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	97	31	47	63	0	59
normalized size	1	1.	4.04	1.29	1.96	2.62	0.	2.46
time (sec)	N/A	0.02	0.076	0.004	1.181	1.299	0.	1.125

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	111	155	54	72	0	35
normalized size	1	1.	4.27	5.96	2.08	2.77	0.	1.35
time (sec)	N/A	0.079	0.13	0.098	1.322	1.29	0.	1.145

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	158	0	0	0	0	0
normalized size	1	1.	0.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.289	0.308	0.031	0.	0.	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	374	374	2305	0	0	0	0	0
normalized size	1	1.	6.16	0.	0.	0.	0.	0.
time (sec)	N/A	1.38	4.304	0.039	0.	0.	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	816	816	13117	0	0	0	0	0
normalized size	1	1.	16.07	0.	0.	0.	0.	0.
time (sec)	N/A	4.552	7.357	0.055	0.	0.	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	36	46	109	126	46
normalized size	1	1.	1.	0.77	0.98	2.32	2.68	0.98
time (sec)	N/A	0.062	0.02	0.003	1.063	1.385	7.148	1.161

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	245	245	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.54	0.175	0.076	0.	0.	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	210	210	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.501	0.187	0.054	0.	0.	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	206	206	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.376	0.138	0.055	0.	0.	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	194	194	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.3	0.084	0.052	0.	0.	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	263	218	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.734	0.728	0.079	0.	0.	0.	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	212	212	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.489	0.142	0.053	0.	0.	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	210	210	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.476	0.128	0.053	0.	0.	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	498	498	391	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.609	1.167	0.054	0.	0.	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	323	323	273	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.371	0.633	0.049	0.	0.	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	158	158	181	0	0	0	0	0
normalized size	1	1.	1.15	0.	0.	0.	0.	0.
time (sec)	N/A	0.124	0.331	0.05	0.	0.	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.208	0.073	0.	0.	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	0.285	0.074	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [83] had the largest ratio of [0.4231]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.	22	0.045
2	A	2	1	1.	22	0.045
3	A	2	1	1.	22	0.045
4	A	2	1	1.	22	0.045
5	A	2	1	1.	20	0.05
6	A	8	8	1.	22	0.364
7	A	8	8	1.	22	0.364
8	A	8	8	1.	22	0.364
9	A	7	6	1.	25	0.24
10	A	6	6	1.	25	0.24
11	A	5	5	1.	25	0.2
12	A	7	6	1.	25	0.24
13	A	7	6	1.	25	0.24
14	A	14	8	1.	25	0.32
15	A	14	8	1.	25	0.32

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
16	A	13	7	1.	23	0.304
17	A	13	7	1.	22	0.318
18	A	14	8	1.	25	0.32
19	A	14	8	1.	25	0.32
20	A	7	6	1.	23	0.261
21	A	4	4	1.	23	0.174
22	A	5	5	1.	23	0.217
23	A	7	6	1.	23	0.261
24	A	5	4	1.	23	0.174
25	A	15	9	1.	23	0.391
26	A	15	9	1.	23	0.391
27	A	14	8	1.	23	0.348
28	A	13	7	1.	21	0.333
29	A	13	7	1.	20	0.35
30	A	14	8	1.	23	0.348
31	A	15	9	1.	23	0.391
32	A	5	5	1.	21	0.238
33	A	7	6	1.	21	0.286
34	A	8	7	1.	18	0.389
35	A	6	4	1.	24	0.167
36	A	5	4	1.	24	0.167
37	A	4	4	1.	24	0.167
38	A	3	3	1.	24	0.125
39	A	3	3	1.	24	0.125
40	A	3	3	1.	24	0.125
41	A	4	4	1.	24	0.167
42	A	5	4	1.	24	0.167
43	A	8	5	1.	25	0.2
44	A	5	5	1.	25	0.2
45	A	7	4	1.	25	0.16
46	A	4	3	1.	23	0.13
47	A	7	4	1.	22	0.182

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
48	A	7	6	1.	25	0.24
49	A	8	5	1.	25	0.2
50	A	5	4	1.	25	0.16
51	A	8	5	1.	25	0.2
52	A	20	7	1.	23	0.304
53	A	5	5	1.	23	0.217
54	A	21	7	1.	23	0.304
55	A	4	3	1.	21	0.143
56	A	19	6	1.	20	0.3
57	A	7	6	1.	23	0.261
58	A	20	7	1.	23	0.304
59	A	11	8	1.	23	0.348
60	A	21	9	1.	23	0.391
61	A	7	6	1.	25	0.24
62	A	7	6	1.	25	0.24
63	A	7	6	1.	23	0.261
64	A	7	6	1.	22	0.273
65	A	7	6	1.	25	0.24
66	A	7	7	1.	25	0.28
67	A	7	6	1.01	25	0.24
68	A	7	6	1.	25	0.24
69	A	7	6	1.	25	0.24
70	A	7	6	1.	25	0.24
71	A	7	6	1.	25	0.24
72	A	7	6	1.	23	0.261
73	A	7	6	1.	22	0.273
74	A	7	6	1.	25	0.24
75	A	8	7	1.	25	0.28
76	A	7	6	1.	25	0.24
77	A	7	6	1.	25	0.24
78	A	7	6	1.	25	0.24
79	A	11	7	1.	29	0.241

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
80	A	10	7	1.	29	0.241
81	A	8	7	1.	29	0.241
82	A	8	7	1.	27	0.259
83	A	16	11	1.	26	0.423
84	A	16	11	1.	29	0.379
85	A	24	12	1.	29	0.414
86	A	0	0	0.	0	0.
87	A	13	4	1.	26	0.154
88	A	10	4	1.	26	0.154
89	A	7	4	1.	24	0.167
90	A	6	3	1.	26	0.115
91	A	8	3	1.	26	0.115
92	A	10	3	1.	26	0.115
93	A	1	1	1.	19	0.053
94	A	2	2	1.	24	0.083
95	A	2	2	1.	26	0.077
96	A	2	2	1.	30	0.067
97	A	1	1	1.	21	0.048
98	A	2	2	1.	26	0.077
99	A	2	2	1.	28	0.071
100	A	2	2	1.	32	0.062
101	A	1	1	1.	18	0.056
102	A	3	3	1.	23	0.13
103	A	3	3	1.	25	0.12
104	A	3	3	1.	29	0.103
105	A	1	1	1.	19	0.053
106	A	2	2	1.	24	0.083
107	A	2	2	1.	26	0.077
108	A	2	2	1.	30	0.067
109	A	1	1	1.	19	0.053
110	A	2	2	1.	24	0.083
111	A	2	2	1.	26	0.077

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
112	A	2	2	1.	30	0.067
113	A	1	1	1.	21	0.048
114	A	2	2	1.	26	0.077
115	A	2	2	1.	28	0.071
116	A	2	2	1.	32	0.062
117	A	1	1	1.	21	0.048
118	A	2	2	1.	26	0.077
119	A	2	2	1.	28	0.071
120	A	2	2	1.	32	0.062
121	A	1	1	1.	18	0.056
122	A	4	3	0.94	23	0.13
123	A	4	3	0.94	25	0.12
124	A	4	3	1.	29	0.103
125	A	1	1	1.	18	0.056
126	A	3	3	1.	23	0.13
127	A	3	3	1.	25	0.12
128	A	3	3	1.	29	0.103
129	A	1	1	1.	19	0.053
130	A	2	2	1.	24	0.083
131	A	2	2	1.	26	0.077
132	A	2	2	1.	30	0.067
133	A	1	1	1.	21	0.048
134	A	2	2	1.	26	0.077
135	A	2	2	1.	28	0.071
136	A	2	2	1.	32	0.062
137	A	1	1	1.	18	0.056
138	A	1	1	1.	23	0.043
139	A	1	1	1.	25	0.04
140	A	2	2	1.	29	0.069
141	A	4	2	1.	29	0.069
142	A	5	3	1.	29	0.103
143	A	6	3	1.	29	0.103

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
144	A	3	3	1.	59	0.051
145	A	5	3	1.	31	0.097
146	A	5	3	1.	29	0.103
147	A	5	3	1.	27	0.111
148	A	5	3	1.	26	0.115
149	A	8	5	1.	29	0.172
150	A	5	3	1.	29	0.103
151	A	5	3	1.	29	0.103
152	A	10	4	1.	31	0.129
153	A	7	4	1.	29	0.138
154	A	2	2	1.	22	0.091
155	A	0	0	0.	0	0.
156	A	0	0	0.	0	0.

Chapter 3

Listing of integrals

3.1 $\int \left(d+ex^3\right)^5 \left(a+bx^3+cx^6\right) dx$

Optimal. Leaf size=163

$$\frac{1}{16}e^3x^{16}\left(e(ae+5bd)+10cd^2\right)+\frac{5}{13}de^2x^{13}\left(e(ae+2bd)+2cd^2\right)+\frac{1}{2}d^2ex^{10}\left(2e(ae+bd)+cd^2\right)+\frac{1}{7}d^3x^7\left(5e(2ae+bd)+\right.$$

```
[Out] a*d^5*x + (d^4*(b*d + 5*a*e)*x^4)/4 + (d^3*(c*d^2 + 5*e*(b*d + 2*a*e))*x^7)
/7 + (d^2*e*(c*d^2 + 2*e*(b*d + a*e))*x^10)/2 + (5*d*e^2*(2*c*d^2 + e*(2*b*
d + a*e))*x^13)/13 + (e^3*(10*c*d^2 + e*(5*b*d + a*e))*x^16)/16 + (e^4*(5*c
*d + b*e)*x^19)/19 + (c*e^5*x^22)/22
```

Rubi [A] time = 0.185171, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1407}

$$\frac{1}{16}e^3x^{16}\left(e(ae+5bd)+10cd^2\right)+\frac{5}{13}de^2x^{13}\left(e(ae+2bd)+2cd^2\right)+\frac{1}{2}d^2ex^{10}\left(2e(ae+bd)+cd^2\right)+\frac{1}{7}d^3x^7\left(5e(2ae+bd)+\right.$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^3)^5*(a + b*x^3 + c*x^6),x]
```


```
[Out] a*d^5*x + (d^4*(b*d + 5*a*e)*x^4)/4 + (d^3*(c*d^2 + 5*e*(b*d + 2*a*e))*x^7)
/7 + (d^2*e*(c*d^2 + 2*e*(b*d + a*e))*x^10)/2 + (5*d*e^2*(2*c*d^2 + e*(2*b*
d + a*e))*x^13)/13 + (e^3*(10*c*d^2 + e*(5*b*d + a*e))*x^16)/16 + (e^4*(5*c
```

```
*d + b*e)*x^19)/19 + (c*e^5*x^22)/22
```

Rule 1407

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2
_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))
, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \left(d+ex^3\right)^5 \left(a+bx^3+cx^6\right) dx = \int \left(ad^5+d^4(bd+5ae)x^3+d^3\left(cd^2+5e(bd+2ae)\right)x^6+5d^2e\left(cd^2+2e(bd+ae)\right)x^9+\right.$$
$$= ad^5x+\frac{1}{4}d^4(bd+5ae)x^4+\frac{1}{7}d^3\left(cd^2+5e(bd+2ae)\right)x^7+\frac{1}{2}d^2e\left(cd^2+2e(bd+ae)\right)x^{10}$$

Mathematica [A] time = 0.0481313, size = 164, normalized size = 1.01

$$\frac{1}{16}e^3x^{16}\left(ae^2+5bde+10cd^2\right)+\frac{5}{13}de^2x^{13}\left(ae^2+2bde+2cd^2\right)+\frac{1}{2}d^2ex^{10}\left(2ae^2+2bde+cd^2\right)+\frac{1}{7}d^3x^7\left(10ae^2+5bde+c\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^3)^5*(a + b*x^3 + c*x^6),x]
```

```
[Out] a*d^5*x + (d^4*(b*d + 5*a*e)*x^4)/4 + (d^3*(c*d^2 + 5*b*d*e + 10*a*e^2)*x^7
)/7 + (d^2*e*(c*d^2 + 2*b*d*e + 2*a*e^2)*x^10)/2 + (5*d*e^2*(2*c*d^2 + 2*b*
d*e + a*e^2)*x^13)/13 + (e^3*(10*c*d^2 + 5*b*d*e + a*e^2)*x^16)/16 + (e^4*(
5*c*d + b*e)*x^19)/19 + (c*e^5*x^22)/22
```

Maple [A] time = 0.001, size = 169, normalized size = 1.

$$\frac{ce^5x^{22}}{22}+\frac{\left(e^5b+5\,de^4c\right)x^{19}}{19}+\frac{\left(e^5a+5\,de^4b+10\,d^2e^3c\right)x^{16}}{16}+\frac{\left(5\,de^4a+10\,d^2e^3b+10\,d^3e^2c\right)x^{13}}{13}+\frac{\left(10\,d^2e^3a+10\,d^3e^2b+\right.}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^3+d)^5*(c*x^6+b*x^3+a),x)
```

```
[Out] 1/22*c*e^5*x^22+1/19*(b*e^5+5*c*d*e^4)*x^19+1/16*(a*e^5+5*b*d*e^4+10*c*d^2*
e^3)*x^16+1/13*(5*a*d*e^4+10*b*d^2*e^3+10*c*d^3*e^2)*x^13+1/10*(10*a*d^2*e^
3+10*b*d^3*e^2+5*c*d^4*e)*x^10+1/7*(10*a*d^3*e^2+5*b*d^4*e+c*d^5)*x^7+1/4*(
5*a*d^4*e+b*d^5)*x^4+a*d^5*x
```

Maxima [A] time = 1.00563, size = 224, normalized size = 1.37

$$\frac{1}{22}ce^5x^{22} + \frac{1}{19}\left(5\,cde^4 + be^5\right)x^{19} + \frac{1}{16}\left(10\,cd^2e^3 + 5\,bde^4 + ae^5\right)x^{16} + \frac{5}{13}\left(2\,cd^3e^2 + 2\,bd^2e^3 + ade^4\right)x^{13} + \frac{1}{2}\left(cd^4e + 2\,bd^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^5*(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/22*c*e^5*x^22 + 1/19*(5*c*d*e^4 + b*e^5)*x^19 + 1/16*(10*c*d^2*e^3 + 5*b*
d*e^4 + a*e^5)*x^16 + 5/13*(2*c*d^3*e^2 + 2*b*d^2*e^3 + a*d*e^4)*x^13 + 1/2
*(c*d^4*e + 2*b*d^3*e^2 + 2*a*d^2*e^3)*x^10 + 1/7*(c*d^5 + 5*b*d^4*e + 10*a
*d^3*e^2)*x^7 + a*d^5*x + 1/4*(b*d^5 + 5*a*d^4*e)*x^4
```

Fricas [A] time = 1.13949, size = 448, normalized size = 2.75

$$\frac{1}{22}x^{22}e^5c + \frac{5}{19}x^{19}e^4dc + \frac{1}{19}x^{19}e^5b + \frac{5}{8}x^{16}e^3d^2c + \frac{5}{16}x^{16}e^4db + \frac{1}{16}x^{16}e^5a + \frac{10}{13}x^{13}e^2d^3c + \frac{10}{13}x^{13}e^3d^2b + \frac{5}{13}x^{13}e^4da + \frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^5*(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/22*x^22*e^5*c + 5/19*x^19*e^4*d*c + 1/19*x^19*e^5*b + 5/8*x^16*e^3*d^2*c
+ 5/16*x^16*e^4*d*b + 1/16*x^16*e^5*a + 10/13*x^13*e^2*d^3*c + 10/13*x^13*e
^3*d^2*b + 5/13*x^13*e^4*d*a + 1/2*x^10*e*d^4*c + x^10*e^2*d^3*b + x^10*e^3
*d^2*a + 1/7*x^7*d^5*c + 5/7*x^7*e*d^4*b + 10/7*x^7*e^2*d^3*a + 1/4*x^4*d^5
*b + 5/4*x^4*e*d^4*a + x*d^5*a
```

Sympy [A] time = 0.095765, size = 187, normalized size = 1.15

$$ad^5x + \frac{ce^5x^{22}}{22} + x^{19}\left(\frac{be^5}{19} + \frac{5cde^4}{19}\right) + x^{16}\left(\frac{ae^5}{16} + \frac{5bde^4}{16} + \frac{5cd^2e^3}{8}\right) + x^{13}\left(\frac{5ade^4}{13} + \frac{10bd^2e^3}{13} + \frac{10cd^3e^2}{13}\right) + x^{10}\left(ad^2e^3 + b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**3+d)**5*(c*x**6+b*x**3+a),x)
```

```
[Out] a*d**5*x + c*e**5*x**22/22 + x**19*(b*e**5/19 + 5*c*d*e**4/19) + x**16*(a*e
**5/16 + 5*b*d*e**4/16 + 5*c*d**2*e**3/8) + x**13*(5*a*d*e**4/13 + 10*b*d**
2*e**3/13 + 10*c*d**3*e**2/13) + x**10*(a*d**2*e**3 + b*d**3*e**2 + c*d**4*
e/2) + x**7*(10*a*d**3*e**2/7 + 5*b*d**4*e/7 + c*d**5/7) + x**4*(5*a*d**4*e
/4 + b*d**5/4)
```

Giac [A] time = 1.13347, size = 234, normalized size = 1.44

$$\frac{1}{22}cx^{22}e^5 + \frac{5}{19}cdx^{19}e^4 + \frac{1}{19}bx^{19}e^5 + \frac{5}{8}cd^2x^{16}e^3 + \frac{5}{16}bdx^{16}e^4 + \frac{1}{16}ax^{16}e^5 + \frac{10}{13}cd^3x^{13}e^2 + \frac{10}{13}bd^2x^{13}e^3 + \frac{5}{13}adx^{13}e^4 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^5*(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/22*c*x^22*e^5 + 5/19*c*d*x^19*e^4 + 1/19*b*x^19*e^5 + 5/8*c*d^2*x^16*e^3
+ 5/16*b*d*x^16*e^4 + 1/16*a*x^16*e^5 + 10/13*c*d^3*x^13*e^2 + 10/13*b*d^2*
x^13*e^3 + 5/13*a*d*x^13*e^4 + 1/2*c*d^4*x^10*e + b*d^3*x^10*e^2 + a*d^2*x^
10*e^3 + 1/7*c*d^5*x^7 + 5/7*b*d^4*x^7*e + 10/7*a*d^3*x^7*e^2 + 1/4*b*d^5*x
^4 + 5/4*a*d^4*x^4*e + a*d^5*x
```

3.2 $\int \left(d+ex^3\right)^4 \left(a+bx^3+cx^6\right) dx$

Optimal. Leaf size=135

$$\frac{1}{13}e^2x^{13}\left(e(ae+4bd)+6cd^2\right)+\frac{1}{7}d^2x^7\left(6ae^2+4bde+cd^2\right)+\frac{1}{5}dex^{10}\left(e(2ae+3bd)+2cd^2\right)+\frac{1}{4}d^3x^4(4ae+bd)+ad^4x+$$

```
[Out] a*d^4*x + (d^3*(b*d + 4*a*e)*x^4)/4 + (d^2*(c*d^2 + 4*b*d*e + 6*a*e^2)*x^7)
/7 + (d*e*(2*c*d^2 + e*(3*b*d + 2*a*e))*x^10)/5 + (e^2*(6*c*d^2 + e*(4*b*d
+ a*e))*x^13)/13 + (e^3*(4*c*d + b*e)*x^16)/16 + (c*e^4*x^19)/19
```

Rubi [A] time = 0.125073, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1407}

$$\frac{1}{13}e^2x^{13}\left(e(ae+4bd)+6cd^2\right)+\frac{1}{7}d^2x^7\left(6ae^2+4bde+cd^2\right)+\frac{1}{5}dex^{10}\left(e(2ae+3bd)+2cd^2\right)+\frac{1}{4}d^3x^4(4ae+bd)+ad^4x+$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^3)^4*(a + b*x^3 + c*x^6),x]
```

```
[Out] a*d^4*x + (d^3*(b*d + 4*a*e)*x^4)/4 + (d^2*(c*d^2 + 4*b*d*e + 6*a*e^2)*x^7)
/7 + (d*e*(2*c*d^2 + e*(3*b*d + 2*a*e))*x^10)/5 + (e^2*(6*c*d^2 + e*(4*b*d
+ a*e))*x^13)/13 + (e^3*(4*c*d + b*e)*x^16)/16 + (c*e^4*x^19)/19
```

Rule 1407

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2
_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))
, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \left(d+ex^3\right)^4 \left(a+bx^3+cx^6\right) dx = \int \left(ad^4+d^3(bd+4ae)x^3+d^2\left(cd^2+4bde+6ae^2\right)x^6+2de\left(2cd^2+e(3bd+2ae)\right)x^9$$

$$= ad^4x+\frac{1}{4}d^3(bd+4ae)x^4+\frac{1}{7}d^2\left(cd^2+4bde+6ae^2\right)x^7+\frac{1}{5}de\left(2cd^2+e(3bd+2ae)\right)x$$

Mathematica [A] time = 0.0361796, size = 135, normalized size = 1.

$$\frac{1}{13}e^2x^{13}\left(ae^2+4bde+6cd^2\right)+\frac{1}{5}dex^{10}\left(2ae^2+3bde+2cd^2\right)+\frac{1}{7}d^2x^7\left(6ae^2+4bde+cd^2\right)+\frac{1}{4}d^3x^4(4ae+bd)+ad^4x+\frac{1}{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^3)^4*(a + b*x^3 + c*x^6),x]
```

```
[Out] a*d^4*x + (d^3*(b*d + 4*a*e)*x^4)/4 + (d^2*(c*d^2 + 4*b*d*e + 6*a*e^2)*x^7)
/7 + (d*e*(2*c*d^2 + 3*b*d*e + 2*a*e^2)*x^10)/5 + (e^2*(6*c*d^2 + 4*b*d*e +
 a*e^2)*x^13)/13 + (e^3*(4*c*d + b*e)*x^16)/16 + (c*e^4*x^19)/19
```

Maple [A] time = 0.001, size = 136, normalized size = 1.

$$\frac{ce^4x^{19}}{19}+\frac{\left(e^4b+4\,de^3c\right)x^{16}}{16}+\frac{\left(e^4a+4\,de^3b+6\,e^2d^2c\right)x^{13}}{13}+\frac{\left(4\,de^3a+6\,e^2d^2b+4\,d^3ec\right)x^{10}}{10}+\frac{\left(6\,e^2d^2a+4\,d^3eb+cd^4\right)x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^3+d)^4*(c*x^6+b*x^3+a),x)
```

```
[Out] 1/19*c*e^4*x^19+1/16*(b*e^4+4*c*d*e^3)*x^16+1/13*(a*e^4+4*b*d*e^3+6*c*d^2*e
^2)*x^13+1/10*(4*a*d*e^3+6*b*d^2*e^2+4*c*d^3*e)*x^10+1/7*(6*a*d^2*e^2+4*b*d
^3*e+c*d^4)*x^7+1/4*(4*a*d^3*e+b*d^4)*x^4+a*d^4*x
```

Maxima [A] time = 0.98309, size = 182, normalized size = 1.35

$$\frac{1}{19}\,ce^4x^{19}+\frac{1}{16}\left(4\,cde^3+be^4\right)x^{16}+\frac{1}{13}\left(6\,cd^2e^2+4\,bde^3+ae^4\right)x^{13}+\frac{1}{5}\left(2\,cd^3e+3\,bd^2e^2+2\,ade^3\right)x^{10}+\frac{1}{7}\left(cd^4+4\,bd^3e+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^4*(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/19*c*e^4*x^19 + 1/16*(4*c*d*e^3 + b*e^4)*x^16 + 1/13*(6*c*d^2*e^2 + 4*b*d
*e^3 + a*e^4)*x^13 + 1/5*(2*c*d^3*e + 3*b*d^2*e^2 + 2*a*d*e^3)*x^10 + 1/7*(
c*d^4 + 4*b*d^3*e + 6*a*d^2*e^2)*x^7 + a*d^4*x + 1/4*(b*d^4 + 4*a*d^3*e)*x^
4
```

Fricas [A] time = 1.16381, size = 360, normalized size = 2.67

$$\frac{1}{19}x^{19}e^4c + \frac{1}{4}x^{16}e^3dc + \frac{1}{16}x^{16}e^4b + \frac{6}{13}x^{13}e^2d^2c + \frac{4}{13}x^{13}e^3db + \frac{1}{13}x^{13}e^4a + \frac{2}{5}x^{10}ed^3c + \frac{3}{5}x^{10}e^2d^2b + \frac{2}{5}x^{10}e^3da + \frac{1}{7}x^7d^4c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^4*(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/19*x^19*e^4*c + 1/4*x^16*e^3*d*c + 1/16*x^16*e^4*b + 6/13*x^13*e^2*d^2*c
+ 4/13*x^13*e^3*d*b + 1/13*x^13*e^4*a + 2/5*x^10*e*d^3*c + 3/5*x^10*e^2*d^2
*b + 2/5*x^10*e^3*d*a + 1/7*x^7*d^4*c + 4/7*x^7*e*d^3*b + 6/7*x^7*e^2*d^2*a
 + 1/4*x^4*d^4*b + x^4*e*d^3*a + x*d^4*a
```

Sympy [A] time = 0.089108, size = 151, normalized size = 1.12

$$ad^4x + \frac{ce^4x^{19}}{19} + x^{16}\left(\frac{be^4}{16} + \frac{cde^3}{4}\right) + x^{13}\left(\frac{ae^4}{13} + \frac{4bde^3}{13} + \frac{6cd^2e^2}{13}\right) + x^{10}\left(\frac{2ade^3}{5} + \frac{3bd^2e^2}{5} + \frac{2cd^3e}{5}\right) + x^7\left(\frac{6ad^2e^2}{7} + \frac{4bd^3}{7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**3+d)**4*(c*x**6+b*x**3+a),x)
```

```
[Out] a*d**4*x + c*e**4*x**19/19 + x**16*(b*e**4/16 + c*d*e**3/4) + x**13*(a*e**4
/13 + 4*b*d*e**3/13 + 6*c*d**2*e**2/13) + x**10*(2*a*d*e**3/5 + 3*b*d**2*e*
*2/5 + 2*c*d**3*e/5) + x**7*(6*a*d**2*e**2/7 + 4*b*d**3*e/7 + c*d**4/7) + x
**4*(a*d**3*e + b*d**4/4)
```

Giac [A] time = 1.11388, size = 190, normalized size = 1.41

$$\frac{1}{19}cx^{19}e^4 + \frac{1}{4}cdx^{16}e^3 + \frac{1}{16}bx^{16}e^4 + \frac{6}{13}cd^2x^{13}e^2 + \frac{4}{13}bdx^{13}e^3 + \frac{1}{13}ax^{13}e^4 + \frac{2}{5}cd^3x^{10}e + \frac{3}{5}bd^2x^{10}e^2 + \frac{2}{5}adx^{10}e^3 + \frac{1}{7}c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^4*(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/19*c*x^19*e^4 + 1/4*c*d*x^16*e^3 + 1/16*b*x^16*e^4 + 6/13*c*d^2*x^13*e^2
+ 4/13*b*d*x^13*e^3 + 1/13*a*x^13*e^4 + 2/5*c*d^3*x^10*e + 3/5*b*d^2*x^10*e
```

```
^2 + 2/5*a*d*x^10*e^3 + 1/7*c*d^4*x^7 + 4/7*b*d^3*x^7*e + 6/7*a*d^2*x^7*e^2
 + 1/4*b*d^4*x^4 + a*d^3*x^4*e + a*d^4*x
```

3.3 $\int \left(d+ex^3\right)^3 \left(a+bx^3+cx^6\right) dx$

Optimal. Leaf size=103

$$\frac{1}{10}ex^{10}\left(e(ae+3bd)+3cd^2\right)+\frac{1}{7}dx^7\left(3e(ae+bd)+cd^2\right)+\frac{1}{4}d^2x^4(3ae+bd)+ad^3x+\frac{1}{13}e^2x^{13}(be+3cd)+\frac{1}{16}ce^3x^{16}$$

```
[Out] a*d^3*x + (d^2*(b*d + 3*a*e)*x^4)/4 + (d*(c*d^2 + 3*e*(b*d + a*e))*x^7)/7 +
 (e*(3*c*d^2 + e*(3*b*d + a*e))*x^10)/10 + (e^2*(3*c*d + b*e)*x^13)/13 + (c
*e^3*x^16)/16
```

Rubi [A] time = 0.0969754, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1407}

$$\frac{1}{10}ex^{10}\left(e(ae+3bd)+3cd^2\right)+\frac{1}{7}dx^7\left(3e(ae+bd)+cd^2\right)+\frac{1}{4}d^2x^4(3ae+bd)+ad^3x+\frac{1}{13}e^2x^{13}(be+3cd)+\frac{1}{16}ce^3x^{16}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^3)^3*(a + b*x^3 + c*x^6),x]
```

```
[Out] a*d^3*x + (d^2*(b*d + 3*a*e)*x^4)/4 + (d*(c*d^2 + 3*e*(b*d + a*e))*x^7)/7 +
 (e*(3*c*d^2 + e*(3*b*d + a*e))*x^10)/10 + (e^2*(3*c*d + b*e)*x^13)/13 + (c
*e^3*x^16)/16
```

Rule 1407

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2
_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))
, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \left(d+ex^3\right)^3 \left(a+bx^3+cx^6\right) dx = \int \left(ad^3+d^2(bd+3ae)x^3+d\left(cd^2+3e(bd+ae)\right)x^6+e\left(3cd^2+e(3bd+ae)\right)x^9+e^2(\right.$$
$$= ad^3x+\frac{1}{4}d^2(bd+3ae)x^4+\frac{1}{7}d\left(cd^2+3e(bd+ae)\right)x^7+\frac{1}{10}e\left(3cd^2+e(3bd+ae)\right)x^{10}$$

Mathematica [A] time = 0.0285729, size = 104, normalized size = 1.01

$$\frac{1}{10}ex^{10}\left(ae^2+3bde+3cd^2\right)+\frac{1}{7}dx^7\left(3ae^2+3bde+cd^2\right)+\frac{1}{4}d^2x^4(3ae+bd)+ad^3x+\frac{1}{13}e^2x^{13}(be+3cd)+\frac{1}{16}ce^3x^{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^3)^3*(a + b*x^3 + c*x^6),x]
```

```
[Out] a*d^3*x + (d^2*(b*d + 3*a*e)*x^4)/4 + (d*(c*d^2 + 3*b*d*e + 3*a*e^2)*x^7)/7
 + (e*(3*c*d^2 + 3*b*d*e + a*e^2)*x^10)/10 + (e^2*(3*c*d + b*e)*x^13)/13 +
(c*e^3*x^16)/16
```

Maple [A] time = 0.002, size = 103, normalized size = 1.

$$\frac{ce^3x^{16}}{16}+\frac{\left(e^3b+3\,cde^2\right)x^{13}}{13}+\frac{\left(e^3a+3\,de^2b+3\,d^2ec\right)x^{10}}{10}+\frac{\left(3\,ade^2+3\,bd^2e+cd^3\right)x^7}{7}+\frac{\left(3\,d^2ea+d^3b\right)x^4}{4}+ad^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^3+d)^3*(c*x^6+b*x^3+a),x)
```

```
[Out] 1/16*c*e^3*x^16+1/13*(b*e^3+3*c*d*e^2)*x^13+1/10*(a*e^3+3*b*d*e^2+3*c*d^2*e
)*x^10+1/7*(3*a*d*e^2+3*b*d^2*e+c*d^3)*x^7+1/4*(3*a*d^2*e+b*d^3)*x^4+a*d^3*
x
```

Maxima [A] time = 1.05264, size = 138, normalized size = 1.34

$$\frac{1}{16}\,ce^3x^{16}+\frac{1}{13}\left(3\,cde^2+be^3\right)x^{13}+\frac{1}{10}\left(3\,cd^2e+3\,bde^2+ae^3\right)x^{10}+\frac{1}{7}\left(cd^3+3\,bd^2e+3\,ade^2\right)x^7+ad^3x+\frac{1}{4}\left(bd^3+3\,ad^2e\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^3*(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/16*c*e^3*x^16 + 1/13*(3*c*d*e^2 + b*e^3)*x^13 + 1/10*(3*c*d^2*e + 3*b*d*e
^2 + a*e^3)*x^10 + 1/7*(c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*x^7 + a*d^3*x + 1/4*
(b*d^3 + 3*a*d^2*e)*x^4
```

Fricas [A] time = 1.15749, size = 282, normalized size = 2.74

$$\frac{1}{16}x^{16}e^3c + \frac{3}{13}x^{13}e^2dc + \frac{1}{13}x^{13}e^3b + \frac{3}{10}x^{10}ed^2c + \frac{3}{10}x^{10}e^2db + \frac{1}{10}x^{10}e^3a + \frac{1}{7}x^7d^3c + \frac{3}{7}x^7ed^2b + \frac{3}{7}x^7e^2da + \frac{1}{4}x^4d^3b + \frac{3}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^3*(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/16*x^16*e^3*c + 3/13*x^13*e^2*d*c + 1/13*x^13*e^3*b + 3/10*x^10*e*d^2*c +
 3/10*x^10*e^2*d*b + 1/10*x^10*e^3*a + 1/7*x^7*d^3*c + 3/7*x^7*e*d^2*b + 3/
7*x^7*e^2*d*a + 1/4*x^4*d^3*b + 3/4*x^4*e*d^2*a + x*d^3*a
```

Sympy [A] time = 0.082, size = 117, normalized size = 1.14

$$ad^3x + \frac{ce^3x^{16}}{16} + x^{13}\left(\frac{be^3}{13} + \frac{3cde^2}{13}\right) + x^{10}\left(\frac{ae^3}{10} + \frac{3bde^2}{10} + \frac{3cd^2e}{10}\right) + x^7\left(\frac{3ade^2}{7} + \frac{3bd^2e}{7} + \frac{cd^3}{7}\right) + x^4\left(\frac{3ad^2e}{4} + \frac{bd^3}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**3+d)**3*(c*x**6+b*x**3+a),x)
```

```
[Out] a*d**3*x + c*e**3*x**16/16 + x**13*(b*e**3/13 + 3*c*d*e**2/13) + x**10*(a*e
**3/10 + 3*b*d*e**2/10 + 3*c*d**2*e/10) + x**7*(3*a*d*e**2/7 + 3*b*d**2*e/7
 + c*d**3/7) + x**4*(3*a*d**2*e/4 + b*d**3/4)
```

Giac [A] time = 1.07675, size = 147, normalized size = 1.43

$$\frac{1}{16}cx^{16}e^3 + \frac{3}{13}cdx^{13}e^2 + \frac{1}{13}bx^{13}e^3 + \frac{3}{10}cd^2x^{10}e + \frac{3}{10}bdx^{10}e^2 + \frac{1}{10}ax^{10}e^3 + \frac{1}{7}cd^3x^7 + \frac{3}{7}bd^2x^7e + \frac{3}{7}adx^7e^2 + \frac{1}{4}bd^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^3*(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/16*c*x^16*e^3 + 3/13*c*d*x^13*e^2 + 1/13*b*x^13*e^3 + 3/10*c*d^2*x^10*e +
 3/10*b*d*x^10*e^2 + 1/10*a*x^10*e^3 + 1/7*c*d^3*x^7 + 3/7*b*d^2*x^7*e + 3/
7*a*d*x^7*e^2 + 1/4*b*d^3*x^4 + 3/4*a*d^2*x^4*e + a*d^3*x
```

3.4 $\int \left(d+e x^3\right)^2 \left(a+b x^3+c x^6\right) \, dx$

Optimal. Leaf size=73

$$\frac{1}{7} x^7 \left(e(a e+2 b d)+c d^2\right)+\frac{1}{4} d x^4 (2 a e+b d)+a d^2 x+\frac{1}{10} e x^{10} (b e+2 c d)+\frac{1}{13} c e^2 x^{13}$$

```
[Out] a*d^2*x + (d*(b*d + 2*a*e)*x^4)/4 + ((c*d^2 + e*(2*b*d + a*e))*x^7)/7 + (e*
(2*c*d + b*e)*x^10)/10 + (c*e^2*x^13)/13
```

Rubi [A] time = 0.0622015, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1407}

$$\frac{1}{7} x^7 \left(e(a e+2 b d)+c d^2\right)+\frac{1}{4} d x^4 (2 a e+b d)+a d^2 x+\frac{1}{10} e x^{10} (b e+2 c d)+\frac{1}{13} c e^2 x^{13}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^3)^2*(a + b*x^3 + c*x^6),x]
```

```
[Out] a*d^2*x + (d*(b*d + 2*a*e)*x^4)/4 + ((c*d^2 + e*(2*b*d + a*e))*x^7)/7 + (e*
(2*c*d + b*e)*x^10)/10 + (c*e^2*x^13)/13
```

Rule 1407

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2
_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))
, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(d+e x^3\right)^2 \left(a+b x^3+c x^6\right) \, dx &= \int \left(a d^2+d(b d+2 a e) x^3+\left(c d^2+e(2 b d+a e)\right) x^6+e(2 c d+b e) x^9+c e^2 x^{12}\right) \, dx\\ &= a d^2 x+\frac{1}{4} d(b d+2 a e) x^4+\frac{1}{7} \left(c d^2+e(2 b d+a e)\right) x^7+\frac{1}{10} e(2 c d+b e) x^{10}+\frac{1}{13} c e^2 x^{13}\end{aligned}$$

Mathematica [A] time = 0.0218051, size = 73, normalized size = 1.

$$\frac{1}{7} x^7 \left(a e^2+2 b d e+c d^2\right)+\frac{1}{4} d x^4 (2 a e+b d)+a d^2 x+\frac{1}{10} e x^{10} (b e+2 c d)+\frac{1}{13} c e^2 x^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^3)^2*(a + b*x^3 + c*x^6),x]
```

```
[Out] a*d^2*x + (d*(b*d + 2*a*e)*x^4)/4 + ((c*d^2 + 2*b*d*e + a*e^2)*x^7)/7 + (e*
(2*c*d + b*e)*x^10)/10 + (c*e^2*x^13)/13
```

Maple [A] time = 0., size = 70, normalized size = 1.

$$\frac{ce^2x^{13}}{13}+\frac{\left(be^2+2\,dec\right)x^{10}}{10}+\frac{\left(ae^2+2\,bde+cd^2\right)x^7}{7}+\frac{\left(2\,dea+bd^2\right)x^4}{4}+ad^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^3+d)^2*(c*x^6+b*x^3+a),x)
```

```
[Out] 1/13*c*e^2*x^13+1/10*(b*e^2+2*c*d*e)*x^10+1/7*(a*e^2+2*b*d*e+c*d^2)*x^7+1/4
*(2*a*d*e+b*d^2)*x^4+a*d^2*x
```

Maxima [A] time = 0.962235, size = 93, normalized size = 1.27

$$\frac{1}{13}\,ce^2x^{13}+\frac{1}{10}\,\left(2\,cde+be^2\right)x^{10}+\frac{1}{7}\,\left(cd^2+2\,bde+ae^2\right)x^7+\frac{1}{4}\,\left(bd^2+2\,ade\right)x^4+ad^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^2*(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/13*c*e^2*x^13 + 1/10*(2*c*d*e + b*e^2)*x^10 + 1/7*(c*d^2 + 2*b*d*e + a*e^
2)*x^7 + 1/4*(b*d^2 + 2*a*d*e)*x^4 + a*d^2*x
```

Fricas [A] time = 1.09655, size = 192, normalized size = 2.63

$$\frac{1}{13}x^{13}e^2c+\frac{1}{5}x^{10}edc+\frac{1}{10}x^{10}e^2b+\frac{1}{7}x^7d^2c+\frac{2}{7}x^7edb+\frac{1}{7}x^7e^2a+\frac{1}{4}x^4d^2b+\frac{1}{2}x^4eda+xd^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^2*(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/13*x^13*e^2*c + 1/5*x^10*e*d*c + 1/10*x^10*e^2*b + 1/7*x^7*d^2*c + 2/7*x^
7*e*d*b + 1/7*x^7*e^2*a + 1/4*x^4*d^2*b + 1/2*x^4*e*d*a + x*d^2*a
```

Sympy [A] time = 0.074794, size = 75, normalized size = 1.03

$$ad^2x + \frac{ce^2x^{13}}{13} + x^{10}\left(\frac{be^2}{10} + \frac{cde}{5}\right) + x^7\left(\frac{ae^2}{7} + \frac{2bde}{7} + \frac{cd^2}{7}\right) + x^4\left(\frac{ade}{2} + \frac{bd^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**3+d)**2*(c*x**6+b*x**3+a),x)
```

```
[Out] a*d**2*x + c*e**2*x**13/13 + x**10*(b*e**2/10 + c*d*e/5) + x**7*(a*e**2/7 +
 2*b*d*e/7 + c*d**2/7) + x**4*(a*d*e/2 + b*d**2/4)
```

Giac [A] time = 1.16215, size = 103, normalized size = 1.41

$$\frac{1}{13}\,cx^{13}e^2 + \frac{1}{5}\,cdx^{10}e + \frac{1}{10}\,bx^{10}e^2 + \frac{1}{7}\,cd^2x^7 + \frac{2}{7}\,bdx^7e + \frac{1}{7}\,ax^7e^2 + \frac{1}{4}\,bd^2x^4 + \frac{1}{2}\,adx^4e + ad^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^2*(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/13*c*x^13*e^2 + 1/5*c*d*x^10*e + 1/10*b*x^10*e^2 + 1/7*c*d^2*x^7 + 2/7*b*
d*x^7*e + 1/7*a*x^7*e^2 + 1/4*b*d^2*x^4 + 1/2*a*d*x^4*e + a*d^2*x
```

3.5 $\int \left(d+ex^3\right) \left(a+bx^3+cx^6\right) \, dx$

Optimal. Leaf size=42

$$\frac{1}{4}x^4(ae+bd)+adx+\frac{1}{7}x^7(be+cd)+\frac{1}{10}cex^{10}$$

```
[Out] a*d*x + ((b*d + a*e)*x^4)/4 + ((c*d + b*e)*x^7)/7 + (c*e*x^10)/10
```

Rubi [A] time = 0.0278242, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {1407}

$$\frac{1}{4}x^4(ae+bd)+adx+\frac{1}{7}x^7(be+cd)+\frac{1}{10}cex^{10}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^3)*(a + b*x^3 + c*x^6),x]
```

```
[Out] a*d*x + ((b*d + a*e)*x^4)/4 + ((c*d + b*e)*x^7)/7 + (c*e*x^10)/10
```

Rule 1407

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2
_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))
, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(d+ex^3\right)\left(a+bx^3+cx^6\right)\,dx &= \int \left(ad+(bd+ae)x^3+(cd+be)x^6+cex^9\right)\,dx\\ &= adx+\frac{1}{4}(bd+ae)x^4+\frac{1}{7}(cd+be)x^7+\frac{1}{10}cex^{10}\end{aligned}$$

Mathematica [A] time = 0.0085175, size = 42, normalized size = 1.

$$\frac{1}{4}x^4(ae+bd)+adx+\frac{1}{7}x^7(be+cd)+\frac{1}{10}cex^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^3)*(a + b*x^3 + c*x^6),x]
```

```
[Out] a*d*x + ((b*d + a*e)*x^4)/4 + ((c*d + b*e)*x^7)/7 + (c*e*x^10)/10
```

Maple [A] time = 0., size = 37, normalized size = 0.9

$$adx + \frac{(ae + bd)\,x^4}{4} + \frac{(be + cd)\,x^7}{7} + \frac{cex^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^3+d)*(c*x^6+b*x^3+a),x)
```

```
[Out] a*d*x+1/4*(a*e+b*d)*x^4+1/7*(b*e+c*d)*x^7+1/10*c*e*x^10
```

Maxima [A] time = 1.05804, size = 49, normalized size = 1.17

$$\frac{1}{10}\,cex^{10} + \frac{1}{7}\,(cd + be)x^7 + \frac{1}{4}\,(bd + ae)x^4 + adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)*(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/10*c*e*x^10 + 1/7*(c*d + b*e)*x^7 + 1/4*(b*d + a*e)*x^4 + a*d*x
```

Fricas [A] time = 1.12813, size = 107, normalized size = 2.55

$$\frac{1}{10}x^{10}ec + \frac{1}{7}x^7dc + \frac{1}{7}x^7eb + \frac{1}{4}x^4db + \frac{1}{4}x^4ea + xda$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)*(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/10*x^10*e*c + 1/7*x^7*d*c + 1/7*x^7*e*b + 1/4*x^4*d*b + 1/4*x^4*e*a + x*d
*a
```

Sympy [A] time = 0.062923, size = 39, normalized size = 0.93

$$adx + \frac{cex^{10}}{10} + x^7\left(\frac{be}{7} + \frac{cd}{7}\right) + x^4\left(\frac{ae}{4} + \frac{bd}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**3+d)*(c*x**6+b*x**3+a),x)
```

```
[Out] a*d*x + c*e*x**10/10 + x**7*(b*e/7 + c*d/7) + x**4*(a*e/4 + b*d/4)
```

Giac [A] time = 1.09884, size = 58, normalized size = 1.38

$$\frac{1}{10}\,cx^{10}e + \frac{1}{7}\,cdx^7 + \frac{1}{7}\,bx^7e + \frac{1}{4}\,bdx^4 + \frac{1}{4}\,ax^4e + adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)*(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/10*c*x^10*e + 1/7*c*d*x^7 + 1/7*b*x^7*e + 1/4*b*d*x^4 + 1/4*a*x^4*e + a*d
*x
```

3.6 $\int \frac{a+bx^3+cx^6}{d+ex^3}\,dx$

Optimal. Leaf size=188

$$-\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(ae^2-bde+cd^2\right)}{6d^{2/3}e^{7/3}}+\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(ae^2-bde+cd^2\right)}{3d^{2/3}e^{7/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)\left(ae^2-bde+cd^2\right)}{\sqrt{3}d^{2/3}e^{7/3}}$$

```
[Out] -(((c*d - b*e)*x)/e^2) + (c*x^4)/(4*e) - ((c*d^2 - b*d*e + a*e^2)*ArcTan[(d
^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(Sqrt[3]*d^(2/3)*e^(7/3)) + ((c*d
^2 - b*d*e + a*e^2)*Log[d^(1/3) + e^(1/3)*x])/(3*d^(2/3)*e^(7/3)) - ((c*d^2
 - b*d*e + a*e^2)*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/(6*d^(2/3
)*e^(7/3))
```

Rubi [A] time = 0.211115, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1411, 388, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(ae^2-bde+cd^2\right)}{6d^{2/3}e^{7/3}}+\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(ae^2-bde+cd^2\right)}{3d^{2/3}e^{7/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)\left(ae^2-bde+cd^2\right)}{\sqrt{3}d^{2/3}e^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)/(d + e*x^3),x]
```



```
[Out] -(((c*d - b*e)*x)/e^2) + (c*x^4)/(4*e) - ((c*d^2 - b*d*e + a*e^2)*ArcTan[(d
^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(Sqrt[3]*d^(2/3)*e^(7/3)) + ((c*d
^2 - b*d*e + a*e^2)*Log[d^(1/3) + e^(1/3)*x])/(3*d^(2/3)*e^(7/3)) - ((c*d^2
 - b*d*e + a*e^2)*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/(6*d^(2/3
)*e^(7/3))
```


Rule 1411

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[(c*x^(n + 1)*(d + e*x^n)^(q + 1))/(e*(n*(q + 2) + 1))
, x] + Dist[1/(e*(n*(q + 2) + 1)), Int[(d + e*x^n)^q*(a*e*(n*(q + 2) + 1) -
 (c*d*(n + 1) - b*e*(n*(q + 2) + 1))*x^n), x], x] /; FreeQ[{a, b, c, d, e,
n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e
^2, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{a+bx^3+cx^6}{d+ex^3}\,dx = \frac{cx^4}{4e} + \frac{\int \frac{4ae-(4cd-4be)x^3}{d+ex^3}\,dx}{4e}$$

$$= -\frac{(cd-be)x}{e^2} + \frac{cx^4}{4e} - \left(-a - \frac{d(cd-be)}{e^2}\right)\int \frac{1}{d+ex^3}\,dx$$

$$= -\frac{(cd-be)x}{e^2} + \frac{cx^4}{4e} + \frac{\left(a+\frac{d(cd-be)}{e^2}\right)\int \frac{1}{\sqrt[3]{d}+\sqrt[3]{e}x}\,dx}{3d^{2/3}} + \frac{\left(a+\frac{d(cd-be)}{e^2}\right)\int \frac{2\sqrt[3]{d}-\sqrt[3]{e}x}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}\,dx}{3d^{2/3}}$$

$$= -\frac{(cd-be)x}{e^2} + \frac{cx^4}{4e} + \frac{\left(cd^2-bde+ae^2\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}e^{7/3}} - \frac{\left(cd^2-bde+ae^2\right)\int \frac{-\sqrt[3]{d}\sqrt[3]{e}+2e^{2/3}x}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}\,dx}{6d^{2/3}e^{7/3}}$$

$$= -\frac{(cd-be)x}{e^2} + \frac{cx^4}{4e} + \frac{\left(cd^2-bde+ae^2\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}e^{7/3}} - \frac{\left(cd^2-bde+ae^2\right)\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e\right.}{6d^{2/3}e^{7/3}}$$

$$= -\frac{(cd-be)x}{e^2} + \frac{cx^4}{4e} - \frac{\left(cd^2-bde+ae^2\right)\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}d^{2/3}e^{7/3}} + \frac{\left(cd^2-bde+ae^2\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{3d^{2/3}e^{7/3}} - \frac{(c}{}$$

Mathematica [A] time = 0.15456, size = 176, normalized size = 0.94

$$\frac{-\frac{2\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(e(ae-bd)+cd^2\right)}{d^{2/3}} + \frac{4\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(e(ae-bd)+cd^2\right)}{d^{2/3}} - \frac{4\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)\left(e(ae-bd)+cd^2\right)}{d^{2/3}} + 12\sqrt[3]{e}x(be-cd)+3ce^{4/3}x^4}{12e^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)/(d + e*x^3),x]
```

```
[Out] (12*e^(1/3)*(-(c*d) + b*e)*x + 3*c*e^(4/3)*x^4 - (4*Sqrt[3]*(c*d^2 + e*(-(b
*d) + a*e))*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3))/Sqrt[3]])/d^(2/3) + (4*(c*d^
2 + e*(-(b*d) + a*e))*Log[d^(1/3) + e^(1/3)*x])/d^(2/3) - (2*(c*d^2 + e*(-(
b*d) + a*e))*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/d^(2/3))/(12*e
^(7/3))
```

Maple [B] time = 0.004, size = 313, normalized size = 1.7

$$\frac{cx^4}{4e}+\frac{bx}{e}-\frac{cdx}{e^2}+\frac{a}{3e}\ln\left(x+\sqrt[3]{\frac{d}{e}}\right)\left(\frac{d}{e}\right)^{-\frac{2}{3}}-\frac{bd}{3e^2}\ln\left(x+\sqrt[3]{\frac{d}{e}}\right)\left(\frac{d}{e}\right)^{-\frac{2}{3}}+\frac{cd^2}{3e^3}\ln\left(x+\sqrt[3]{\frac{d}{e}}\right)\left(\frac{d}{e}\right)^{-\frac{2}{3}}-\frac{a}{6e}\ln\left(x^2-\sqrt[3]{\frac{d}{e}}x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)/(e*x^3+d),x)
```

```
[Out] 1/4*c*x^4/e+1/e*b*x-1/e^2*c*d*x+1/3/e/(d/e)^(2/3)*ln(x+(d/e)^(1/3))*a-1/3/e
^2/(d/e)^(2/3)*ln(x+(d/e)^(1/3))*b*d+1/3/e^3/(d/e)^(2/3)*ln(x+(d/e)^(1/3))*
c*d^2-1/6/e/(d/e)^(2/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))*a+1/6/e^2/(d/e)^(
2/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))*b*d-1/6/e^3/(d/e)^(2/3)*ln(x^2-(d/e)
^(1/3)*x+(d/e)^(2/3))*c*d^2+1/3/e/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2
/(d/e)^(1/3)*x-1))*a-1/3/e^2/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e
)^(1/3)*x-1))*b*d+1/3/e^3/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(
1/3)*x-1))*c*d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.43311, size = 1096, normalized size = 5.83

$$\left[\frac{3\,cd^2e^2x^4+6\sqrt{\frac{1}{3}}\left(cd^3e-bd^2e^2+ade^3\right)\sqrt{-\frac{\left(d^2e\right)^{\frac{1}{3}}}{e}}\log\left(\frac{2\,dex^3-3\left(d^2e\right)^{\frac{1}{3}}dx-d^2+3\sqrt{\frac{1}{3}}\left(2\,dex^2+\left(d^2e\right)^{\frac{2}{3}}x-\left(d^2e\right)^{\frac{1}{3}}d\right)\sqrt{-\frac{\left(d^2e\right)^{\frac{1}{3}}}{e}}}{ex^3+d}\right)-2\left(cd^2-b\right.}{12\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d),x, algorithm="fricas")
```

```
[Out] [1/12*(3*c*d^2*e^2*x^4 + 6*sqrt(1/3)*(c*d^3*e - b*d^2*e^2 + a*d*e^3)*sqrt(-
(d^2*e)^(1/3)/e)*log((2*d*e*x^3 - 3*(d^2*e)^(1/3)*d*x - d^2 + 3*sqrt(1/3)*(
2*d*e*x^2 + (d^2*e)^(2/3)*x - (d^2*e)^(1/3)*d)*sqrt(-(d^2*e)^(1/3)/e))/(e*x
^3 + d)) - 2*(c*d^2 - b*d*e + a*e^2)*(d^2*e)^(2/3)*log(d*e*x^2 - (d^2*e)^(2
/3)*x + (d^2*e)^(1/3)*d) + 4*(c*d^2 - b*d*e + a*e^2)*(d^2*e)^(2/3)*log(d*e*
x + (d^2*e)^(2/3)) - 12*(c*d^3*e - b*d^2*e^2)*x)/(d^2*e^3), 1/12*(3*c*d^2*e
^2*x^4 + 12*sqrt(1/3)*(c*d^3*e - b*d^2*e^2 + a*d*e^3)*sqrt((d^2*e)^(1/3)/e)
*arctan(sqrt(1/3)*(2*(d^2*e)^(2/3)*x - (d^2*e)^(1/3)*d)*sqrt((d^2*e)^(1/3)/
e)/d^2) - 2*(c*d^2 - b*d*e + a*e^2)*(d^2*e)^(2/3)*log(d*e*x^2 - (d^2*e)^(2/
3)*x + (d^2*e)^(1/3)*d) + 4*(c*d^2 - b*d*e + a*e^2)*(d^2*e)^(2/3)*log(d*e*x
 + (d^2*e)^(2/3)) - 12*(c*d^3*e - b*d^2*e^2)*x)/(d^2*e^3)]
```

Sympy [A] time = 1.07036, size = 175, normalized size = 0.93

$$\frac{cx^4}{4e} + \operatorname{RootSum}\left(27t^3d^2e^7 - a^3e^6 + 3a^2bde^5 - 3a^2cd^2e^4 - 3ab^2d^2e^4 + 6abcd^3e^3 - 3ac^2d^4e^2 + b^3d^3e^3 - 3b^2cd^4e^2 + 3bc^2d^5e\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)/(e*x**3+d),x)
```

```
[Out] c*x**4/(4*e) + RootSum(27*_t**3*d**2*e**7 - a**3*e**6 + 3*a**2*b*d*e**5 - 3
*a**2*c*d**2*e**4 - 3*a*b**2*d**2*e**4 + 6*a*b*c*d**3*e**3 - 3*a*c**2*d**4*
e**2 + b**3*d**3*e**3 - 3*b**2*c*d**4*e**2 + 3*b*c**2*d**5*e - c**3*d**6, L
ambda(_t, _t*log(3*_t*d*e**2/(a*e**2 - b*d*e + c*d**2) + x))) + x*(b*e - c*
d)/e**2
```

Giac [A] time = 1.10266, size = 279, normalized size = 1.48

$$\frac{\sqrt{3}\left(\left(-de^2\right)^{\frac{1}{3}}cd^2 - \left(-de^2\right)^{\frac{1}{3}}bde + \left(-de^2\right)^{\frac{1}{3}}ae^2\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-de^{(-1)}\right)^{\frac{1}{3}}\right)}{3\left(-de^{(-1)}\right)^{\frac{1}{3}}}\right)e^{(-3)}}{3d} - \frac{\left(cd^2e^2 - bde^3 + ae^4\right)\left(-de^{(-1)}\right)^{\frac{1}{3}}e^{(-4)}\log\left(\right.}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*((-d*e^2)^(1/3)*c*d^2 - (-d*e^2)^(1/3)*b*d*e + (-d*e^2)^(1/3)*a
*e^2)*arctan(1/3*sqrt(3)*(2*x + (-d*e^(-1))^(1/3))/(-d*e^(-1))^(1/3))*e^(-3
)/d - 1/3*(c*d^2*e^2 - b*d*e^3 + a*e^4)*(-d*e^(-1))^(1/3)*e^(-4)*log(abs(x
- (-d*e^(-1))^(1/3)))/d + 1/4*(c*x^4*e^3 - 4*c*d*x*e^2 + 4*b*x*e^3)*e^(-4)
+ 1/6*((-d*e^2)^(1/3)*c*d^2 - (-d*e^2)^(1/3)*b*d*e + (-d*e^2)^(1/3)*a*e^2)*
e^(-3)*log(x^2 + (-d*e^(-1))^(1/3)*x + (-d*e^(-1))^(2/3))/d
```

3.7 $\int \frac{a+bx^3+cx^6}{\left(d+ex^3\right)^2} dx$

Optimal. Leaf size=213

$$\frac{x\left(ae^2-bde+cd^2\right)}{3de^2\left(d+ex^3\right)}+\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(4cd^2-e(2ae+bd)\right)}{18d^{5/3}e^{7/3}}-\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(4cd^2-e(2ae+bd)\right)}{9d^{5/3}e^{7/3}}+\frac{\tan^{-1}\left(\right.}{}$$

```
[Out] (c*x)/e^2 + ((c*d^2 - b*d*e + a*e^2)*x)/(3*d*e^2*(d + e*x^3)) + ((4*c*d^2 -
 e*(b*d + 2*a*e))*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(3*Sqr
t[3]*d^(5/3)*e^(7/3)) - ((4*c*d^2 - e*(b*d + 2*a*e))*Log[d^(1/3) + e^(1/3)*
x])/(9*d^(5/3)*e^(7/3)) + ((4*c*d^2 - e*(b*d + 2*a*e))*Log[d^(2/3) - d^(1/3
)*e^(1/3)*x + e^(2/3)*x^2])/(18*d^(5/3)*e^(7/3))
```

Rubi [A] time = 0.225653, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1409, 388, 200, 31, 634, 617, 204, 628}

$$\frac{x\left(ae^2-bde+cd^2\right)}{3de^2\left(d+ex^3\right)}+\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(4cd^2-e(2ae+bd)\right)}{18d^{5/3}e^{7/3}}-\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(4cd^2-e(2ae+bd)\right)}{9d^{5/3}e^{7/3}}+\frac{\tan^{-1}\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)/(d + e*x^3)^2,x]
```

```
[Out] (c*x)/e^2 + ((c*d^2 - b*d*e + a*e^2)*x)/(3*d*e^2*(d + e*x^3)) + ((4*c*d^2 -
 e*(b*d + 2*a*e))*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(3*Sqr
t[3]*d^(5/3)*e^(7/3)) - ((4*c*d^2 - e*(b*d + 2*a*e))*Log[d^(1/3) + e^(1/3)*
x])/(9*d^(5/3)*e^(7/3)) + ((4*c*d^2 - e*(b*d + 2*a*e))*Log[d^(2/3) - d^(1/3
)*e^(1/3)*x + e^(2/3)*x^2])/(18*d^(5/3)*e^(7/3))
```

Rule 1409

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> -Simp[((c*d^2 - b*d*e + a*e^2)*x*(d + e*x^n)^(q + 1))/(d*e
^2*n*(q + 1)), x] + Dist[1/(n*(q + 1)*d*e^2), Int[(d + e*x^n)^(q + 1)*Simp[
c*d^2 - b*d*e + a*e^2*(n*(q + 1) + 1) + c*d*e*n*(q + 1)*x^n, x], x], x] /;
FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[
```

```
c*d^2 - b*d*e + a*e^2, 0] && LtQ[q, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{a+bx^3+cx^6}{\left(d+ex^3\right)^2}\,dx = \frac{\left(cd^2-bde+ae^2\right)x}{3de^2\left(d+ex^3\right)} - \frac{\int \frac{cd^2-e(bd+2ae)-3cdex^3}{d+ex^3}\,dx}{3de^2}$$

$$= \frac{cx}{e^2} + \frac{\left(cd^2-bde+ae^2\right)x}{3de^2\left(d+ex^3\right)} - \frac{\left(4cd^2-e(bd+2ae)\right)\int \frac{1}{d+ex^3}\,dx}{3de^2}$$

$$= \frac{cx}{e^2} + \frac{\left(cd^2-bde+ae^2\right)x}{3de^2\left(d+ex^3\right)} - \frac{\left(4cd^2-e(bd+2ae)\right)\int \frac{1}{\sqrt[3]{d}+\sqrt[3]{e}x}\,dx}{9d^{5/3}e^2} - \frac{\left(4cd^2-e(bd+2ae)\right)\int \frac{2\sqrt[3]{d}-\sqrt[3]{e}}{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+}}{9d^{5/3}e^2}$$

$$= \frac{cx}{e^2} + \frac{\left(cd^2-bde+ae^2\right)x}{3de^2\left(d+ex^3\right)} - \frac{\left(4cd^2-e(bd+2ae)\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{9d^{5/3}e^{7/3}} + \frac{\left(4cd^2-e(bd+2ae)\right)\int \frac{-\sqrt[3]{d}\sqrt[3]{}}{d^{2/3}-\sqrt[3]{d}}}{18d^{5/3}e^{7/3}}$$

$$= \frac{cx}{e^2} + \frac{\left(cd^2-bde+ae^2\right)x}{3de^2\left(d+ex^3\right)} - \frac{\left(4cd^2-e(bd+2ae)\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{9d^{5/3}e^{7/3}} + \frac{\left(4cd^2-e(bd+2ae)\right)\log\left(d^{2/3}\right.}{18d^{5/3}e^{7/3}}$$

$$= \frac{cx}{e^2} + \frac{\left(cd^2-bde+ae^2\right)x}{3de^2\left(d+ex^3\right)} + \frac{\left(4cd^2-e(bd+2ae)\right)\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{3\sqrt{3}d^{5/3}e^{7/3}} - \frac{\left(4cd^2-e(bd+2ae)\right)\log\left(\sqrt[3]{d}\right.}{9d^{5/3}e^{7/3}}$$

Mathematica [A] time = 0.19949, size = 199, normalized size = 0.93

$$\frac{\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(4cd^2-e(2ae+bd)\right)}{d^{5/3}} + \frac{6\sqrt[3]{e}x\left(e(ae-bd)+cd^2\right)}{d\left(d+ex^3\right)} - \frac{2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(4cd^2-e(2ae+bd)\right)}{d^{5/3}} + \frac{2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{e}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)\left(4cd^2-e(2ae+bd)\right)}{d^{5/3}} + 18c}{18e^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)/(d + e*x^3)^2,x]
```

```
[Out] (18*c*e^(1/3)*x + (6*e^(1/3)*(c*d^2 + e*(-(b*d) + a*e))*x)/(d*(d + e*x^3))
+ (2*Sqrt[3]*(4*c*d^2 - e*(b*d + 2*a*e))*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3))
/Sqrt[3]])/d^(5/3) - (2*(4*c*d^2 - e*(b*d + 2*a*e))*Log[d^(1/3) + e^(1/3)*x
])/d^(5/3) + ((4*c*d^2 - e*(b*d + 2*a*e))*Log[d^(2/3) - d^(1/3)*e^(1/3)*x +
```

```
e^(2/3)*x^2])/d^(5/3))/(18*e^(7/3))
```

Maple [A] time = 0.008, size = 345, normalized size = 1.6

$$\frac{cx}{e^2}+\frac{ax}{3d\left(ex^3+d\right)}-\frac{bx}{3e\left(ex^3+d\right)}+\frac{dxc}{3e^2\left(ex^3+d\right)}+\frac{2a}{9de}\ln\left(x+\sqrt[3]{\frac{d}{e}}\right)\left(\frac{d}{e}\right)^{-\frac{2}{3}}+\frac{b}{9e^2}\ln\left(x+\sqrt[3]{\frac{d}{e}}\right)\left(\frac{d}{e}\right)^{-\frac{2}{3}}-\frac{4cd}{9e^3}\ln\left(x-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)/(e*x^3+d)^2,x)
```

```
[Out] c*x/e^2+1/3/d*x/(e*x^3+d)*a-1/3/e*x/(e*x^3+d)*b+1/3/e^2*d*x/(e*x^3+d)*c+2/9
/e/d/(d/e)^(2/3)*ln(x+(d/e)^(1/3))*a+1/9/e^2/(d/e)^(2/3)*ln(x+(d/e)^(1/3))*
b-4/9/e^3*d/(d/e)^(2/3)*ln(x+(d/e)^(1/3))*c-1/9/e/d/(d/e)^(2/3)*ln(x^2-(d/e
)^(1/3)*x+(d/e)^(2/3))*a-1/18/e^2/(d/e)^(2/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2
/3))*b+2/9/e^3*d/(d/e)^(2/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))*c+2/9/e/d/(d
/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*a+1/9/e^2/(d/e)^(
2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*b-4/9/e^3*d/(d/e)^(2/3
)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.38885, size = 1548, normalized size = 7.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^2,x, algorithm="fricas")
```

```
[Out] [1/18*(18*c*d^3*e^2*x^4 - 3*sqrt(1/3)*(4*c*d^4*e - b*d^3*e^2 - 2*a*d^2*e^3
+ (4*c*d^3*e^2 - b*d^2*e^3 - 2*a*d*e^4)*x^3)*sqrt(-(d^2*e)^(1/3)/e)*log((2*
d*e*x^3 - 3*(d^2*e)^(1/3)*d*x - d^2 + 3*sqrt(1/3)*(2*d*e*x^2 + (d^2*e)^(2/3
)*x - (d^2*e)^(1/3)*d)*sqrt(-(d^2*e)^(1/3)/e))/(e*x^3 + d)) + (4*c*d^3 - b*
d^2*e - 2*a*d*e^2 + (4*c*d^2*e - b*d*e^2 - 2*a*e^3)*x^3)*(d^2*e)^(2/3)*log(
d*e*x^2 - (d^2*e)^(2/3)*x + (d^2*e)^(1/3)*d) - 2*(4*c*d^3 - b*d^2*e - 2*a*d
*e^2 + (4*c*d^2*e - b*d*e^2 - 2*a*e^3)*x^3)*(d^2*e)^(2/3)*log(d*e*x + (d^2*
e)^(2/3)) + 6*(4*c*d^4*e - b*d^3*e^2 + a*d^2*e^3)*x)/(d^3*e^4*x^3 + d^4*e^3
), 1/18*(18*c*d^3*e^2*x^4 - 6*sqrt(1/3)*(4*c*d^4*e - b*d^3*e^2 - 2*a*d^2*e^
3 + (4*c*d^3*e^2 - b*d^2*e^3 - 2*a*d*e^4)*x^3)*sqrt((d^2*e)^(1/3)/e)*arctan
(sqrt(1/3)*(2*(d^2*e)^(2/3)*x - (d^2*e)^(1/3)*d)*sqrt((d^2*e)^(1/3)/e)/d^2)
 + (4*c*d^3 - b*d^2*e - 2*a*d*e^2 + (4*c*d^2*e - b*d*e^2 - 2*a*e^3)*x^3)*(d
^2*e)^(2/3)*log(d*e*x^2 - (d^2*e)^(2/3)*x + (d^2*e)^(1/3)*d) - 2*(4*c*d^3 -
 b*d^2*e - 2*a*d*e^2 + (4*c*d^2*e - b*d*e^2 - 2*a*e^3)*x^3)*(d^2*e)^(2/3)*l
og(d*e*x + (d^2*e)^(2/3)) + 6*(4*c*d^4*e - b*d^3*e^2 + a*d^2*e^3)*x)/(d^3*e
^4*x^3 + d^4*e^3)]
```

Sympy [A] time = 1.98364, size = 206, normalized size = 0.97

$$\frac{cx}{e^2} + \frac{x\left(ae^2 - bde + cd^2\right)}{3d^2e^2 + 3de^3x^3} + \operatorname{RootSum}\left(729t^3d^5e^7 - 8a^3e^6 - 12a^2bde^5 + 48a^2cd^2e^4 - 6ab^2d^2e^4 + 48abcd^3e^3 - 96ac^2d^4e^2 - \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)/(e*x**3+d)**2,x)
```

```
[Out] c*x/e**2 + x*(a*e**2 - b*d*e + c*d**2)/(3*d**2*e**2 + 3*d*e**3*x**3) + Root
Sum(729*_t**3*d**5*e**7 - 8*a**3*e**6 - 12*a**2*b*d*e**5 + 48*a**2*c*d**2*e
**4 - 6*a*b**2*d**2*e**4 + 48*a*b*c*d**3*e**3 - 96*a*c**2*d**4*e**2 - b**3*
d**3*e**3 + 12*b**2*c*d**4*e**2 - 48*b*c**2*d**5*e + 64*c**3*d**6, Lambda(_
t, _t*log(9*_t*d**2*e**2/(2*a*e**2 + b*d*e - 4*c*d**2) + x)))
```

Giac [A] time = 1.12166, size = 306, normalized size = 1.44

$$cxe^{(-2)} - \frac{\sqrt{3}\left(4\left(-de^2\right)^{\frac{1}{3}}cd^2 - \left(-de^2\right)^{\frac{1}{3}}bde - 2\left(-de^2\right)^{\frac{1}{3}}ae^2\right)\arctan\left(\frac{\sqrt{3}\left(2x + \left(-de^{(-1)}\right)^{\frac{1}{3}}\right)}{3\left(-de^{(-1)}\right)^{\frac{1}{3}}}\right)e^{(-3)}}{9d^2} + \frac{\left(4cd^2 - bde - 2ae^2\right)\left(-de^{(-\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^2,x, algorithm="giac")
```

```
[Out] c*x*e^(-2) - 1/9*sqrt(3)*(4*(-d*e^2)^(1/3)*c*d^2 - (-d*e^2)^(1/3)*b*d*e - 2
*(-d*e^2)^(1/3)*a*e^2)*arctan(1/3*sqrt(3)*(2*x + (-d*e^(-1))^(1/3))/(-d*e^(
-1))^(1/3))*e^(-3)/d^2 + 1/9*(4*c*d^2 - b*d*e - 2*a*e^2)*(-d*e^(-1))^(1/3)*
e^(-2)*log(abs(x - (-d*e^(-1))^(1/3)))/d^2 - 1/18*(4*(-d*e^2)^(1/3)*c*d^2 -
 (-d*e^2)^(1/3)*b*d*e - 2*(-d*e^2)^(1/3)*a*e^2)*e^(-3)*log(x^2 + (-d*e^(-1)
)^(1/3)*x + (-d*e^(-1))^(2/3))/d^2 + 1/3*(c*d^2*x - b*d*x*e + a*x*e^2)*e^(-
2)/((x^3*e + d)*d)
```

3.8 $\int \frac{a+bx^3+cx^6}{\left(d+ex^3\right)^3}\,dx$

Optimal. Leaf size=242

$$-\frac{x\left(7cd^2-e(5ae+bd)\right)}{18d^2e^2\left(d+ex^3\right)}+\frac{x\left(ae^2-bde+cd^2\right)}{6de^2\left(d+ex^3\right)^2}-\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(e(5ae+bd)+2cd^2\right)}{54d^{8/3}e^{7/3}}+\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(e(5\right.}{27d^{8/3}}$$

```
[Out] ((c*d^2 - b*d*e + a*e^2)*x)/(6*d*e^2*(d + e*x^3)^2) - ((7*c*d^2 - e*(b*d +
5*a*e))*x)/(18*d^2*e^2*(d + e*x^3)) - ((2*c*d^2 + e*(b*d + 5*a*e))*ArcTan[(
d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(9*Sqrt[3]*d^(8/3)*e^(7/3)) + ((
2*c*d^2 + e*(b*d + 5*a*e))*Log[d^(1/3) + e^(1/3)*x])/(27*d^(8/3)*e^(7/3)) -
 ((2*c*d^2 + e*(b*d + 5*a*e))*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2
])/(54*d^(8/3)*e^(7/3))
```

Rubi [A] time = 0.262446, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1409, 385, 200, 31, 634, 617, 204, 628}

$$-\frac{x\left(7cd^2-e(5ae+bd)\right)}{18d^2e^2\left(d+ex^3\right)}+\frac{x\left(ae^2-bde+cd^2\right)}{6de^2\left(d+ex^3\right)^2}-\frac{\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(e(5ae+bd)+2cd^2\right)}{54d^{8/3}e^{7/3}}+\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(e(5\right.}{27d^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)/(d + e*x^3)^3,x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)*x)/(6*d*e^2*(d + e*x^3)^2) - ((7*c*d^2 - e*(b*d +
5*a*e))*x)/(18*d^2*e^2*(d + e*x^3)) - ((2*c*d^2 + e*(b*d + 5*a*e))*ArcTan[(
d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(9*Sqrt[3]*d^(8/3)*e^(7/3)) + ((
2*c*d^2 + e*(b*d + 5*a*e))*Log[d^(1/3) + e^(1/3)*x])/(27*d^(8/3)*e^(7/3)) -
 ((2*c*d^2 + e*(b*d + 5*a*e))*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2
])/(54*d^(8/3)*e^(7/3))
```

Rule 1409

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> -Simp[((c*d^2 - b*d*e + a*e^2)*x*(d + e*x^n)^(q + 1))/(d*e
^2*n*(q + 1)), x] + Dist[1/(n*(q + 1)*d*e^2), Int[(d + e*x^n)^(q + 1)*Simp[
```

```
c*d^2 - b*d*e + a*e^2*(n*(q + 1) + 1) + c*d*e*n*(q + 1)*x^n, x], x], x] /;
FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{a+bx^3+cx^6}{\left(d+ex^3\right)^3}\,dx = \frac{\left(cd^2-bde+ae^2\right)x}{6de^2\left(d+ex^3\right)^2} - \frac{\int \frac{cd^2-e(bd+5ae)-6cdex^3}{\left(d+ex^3\right)^2}\,dx}{6de^2}$$

$$= \frac{\left(cd^2-bde+ae^2\right)x}{6de^2\left(d+ex^3\right)^2} - \frac{\left(7cd^2-e(bd+5ae)\right)x}{18d^2e^2\left(d+ex^3\right)} + \frac{\left(2cd^2+e(bd+5ae)\right)\int \frac{1}{d+ex^3}\,dx}{9d^2e^2}$$

$$= \frac{\left(cd^2-bde+ae^2\right)x}{6de^2\left(d+ex^3\right)^2} - \frac{\left(7cd^2-e(bd+5ae)\right)x}{18d^2e^2\left(d+ex^3\right)} + \frac{\left(2cd^2+e(bd+5ae)\right)\int \frac{1}{\sqrt[3]{d}+\sqrt[3]{e}x}\,dx}{27d^{8/3}e^2} + \frac{\left(2cd^2+e(bd\right.}{}$$

$$= \frac{\left(cd^2-bde+ae^2\right)x}{6de^2\left(d+ex^3\right)^2} - \frac{\left(7cd^2-e(bd+5ae)\right)x}{18d^2e^2\left(d+ex^3\right)} + \frac{\left(2cd^2+e(bd+5ae)\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{27d^{8/3}e^{7/3}} - \frac{\left(2cd^2+e(\right.}{}$$

$$= \frac{\left(cd^2-bde+ae^2\right)x}{6de^2\left(d+ex^3\right)^2} - \frac{\left(7cd^2-e(bd+5ae)\right)x}{18d^2e^2\left(d+ex^3\right)} + \frac{\left(2cd^2+e(bd+5ae)\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{27d^{8/3}e^{7/3}} - \frac{\left(2cd^2+e(\right.}{}$$

$$= \frac{\left(cd^2-bde+ae^2\right)x}{6de^2\left(d+ex^3\right)^2} - \frac{\left(7cd^2-e(bd+5ae)\right)x}{18d^2e^2\left(d+ex^3\right)} - \frac{\left(2cd^2+e(bd+5ae)\right)\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}x}{\sqrt{3}\sqrt[3]{d}}\right)}{9\sqrt{3}d^{8/3}e^{7/3}} + \frac{\left(2cd^2+e\right.}{}$$

Mathematica [A] time = 0.271054, size = 209, normalized size = 0.86

$$\frac{-\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2\right)\left(e(5ae+bd)+2cd^2\right)-\frac{3d^{2/3}\sqrt[3]{e}x\left(cd^2\left(4d+7ex^3\right)-e\left(ae\left(8d+5ex^3\right)+bd\left(ex^3-2d\right)\right)\right)}{\left(d+ex^3\right)^2}+2\log\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\left(e(5a\right.}{54d^{8/3}e^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)/(d + e*x^3)^3,x]
```

```
[Out] ((-3*d^(2/3)*e^(1/3)*x*(c*d^2*(4*d + 7*e*x^3) - e*(b*d*(-2*d + e*x^3) + a*e
*(8*d + 5*e*x^3))))/(d + e*x^3)^2 - 2*Sqrt[3]*(2*c*d^2 + e*(b*d + 5*a*e))*A
rcTan[(1 - (2*e^(1/3)*x)/d^(1/3))/Sqrt[3]] + 2*(2*c*d^2 + e*(b*d + 5*a*e))*
Log[d^(1/3) + e^(1/3)*x] - (2*c*d^2 + e*(b*d + 5*a*e))*Log[d^(2/3) - d^(1/3
)*e^(1/3)*x + e^(2/3)*x^2])/(54*d^(8/3)*e^(7/3))
```

Maple [A] time = 0.01, size = 362, normalized size = 1.5

$$\frac{1}{\left(ex^3+d\right)^2}\left(\frac{\left(5\,ae^2+bde-7\,cd^2\right)x^4}{18\,d^2e}+\frac{\left(4\,ae^2-bde-2\,cd^2\right)x}{9\,de^2}\right)+\frac{5\,a}{27\,d^2e}\ln\left(x+\sqrt[3]{\frac{d}{e}}\right)\left(\frac{d}{e}\right)^{-\frac{2}{3}}+\frac{b}{27\,de^2}\ln\left(x+\sqrt[3]{\frac{d}{e}}\right)\left(\frac{d}{e}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)/(e*x^3+d)^3,x)
```

```
[Out] (1/18*(5*a*e^2+b*d*e-7*c*d^2)/d^2/e*x^4+1/9*(4*a*e^2-b*d*e-2*c*d^2)/d/e^2*x
)/(e*x^3+d)^2+5/27/e/d^2/(d/e)^(2/3)*ln(x+(d/e)^(1/3))*a+1/27/e^2/d/(d/e)^(
2/3)*ln(x+(d/e)^(1/3))*b+2/27/e^3/(d/e)^(2/3)*ln(x+(d/e)^(1/3))*c-5/54/e/d^
2/(d/e)^(2/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))*a-1/54/e^2/d/(d/e)^(2/3)*ln
(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))*b-1/27/e^3/(d/e)^(2/3)*ln(x^2-(d/e)^(1/3)*x
+(d/e)^(2/3))*c+5/27/e/d^2/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^
(1/3)*x-1))*a+1/27/e^2/d/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1
/3)*x-1))*b+2/27/e^3/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*
x-1))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.39834, size = 2067, normalized size = 8.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/54*(3*(7*c*d^4*e^2 - b*d^3*e^3 - 5*a*d^2*e^4)*x^4 - 3*sqrt(1/3)*(2*c*d^
5*e + b*d^4*e^2 + 5*a*d^3*e^3 + (2*c*d^3*e^3 + b*d^2*e^4 + 5*a*d*e^5)*x^6 +
 2*(2*c*d^4*e^2 + b*d^3*e^3 + 5*a*d^2*e^4)*x^3)*sqrt(-(d^2*e)^(1/3)/e)*log(
(2*d*e*x^3 - 3*(d^2*e)^(1/3)*d*x - d^2 + 3*sqrt(1/3)*(2*d*e*x^2 + (d^2*e)^(
2/3)*x - (d^2*e)^(1/3)*d)*sqrt(-(d^2*e)^(1/3)/e))/(e*x^3 + d)) + ((2*c*d^2*
e^2 + b*d*e^3 + 5*a*e^4)*x^6 + 2*c*d^4 + b*d^3*e + 5*a*d^2*e^2 + 2*(2*c*d^3
*e + b*d^2*e^2 + 5*a*d*e^3)*x^3)*(d^2*e)^(2/3)*log(d*e*x^2 - (d^2*e)^(2/3)*
x + (d^2*e)^(1/3)*d) - 2*((2*c*d^2*e^2 + b*d*e^3 + 5*a*e^4)*x^6 + 2*c*d^4 +
 b*d^3*e + 5*a*d^2*e^2 + 2*(2*c*d^3*e + b*d^2*e^2 + 5*a*d*e^3)*x^3)*(d^2*e)
^(2/3)*log(d*e*x + (d^2*e)^(2/3)) + 6*(2*c*d^5*e + b*d^4*e^2 - 4*a*d^3*e^3)
*x)/(d^4*e^5*x^6 + 2*d^5*e^4*x^3 + d^6*e^3), -1/54*(3*(7*c*d^4*e^2 - b*d^3*
e^3 - 5*a*d^2*e^4)*x^4 - 6*sqrt(1/3)*(2*c*d^5*e + b*d^4*e^2 + 5*a*d^3*e^3 +
 (2*c*d^3*e^3 + b*d^2*e^4 + 5*a*d*e^5)*x^6 + 2*(2*c*d^4*e^2 + b*d^3*e^3 + 5
*a*d^2*e^4)*x^3)*sqrt((d^2*e)^(1/3)/e)*arctan(sqrt(1/3)*(2*(d^2*e)^(2/3)*x
- (d^2*e)^(1/3)*d)*sqrt((d^2*e)^(1/3)/e)/d^2) + ((2*c*d^2*e^2 + b*d*e^3 + 5
*a*e^4)*x^6 + 2*c*d^4 + b*d^3*e + 5*a*d^2*e^2 + 2*(2*c*d^3*e + b*d^2*e^2 +
5*a*d*e^3)*x^3)*(d^2*e)^(2/3)*log(d*e*x^2 - (d^2*e)^(2/3)*x + (d^2*e)^(1/3)
*d) - 2*((2*c*d^2*e^2 + b*d*e^3 + 5*a*e^4)*x^6 + 2*c*d^4 + b*d^3*e + 5*a*d^
2*e^2 + 2*(2*c*d^3*e + b*d^2*e^2 + 5*a*d*e^3)*x^3)*(d^2*e)^(2/3)*log(d*e*x
+ (d^2*e)^(2/3)) + 6*(2*c*d^5*e + b*d^4*e^2 - 4*a*d^3*e^3)*x)/(d^4*e^5*x^6
+ 2*d^5*e^4*x^3 + d^6*e^3)]
```

Sympy [A] time = 7.55052, size = 246, normalized size = 1.02

$$\frac{x^4\left(5ae^3+bde^2-7cd^2e\right)+x\left(8ade^2-2bd^2e-4cd^3\right)}{18d^4e^2+36d^3e^3x^3+18d^2e^4x^6}+\mathrm{RootSum}\Big(19683t^3d^8e^7-125a^3e^6-75a^2bde^5-150a^2cd^2e^4-15a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)/(e*x**3+d)**3,x)
```

```
[Out] (x**4*(5*a*e**3 + b*d*e**2 - 7*c*d**2*e) + x*(8*a*d*e**2 - 2*b*d**2*e - 4*c
*d**3))/(18*d**4*e**2 + 36*d**3*e**3*x**3 + 18*d**2*e**4*x**6) + RootSum(19
683*_t**3*d**8*e**7 - 125*a**3*e**6 - 75*a**2*b*d*e**5 - 150*a**2*c*d**2*e*
*4 - 15*a*b**2*d**2*e**4 - 60*a*b*c*d**3*e**3 - 60*a*c**2*d**4*e**2 - b**3*
d**3*e**3 - 6*b**2*c*d**4*e**2 - 12*b*c**2*d**5*e - 8*c**3*d**6, Lambda(_t,
 _t*log(27*_t*d**3*e**2/(5*a*e**2 + b*d*e + 2*c*d**2) + x)))
```

Giac [A] time = 1.14512, size = 340, normalized size = 1.4

$$\frac{\sqrt{3}\left(2\left(-de^2\right)^{\frac{1}{3}}cd^2+\left(-de^2\right)^{\frac{1}{3}}bde+5\left(-de^2\right)^{\frac{1}{3}}ae^2\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-de^{(-1)}\right)^{\frac{1}{3}}\right)}{3\left(-de^{(-1)}\right)^{\frac{1}{3}}}\right)e^{(-3)}}{27d^3}-\frac{\left(2cd^2+bde+5ae^2\right)\left(-de^{(-1)}\right)^{\frac{1}{3}}e^{(-}}{27d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^3,x, algorithm="giac")
```

```
[Out] 1/27*sqrt(3)*(2*(-d*e^2)^(1/3)*c*d^2 + (-d*e^2)^(1/3)*b*d*e + 5*(-d*e^2)^(1
/3)*a*e^2)*arctan(1/3*sqrt(3)*(2*x + (-d*e^(-1))^(1/3))/(-d*e^(-1))^(1/3))*
e^(-3)/d^3 - 1/27*(2*c*d^2 + b*d*e + 5*a*e^2)*(-d*e^(-1))^(1/3)*e^(-2)*log(
abs(x - (-d*e^(-1))^(1/3)))/d^3 + 1/54*(2*(-d*e^2)^(1/3)*c*d^2 + (-d*e^2)^(
1/3)*b*d*e + 5*(-d*e^2)^(1/3)*a*e^2)*e^(-3)*log(x^2 + (-d*e^(-1))^(1/3)*x +
 (-d*e^(-1))^(2/3))/d^3 - 1/18*(7*c*d^2*x^4*e - b*d*x^4*e^2 - 5*a*x^4*e^3 +
 4*c*d^3*x + 2*b*d^2*x*e - 8*a*d*x*e^2)*e^(-2)/((x^3*e + d)^2*d^2)
```

3.9 $\int \frac{x^8(d+ex^3)}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=132

$$-\frac{\left(ace+b^2(-e)+bcd\right)\log\left(a+bx^3+cx^6\right)}{6c^3}-\frac{\left(3abce-2ac^2d+b^2cd+b^3(-e)\right)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^3\sqrt{b^2-4ac}}+\frac{x^3(cd-be)}{3c^2}+\frac{ex^6}{6c}$$

```
[Out] ((c*d - b*e)*x^3)/(3*c^2) + (e*x^6)/(6*c) - ((b^2*c*d - 2*a*c^2*d - b^3*e +
 3*a*b*c*e)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*c^3*Sqrt[b^2 - 4*a
*c]) - ((b*c*d - b^2*e + a*c*e)*Log[a + b*x^3 + c*x^6])/(6*c^3)
```

Rubi [A] time = 0.217983, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1474, 800, 634, 618, 206, 628}

$$-\frac{\left(ace+b^2(-e)+bcd\right)\log\left(a+bx^3+cx^6\right)}{6c^3}-\frac{\left(3abce-2ac^2d+b^2cd+b^3(-e)\right)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^3\sqrt{b^2-4ac}}+\frac{x^3(cd-be)}{3c^2}+\frac{ex^6}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] ((c*d - b*e)*x^3)/(3*c^2) + (e*x^6)/(6*c) - ((b^2*c*d - 2*a*c^2*d - b^3*e +
 3*a*b*c*e)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*c^3*Sqrt[b^2 - 4*a
*c]) - ((b*c*d - b^2*e + a*c*e)*Log[a + b*x^3 + c*x^6])/(6*c^3)
```

Rule 1474

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^8 \left(d+ex^3\right)}{a+bx^3+cx^6}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{x^2(d+ex)}{a+bx+cx^2}\,dx,x,x^3\right)$$
$$= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{cd-be}{c^2}+\frac{ex}{c}-\frac{a(cd-be)+\left(bcd-b^2e+ace\right)x}{c^2\left(a+bx+cx^2\right)}\right)dx,x,x^3\right)$$
$$= \frac{(cd-be)x^3}{3c^2}+\frac{ex^6}{6c}-\frac{\operatorname{Subst}\left(\int \frac{a(cd-be)+\left(bcd-b^2e+ace\right)x}{a+bx+cx^2}\,dx,x,x^3\right)}{3c^2}$$
$$= \frac{(cd-be)x^3}{3c^2}+\frac{ex^6}{6c}-\frac{\left(bcd-b^2e+ace\right)\operatorname{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^3\right)}{6c^3}+\frac{\left(b^2cd-2ac^2d-b^3e+3abce\right)}{6c$$
$$= \frac{(cd-be)x^3}{3c^2}+\frac{ex^6}{6c}-\frac{\left(bcd-b^2e+ace\right)\log\left(a+bx^3+cx^6\right)}{6c^3}-\frac{\left(b^2cd-2ac^2d-b^3e+3abce\right)\operatorname{Subst}\left(}{3c^3}$$
$$= \frac{(cd-be)x^3}{3c^2}+\frac{ex^6}{6c}-\frac{\left(b^2cd-2ac^2d-b^3e+3abce\right)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^3\sqrt{b^2-4ac}}-\frac{\left(bcd-b^2e+ace\right)\log\left(a+b\right.}{6c^3}$$

Mathematica [A] time = 0.0654022, size = 126, normalized size = 0.95

$$\frac{\frac{2\left(3abce-2ac^2d+b^2cd+b^3(-e)\right)\tan^{-1}\left(\frac{b+2cx^3}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\left(-ace+b^2e-bcd\right)\log\left(a+bx^3+cx^6\right)+2cx^3(cd-be)+c^2ex^6}{6c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] (2*c*(c*d - b*e)*x^3 + c^2*e*x^6 + (2*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*
c*e)*ArcTan[(b + 2*c*x^3)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (-(b*c*
d) + b^2*e - a*c*e)*Log[a + b*x^3 + c*x^6])/(6*c^3)
```

Maple [B] time = 0.004, size = 260, normalized size = 2.

$$\frac{ex^6}{6\,c}-\frac{bex^3}{3\,c^2}+\frac{dx^3}{3\,c}-\frac{\ln\left(cx^6+bx^3+a\right)ae}{6\,c^2}+\frac{\ln\left(cx^6+bx^3+a\right)b^2e}{6\,c^3}-\frac{\ln\left(cx^6+bx^3+a\right)bd}{6\,c^2}+\frac{abe}{c^2}\arctan\left((2\,cx^3+b)\frac{}{\sqrt{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(e*x^3+d)/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/6*e*x^6/c-1/3/c^2*b*e*x^3+1/3/c*d*x^3-1/6/c^2*ln(c*x^6+b*x^3+a)*a*e+1/6/c
^3*ln(c*x^6+b*x^3+a)*b^2*e-1/6/c^2*ln(c*x^6+b*x^3+a)*b*d+1/c^2/(4*a*c-b^2)^
(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2))*a*b*e-2/3/c/(4*a*c-b^2)^(1/2)*a
rctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2))*a*d-1/3/c^3/(4*a*c-b^2)^(1/2)*arctan((
2*c*x^3+b)/(4*a*c-b^2)^(1/2))*b^3*e+1/3/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x
^3+b)/(4*a*c-b^2)^(1/2))*b^2*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.25384, size = 902, normalized size = 6.83

$$\left[\frac{\left(b^2c^2-4\,ac^3\right)ex^6+2\left(\left(b^2c^2-4\,ac^3\right)d-\left(b^3c-4\,abc^2\right)e\right)x^3+\sqrt{b^2-4\,ac}\left(\left(b^2c-2\,ac^2\right)d-\left(b^3-3\,abc\right)e\right)\log\left(\frac{2\,c^2x^6+2\,bcx^3}{}\right.}{6\left(b^2c^3-4\,ac^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*((b^2*c^2 - 4*a*c^3)*e*x^6 + 2*((b^2*c^2 - 4*a*c^3)*d - (b^3*c - 4*a*b
*c^2)*e)*x^3 + sqrt(b^2 - 4*a*c)*((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e)*
log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c - (2*c*x^3 + b)*sqrt(b^2 - 4*a*c))
/(c*x^6 + b*x^3 + a)) - ((b^3*c - 4*a*b*c^2)*d - (b^4 - 5*a*b^2*c + 4*a^2*c
^2)*e)*log(c*x^6 + b*x^3 + a))/(b^2*c^3 - 4*a*c^4), 1/6*((b^2*c^2 - 4*a*c^3
)*e*x^6 + 2*((b^2*c^2 - 4*a*c^3)*d - (b^3*c - 4*a*b*c^2)*e)*x^3 - 2*sqrt(-b
^2 + 4*a*c)*((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e)*arctan(-(2*c*x^3 + b)
*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^3*c - 4*a*b*c^2)*d - (b^4 - 5*a*b^
2*c + 4*a^2*c^2)*e)*log(c*x^6 + b*x^3 + a))/(b^2*c^3 - 4*a*c^4)]
```

Sympy [B] time = 15.0021, size = 619, normalized size = 4.69

$$\left(-\frac{\sqrt{-4ac+b^2}\left(3abce-2ac^2d-b^3e+b^2cd\right)}{6c^3\left(4ac-b^2\right)}-\frac{ace-b^2e+bcd}{6c^3}\right)\log\left(x^3+\frac{2a^2ce-ab^2e+abcd+12ac^3\left(-\frac{\sqrt{-4ac+b^2}\left(3abce-2a\right.}{6c^3\left(4ac-\right.}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(e*x**3+d)/(c*x**6+b*x**3+a),x)
```

```
[Out] (-sqrt(-4*a*c + b**2)*(3*a*b*c*e - 2*a*c**2*d - b**3*e + b**2*c*d)/(6*c**3*
(4*a*c - b**2)) - (a*c*e - b**2*e + b*c*d)/(6*c**3))*log(x**3 + (2*a**2*c*e
 - a*b**2*e + a*b*c*d + 12*a*c**3*(-sqrt(-4*a*c + b**2)*(3*a*b*c*e - 2*a*c*
*2*d - b**3*e + b**2*c*d)/(6*c**3*(4*a*c - b**2)) - (a*c*e - b**2*e + b*c*d
)/(6*c**3)) - 3*b**2*c**2*(-sqrt(-4*a*c + b**2)*(3*a*b*c*e - 2*a*c**2*d - b
**3*e + b**2*c*d)/(6*c**3*(4*a*c - b**2)) - (a*c*e - b**2*e + b*c*d)/(6*c**
3)))/(3*a*b*c*e - 2*a*c**2*d - b**3*e + b**2*c*d)) + (sqrt(-4*a*c + b**2)*(
3*a*b*c*e - 2*a*c**2*d - b**3*e + b**2*c*d)/(6*c**3*(4*a*c - b**2)) - (a*c*
e - b**2*e + b*c*d)/(6*c**3))*log(x**3 + (2*a**2*c*e - a*b**2*e + a*b*c*d +
 12*a*c**3*(sqrt(-4*a*c + b**2)*(3*a*b*c*e - 2*a*c**2*d - b**3*e + b**2*c*d
)/(6*c**3*(4*a*c - b**2)) - (a*c*e - b**2*e + b*c*d)/(6*c**3)) - 3*b**2*c**
2*(sqrt(-4*a*c + b**2)*(3*a*b*c*e - 2*a*c**2*d - b**3*e + b**2*c*d)/(6*c**3
*(4*a*c - b**2)) - (a*c*e - b**2*e + b*c*d)/(6*c**3)))/(3*a*b*c*e - 2*a*c**
2*d - b**3*e + b**2*c*d)) + e*x**6/(6*c) - x**3*(b*e - c*d)/(3*c**2)
```

Giac [A] time = 1.36096, size = 177, normalized size = 1.34

$$\frac{cx^6e+2\,cdx^3-2\,bx^3e}{6\,c^2}-\frac{\left(bcd-b^2e+ace\right)\log\left(cx^6+bx^3+a\right)}{6\,c^3}+\frac{\left(b^2cd-2\,ac^2d-b^3e+3\,abce\right)\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/6*(c*x^6*e + 2*c*d*x^3 - 2*b*x^3*e)/c^2 - 1/6*(b*c*d - b^2*e + a*c*e)*log
(c*x^6 + b*x^3 + a)/c^3 + 1/3*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)*arc
tan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^3)
```

3.10 $\int \frac{x^5\left(d+ex^3\right)}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=97

$$\frac{\left(2ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^2\sqrt{b^2-4ac}}+\frac{(cd-be)\log\left(a+bx^3+cx^6\right)}{6c^2}+\frac{ex^3}{3c}$$

```
[Out] (e*x^3)/(3*c) + ((b*c*d - b^2*e + 2*a*c*e)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 -
 4*a*c]])/(3*c^2*Sqrt[b^2 - 4*a*c]) + ((c*d - b*e)*Log[a + b*x^3 + c*x^6])/
(6*c^2)
```

Rubi [A] time = 0.119684, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1474, 773, 634, 618, 206, 628}

$$\frac{\left(2ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^2\sqrt{b^2-4ac}}+\frac{(cd-be)\log\left(a+bx^3+cx^6\right)}{6c^2}+\frac{ex^3}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] (e*x^3)/(3*c) + ((b*c*d - b^2*e + 2*a*c*e)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 -
 4*a*c]])/(3*c^2*Sqrt[b^2 - 4*a*c]) + ((c*d - b*e)*Log[a + b*x^3 + c*x^6])/
(6*c^2)
```

Rule 1474

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5\left(d+ex^3\right)}{a+bx^3+cx^6}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{x(d+ex)}{a+bx+cx^2}\,dx,x,x^3\right)\\
&= \frac{ex^3}{3c}+\frac{\operatorname{Subst}\left(\int \frac{-ae+(cd-be)x}{a+bx+cx^2}\,dx,x,x^3\right)}{3c}\\
&= \frac{ex^3}{3c}+\frac{(cd-be)\operatorname{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^3\right)}{6c^2}-\frac{\left(bcd-b^2e+2ace\right)\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^3\right)}{6c^2}\\
&= \frac{ex^3}{3c}+\frac{(cd-be)\log\left(a+bx^3+cx^6\right)}{6c^2}+\frac{\left(bcd-b^2e+2ace\right)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^3\right)}{3c^2}\\
&= \frac{ex^3}{3c}+\frac{\left(bcd-b^2e+2ace\right)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^2\sqrt{b^2-4ac}}+\frac{(cd-be)\log\left(a+bx^3+cx^6\right)}{6c^2}
\end{aligned}
$$

Mathematica [A] time = 0.0702844, size = 93, normalized size = 0.96

$$\frac{\frac{2\left(-2ace+b^2e-bcd\right)\tan^{-1}\left(\frac{b+2cx^3}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+(cd-be)\log\left(a+bx^3+cx^6\right)+2cex^3}{6c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] (2*c*e*x^3 + (2*(-(b*c*d) + b^2*e - 2*a*c*e)*ArcTan[(b + 2*c*x^3)/Sqrt[-b^2
 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (c*d - b*e)*Log[a + b*x^3 + c*x^6])/(6*c^2
)
```

Maple [A] time = 0.003, size = 175, normalized size = 1.8

$$\frac{ex^3}{3c}-\frac{\ln\left(cx^6+bx^3+a\right)be}{6c^2}+\frac{\ln\left(cx^6+bx^3+a\right)d}{6c}-\frac{2ae}{3c}\arctan\left((2cx^3+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}+\frac{b^2e}{3c^2}\arctan\left((2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x^3+d)/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/3*e*x^3/c-1/6/c^2*ln(c*x^6+b*x^3+a)*b*e+1/6/c*ln(c*x^6+b*x^3+a)*d-2/3/c/(
4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2))*a*e+1/3/c^2/(4*a*c-b
^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2))*b^2*e-1/3/c/(4*a*c-b^2)^(1/
2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2))*b*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.92803, size = 664, normalized size = 6.85

$$\left[\frac{2\left(b^2c-4\,ac^2\right)ex^3+\left(bcd-\left(b^2-2\,ac\right)e\right)\sqrt{b^2-4\,ac}\log\left(\frac{2\,c^2x^6+2\,bcx^3+b^2-2\,ac+\left(2\,cx^3+b\right)\sqrt{b^2-4\,ac}}{cx^6+bx^3+a}\right)+\left(\left(b^2c-4\,ac^2\right)d-\left(b^3-4\,ab\right.\right.}{6\left(b^2c^2-4\,ac^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*(b^2*c - 4*a*c^2)*e*x^3 + (b*c*d - (b^2 - 2*a*c)*e)*sqrt(b^2 - 4*a*
c)*log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c + (2*c*x^3 + b)*sqrt(b^2 - 4*a*
c))/(c*x^6 + b*x^3 + a)) + ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*log(c*
x^6 + b*x^3 + a))/(b^2*c^2 - 4*a*c^3), 1/6*(2*(b^2*c - 4*a*c^2)*e*x^3 + 2*(
b*c*d - (b^2 - 2*a*c)*e)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^3 + b)*sqrt(-b^2
 + 4*a*c)/(b^2 - 4*a*c)) + ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*log(c*
x^6 + b*x^3 + a))/(b^2*c^2 - 4*a*c^3)]
```

Sympy [B] time = 9.09558, size = 434, normalized size = 4.47

$$\left(-\frac{\sqrt{-4ac+b^2}\left(2ace-b^2e+bcd\right)}{6c^2\left(4ac-b^2\right)}-\frac{be-cd}{6c^2}\right)\log\left(x^3+\frac{-abe-12ac^2\left(-\frac{\sqrt{-4ac+b^2}(2ace-b^2e+bcd)}{6c^2(4ac-b^2)}-\frac{be-cd}{6c^2}\right)+2acd+3b^2c\left(-\frac{\sqrt{-}}{}\right.}{2ace-b^2e+bcd}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x**3+d)/(c*x**6+b*x**3+a),x)
```

```
[Out] (-sqrt(-4*a*c + b**2)*(2*a*c*e - b**2*e + b*c*d)/(6*c**2*(4*a*c - b**2)) -
(b*e - c*d)/(6*c**2))*log(x**3 + (-a*b*e - 12*a*c**2*(-sqrt(-4*a*c + b**2)*
(2*a*c*e - b**2*e + b*c*d)/(6*c**2*(4*a*c - b**2)) - (b*e - c*d)/(6*c**2))
+ 2*a*c*d + 3*b**2*c*(-sqrt(-4*a*c + b**2)*(2*a*c*e - b**2*e + b*c*d)/(6*c*
*2*(4*a*c - b**2)) - (b*e - c*d)/(6*c**2)))/(2*a*c*e - b**2*e + b*c*d)) + (
sqrt(-4*a*c + b**2)*(2*a*c*e - b**2*e + b*c*d)/(6*c**2*(4*a*c - b**2)) - (b
*e - c*d)/(6*c**2))*log(x**3 + (-a*b*e - 12*a*c**2*(sqrt(-4*a*c + b**2)*(2*
a*c*e - b**2*e + b*c*d)/(6*c**2*(4*a*c - b**2)) - (b*e - c*d)/(6*c**2)) + 2
*a*c*d + 3*b**2*c*(sqrt(-4*a*c + b**2)*(2*a*c*e - b**2*e + b*c*d)/(6*c**2*(
4*a*c - b**2)) - (b*e - c*d)/(6*c**2)))/(2*a*c*e - b**2*e + b*c*d)) + e*x**
```

```
3/(3*c)
```

Giac [A] time = 1.35018, size = 128, normalized size = 1.32

$$\frac{x^3e}{3\,c} + \frac{(cd - be)\log\left(cx^6 + bx^3 + a\right)}{6\,c^2} - \frac{\left(bcd - b^2e + 2\,ace\right)\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2 + 4\,ac}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*x^3*e/c + 1/6*(c*d - b*e)*log(c*x^6 + b*x^3 + a)/c^2 - 1/3*(b*c*d - b^2
*e + 2*a*c*e)*arctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*
c^2)
```

3.11 $\int \frac{x^2(d+ex^3)}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=72

$$\frac{e\log\left(a+bx^3+cx^6\right)}{6c}-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c\sqrt{b^2-4ac}}$$

```
[Out] -((2*c*d - b*e)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*c*Sqrt[b^2 - 4
*a*c]) + (e*Log[a + b*x^3 + c*x^6])/(6*c)
```

Rubi [A] time = 0.0731691, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1468, 634, 618, 206, 628}

$$\frac{e\log\left(a+bx^3+cx^6\right)}{6c}-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] -((2*c*d - b*e)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*c*Sqrt[b^2 - 4
*a*c]) + (e*Log[a + b*x^3 + c*x^6])/(6*c)
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\left(d+ex^3\right)}{a+bx^3+cx^6}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{d+ex}{a+bx+cx^2}\,dx, x, x^3\right)\\
&= \frac{e\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^3\right)}{6c} + \frac{(2cd-be)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^3\right)}{6c}\\
&= \frac{e\log\left(a+bx^3+cx^6\right)}{6c} - \frac{(2cd-be)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^3\right)}{3c}\\
&= -\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c\sqrt{b^2-4ac}} + \frac{e\log\left(a+bx^3+cx^6\right)}{6c}
\end{aligned}$$

Mathematica [A] time = 0.0500176, size = 71, normalized size = 0.99

$$\frac{e\log\left(a+bx^3+cx^6\right) - \frac{2(be-2cd)\tan^{-1}\left(\frac{b+2cx^3}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{6c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] ((-2*(-2*c*d + b*e)*ArcTan[(b + 2*c*x^3)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4
*a*c] + e*Log[a + b*x^3 + c*x^6])/(6*c)
```

Maple [A] time = 0.002, size = 99, normalized size = 1.4

$$\frac{e\ln\left(cx^6+bx^3+a\right)}{6c}+\frac{2d}{3}\arctan\left((2cx^3+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}-\frac{be}{3c}\arctan\left((2cx^3+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^3+d)/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/6*e*ln(c*x^6+b*x^3+a)/c+2/3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b
^2)^(1/2))*d-1/3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2))*e*
b/c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.44533, size = 481, normalized size = 6.68

$$\left[\frac{\left(b^2-4ac\right)e\log\left(cx^6+bx^3+a\right)-\sqrt{b^2-4ac}(2cd-be)\log\left(\frac{2c^2x^6+2bcx^3+b^2-2ac+\left(2cx^3+b\right)\sqrt{b^2-4ac}}{cx^6+bx^3+a}\right)}{6\left(b^2c-4ac^2\right)},\frac{\left(b^2-4ac\right)e\log\left(cx^6+b\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*((b^2 - 4*a*c)*e*log(c*x^6 + b*x^3 + a) - sqrt(b^2 - 4*a*c)*(2*c*d - b
*e)*log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c + (2*c*x^3 + b)*sqrt(b^2 - 4*a
*c))/(c*x^6 + b*x^3 + a)))/(b^2*c - 4*a*c^2), 1/6*((b^2 - 4*a*c)*e*log(c*x^
6 + b*x^3 + a) - 2*sqrt(-b^2 + 4*a*c)*(2*c*d - b*e)*arctan(-(2*c*x^3 + b)*s
qrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/(b^2*c - 4*a*c^2)]
```

Sympy [B] time = 4.38941, size = 287, normalized size = 3.99

$$\left(\frac{e}{6c} - \frac{\sqrt{-4ac+b^2}\,(be-2cd)}{6c\left(4ac-b^2\right)}\right)\log\left(x^3 + \frac{-12ac\left(\frac{e}{6c} - \frac{\sqrt{-4ac+b^2}(be-2cd)}{6c\left(4ac-b^2\right)}\right) + 2ae + 3b^2\left(\frac{e}{6c} - \frac{\sqrt{-4ac+b^2}(be-2cd)}{6c\left(4ac-b^2\right)}\right) - bd}{be-2cd}\right) + \left(\frac{e}{6c} + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**3+d)/(c*x**6+b*x**3+a),x)
```

```
[Out] (e/(6*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(6*c*(4*a*c - b**2)))*log(x**3
 + (-12*a*c*(e/(6*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(6*c*(4*a*c - b**2
))) + 2*a*e + 3*b**2*(e/(6*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(6*c*(4*a
*c - b**2))) - b*d)/(b*e - 2*c*d)) + (e/(6*c) + sqrt(-4*a*c + b**2)*(b*e -
2*c*d)/(6*c*(4*a*c - b**2)))*log(x**3 + (-12*a*c*(e/(6*c) + sqrt(-4*a*c + b
**2)*(b*e - 2*c*d)/(6*c*(4*a*c - b**2))) + 2*a*e + 3*b**2*(e/(6*c) + sqrt(-
4*a*c + b**2)*(b*e - 2*c*d)/(6*c*(4*a*c - b**2))) - b*d)/(b*e - 2*c*d))
```

Giac [A] time = 1.37539, size = 95, normalized size = 1.32

$$\frac{e\log\left(cx^6 + bx^3 + a\right)}{6\,c} + \frac{(2\,cd - be)\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/6*e*log(c*x^6 + b*x^3 + a)/c + 1/3*(2*c*d - b*e)*arctan((2*c*x^3 + b)/sqr
t(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c)
```

3.12 $\int \frac{d+ex^3}{x\left(a+bx^3+cx^6\right)}\,dx$

Optimal. Leaf size=78

$$\frac{(bd-2ae)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a\sqrt{b^2-4ac}}-\frac{d\log\left(a+bx^3+cx^6\right)}{6a}+\frac{d\log(x)}{a}$$

```
[Out] ((b*d - 2*a*e)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*a*Sqrt[b^2 - 4*
a*c]) + (d*Log[x])/a - (d*Log[a + b*x^3 + c*x^6])/(6*a)
```

Rubi [A] time = 0.127953, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1474, 800, 634, 618, 206, 628}

$$\frac{(bd-2ae)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a\sqrt{b^2-4ac}}-\frac{d\log\left(a+bx^3+cx^6\right)}{6a}+\frac{d\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^3)/(x*(a + b*x^3 + c*x^6)),x]
```

```
[Out] ((b*d - 2*a*e)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*a*Sqrt[b^2 - 4*
a*c]) + (d*Log[x])/a - (d*Log[a + b*x^3 + c*x^6])/(6*a)
```

Rule 1474

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex^3}{x\left(a+bx^3+cx^6\right)}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{d+ex}{x\left(a+bx+cx^2\right)}\,dx, x, x^3\right)\\
&= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{d}{ax}+\frac{-bd+ae-cdx}{a\left(a+bx+cx^2\right)}\right)dx, x, x^3\right)\\
&= \frac{d\log(x)}{a}+\frac{\operatorname{Subst}\left(\int \frac{-bd+ae-cdx}{a+bx+cx^2}\,dx, x, x^3\right)}{3a}\\
&= \frac{d\log(x)}{a}-\frac{d\operatorname{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^3\right)}{6a}+\frac{(-bd+2ae)\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^3\right)}{6a}\\
&= \frac{d\log(x)}{a}-\frac{d\log\left(a+bx^3+cx^6\right)}{6a}-\frac{(-bd+2ae)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^3\right)}{3a}\\
&= \frac{(bd-2ae)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a\sqrt{b^2-4ac}}+\frac{d\log(x)}{a}-\frac{d\log\left(a+bx^3+cx^6\right)}{6a}
\end{aligned}$$

Mathematica [C] time = 0.0336629, size = 80, normalized size = 1.03

$$\frac{d\log(x)}{a}-\frac{\text{RootSum}\left[\#1^3b+\#1^6c+a\&,\frac{\#1^3cd\log(x-\#1)-ae\log(x-\#1)+bd\log(x-\#1)}{2\#1^3c+b}\&\right]}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^3)/(x*(a + b*x^3 + c*x^6)),x]
```

```
[Out] (d*Log[x])/a - RootSum[a + b*#1^3 + c*#1^6 & , (b*d*Log[x - #1] - a*e*Log[x
 - #1] + c*d*Log[x - #1]*#1^3)/(b + 2*c*#1^3) & ]/(3*a)
```

Maple [A] time = 0.006, size = 106, normalized size = 1.4

$$\frac{d\ln(x)}{a}-\frac{d\ln\left(cx^6+bx^3+a\right)}{6a}+\frac{2e}{3}\arctan\left((2cx^3+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}-\frac{bd}{3a}\arctan\left((2cx^3+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^3+d)/x/(c*x^6+b*x^3+a),x)
```

```
[Out] d*ln(x)/a-1/6*d*ln(c*x^6+b*x^3+a)/a+2/3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b
)/(4*a*c-b^2)^(1/2))*e-1/3/a/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^
2)^(1/2))*b*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/x/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.19806, size = 556, normalized size = 7.13

$$\left[-\frac{(b^2-4\,ac)d\log\left(cx^6+bx^3+a\right)-6\left(b^2-4\,ac\right)d\log(x)+\sqrt{b^2-4\,ac}(bd-2\,ae)\log\left(\frac{2\,c^2x^6+2\,bcx^3+b^2-2\,ac-\left(2\,cx^3+b\right)\sqrt{b^2-4\,ac}}{cx^6+bx^3+a}\right.}{6\left(ab^2-4\,a^2c\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/x/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] [-1/6*((b^2 - 4*a*c)*d*log(c*x^6 + b*x^3 + a) - 6*(b^2 - 4*a*c)*d*log(x) +
sqrt(b^2 - 4*a*c)*(b*d - 2*a*e)*log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c -
(2*c*x^3 + b)*sqrt(b^2 - 4*a*c))/(c*x^6 + b*x^3 + a)))/(a*b^2 - 4*a^2*c), -
1/6*((b^2 - 4*a*c)*d*log(c*x^6 + b*x^3 + a) - 6*(b^2 - 4*a*c)*d*log(x) - 2*
sqrt(-b^2 + 4*a*c)*(b*d - 2*a*e)*arctan(-(2*c*x^3 + b)*sqrt(-b^2 + 4*a*c)/(
b^2 - 4*a*c)))/(a*b^2 - 4*a^2*c)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**3+d)/x/(c*x**6+b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.38437, size = 103, normalized size = 1.32

$$-\frac{d\log\left(cx^6+bx^3+a\right)}{6\,a}+\frac{d\log\left(|x|\right)}{a}-\frac{(bd-2\,ae)\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\sqrt{-b^2+4\,ac}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/x/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/6*d*log(c*x^6 + b*x^3 + a)/a + d*log(abs(x))/a - 1/3*(b*d - 2*a*e)*arcta
n((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a)
```

3.13 $\int \frac{d+ex^3}{x^4(a+bx^3+cx^6)}\,dx$

Optimal. Leaf size=112

$$-\frac{\left(-abe-2acd+b^2d\right)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a^2\sqrt{b^2-4ac}}+\frac{(bd-ae)\log\left(a+bx^3+cx^6\right)}{6a^2}-\frac{\log(x)(bd-ae)}{a^2}-\frac{d}{3ax^3}$$

```
[Out] -d/(3*a*x^3) - ((b^2*d - 2*a*c*d - a*b*e)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 -
4*a*c]])/(3*a^2*Sqrt[b^2 - 4*a*c]) - ((b*d - a*e)*Log[x])/a^2 + ((b*d - a*e
)*Log[a + b*x^3 + c*x^6])/(6*a^2)
```

Rubi [A] time = 0.197486, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1474, 800, 634, 618, 206, 628}

$$-\frac{\left(-abe-2acd+b^2d\right)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a^2\sqrt{b^2-4ac}}+\frac{(bd-ae)\log\left(a+bx^3+cx^6\right)}{6a^2}-\frac{\log(x)(bd-ae)}{a^2}-\frac{d}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^3)/(x^4*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -d/(3*a*x^3) - ((b^2*d - 2*a*c*d - a*b*e)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 -
4*a*c]])/(3*a^2*Sqrt[b^2 - 4*a*c]) - ((b*d - a*e)*Log[x])/a^2 + ((b*d - a*e
)*Log[a + b*x^3 + c*x^6])/(6*a^2)
```

Rule 1474

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{d+ex^3}{x^4\left(a+bx^3+cx^6\right)}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{d+ex}{x^2\left(a+bx+cx^2\right)}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{d}{ax^2}+\frac{-bd+ae}{a^2x}+\frac{b^2d-acd-abe+c(bd-ae)x}{a^2\left(a+bx+cx^2\right)}\right)dx, x, x^3\right)$$
$$= -\frac{d}{3ax^3}-\frac{(bd-ae)\log(x)}{a^2}+\frac{\text{Subst}\left(\int \frac{b^2d-acd-abe+c(bd-ae)x}{a+bx+cx^2}\,dx, x, x^3\right)}{3a^2}$$
$$= -\frac{d}{3ax^3}-\frac{(bd-ae)\log(x)}{a^2}+\frac{(bd-ae)\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^3\right)}{6a^2}+\frac{\left(b^2d-2acd-abe\right)\text{Subs}}{6a^2}$$
$$= -\frac{d}{3ax^3}-\frac{(bd-ae)\log(x)}{a^2}+\frac{(bd-ae)\log\left(a+bx^3+cx^6\right)}{6a^2}-\frac{\left(b^2d-2acd-abe\right)\text{Subst}\left(\int \frac{1}{b^2-4}\right.}{3a^2}$$
$$= -\frac{d}{3ax^3}-\frac{\left(b^2d-2acd-abe\right)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a^2\sqrt{b^2-4ac}}-\frac{(bd-ae)\log(x)}{a^2}+\frac{(bd-ae)\log\left(a+bx^3+c\right.}{6a^2}$$

Mathematica [C] time = 0.0521102, size = 130, normalized size = 1.16

$$\frac{\text{RootSum}\left[\#1^3b+\#1^6c+a\&,\frac{-\#1^3ace\log(x-\#1)+\#1^3bcd\log(x-\#1)-abe\log(x-\#1)-acd\log(x-\#1)+b^2d\log(x-\#1)}{2\#1^3c+b}\&\right]}{3a^2}+\frac{\log(x)(ae-bd)}{a^2}-\frac{}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^3)/(x^4*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -d/(3*a*x^3) + ((-(b*d) + a*e)*Log[x])/a^2 + RootSum[a + b*#1^3 + c*#1^6 &
, (b^2*d*Log[x - #1] - a*c*d*Log[x - #1] - a*b*e*Log[x - #1] + b*c*d*Log[x
- #1]*#1^3 - a*c*e*Log[x - #1]*#1^3)/(b + 2*c*#1^3) & ]/(3*a^2)
```

Maple [A] time = 0.009, size = 191, normalized size = 1.7

$$-\frac{d}{3\,ax^3}+\frac{\ln(x)\,e}{a}-\frac{b\ln(x)\,d}{a^2}-\frac{\ln\left(cx^6+bx^3+a\right)e}{6\,a}+\frac{\ln\left(cx^6+bx^3+a\right)bd}{6\,a^2}-\frac{be}{3\,a}\arctan\left(\left(2\,cx^3+b\right)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^3+d)/x^4/(c*x^6+b*x^3+a),x)
```

```
[Out] -1/3*d/a/x^3+1/a*ln(x)*e-1/a^2*ln(x)*b*d-1/6/a*ln(c*x^6+b*x^3+a)*e+1/6/a^2*
ln(c*x^6+b*x^3+a)*b*d-1/3/a/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2
)^(1/2))*b*e-2/3/a/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2))*
c*d+1/3/a^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2))*b^2*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/x^4/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 4.11086, size = 845, normalized size = 7.54

$$\left[\frac{\left(abe-\left(b^2-2\,ac\right)d\right)\sqrt{b^2-4\,ac}x^3\log\left(\frac{2\,c^2x^6+2\,bcx^3+b^2-2\,ac+\left(2\,cx^3+b\right)\sqrt{b^2-4\,ac}}{cx^6+bx^3+a}\right)+\left(\left(b^3-4\,abc\right)d-\left(ab^2-4\,a^2c\right)e\right)x^3\log\left(cx^6\right.}{6\left(a^2b^2-4\,a^3c\right)x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/x^4/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*((a*b*e - (b^2 - 2*a*c)*d)*sqrt(b^2 - 4*a*c)*x^3*log((2*c^2*x^6 + 2*b*
c*x^3 + b^2 - 2*a*c + (2*c*x^3 + b)*sqrt(b^2 - 4*a*c))/(c*x^6 + b*x^3 + a))
 + ((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^3*log(c*x^6 + b*x^3 + a) - 6
*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^3*log(x) - 2*(a*b^2 - 4*a^2*c)
*d)/((a^2*b^2 - 4*a^3*c)*x^3), 1/6*(2*(a*b*e - (b^2 - 2*a*c)*d)*sqrt(-b^2 +
 4*a*c)*x^3*arctan(-(2*c*x^3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + ((b^3
 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^3*log(c*x^6 + b*x^3 + a) - 6*((b^3 -
 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^3*log(x) - 2*(a*b^2 - 4*a^2*c)*d)/((a^
2*b^2 - 4*a^3*c)*x^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**3+d)/x**4/(c*x**6+b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36632, size = 173, normalized size = 1.54

$$\frac{(bd-ae)\log\left(cx^6+bx^3+a\right)}{6\,a^2}-\frac{(bd-ae)\log\left(|x|\right)}{a^2}+\frac{\left(b^2d-2\,acd-abe\right)\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}a^2}+\frac{bdx^3-ax^3e-ad}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/x^4/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/6*(b*d - a*e)*log(c*x^6 + b*x^3 + a)/a^2 - (b*d - a*e)*log(abs(x))/a^2 +
1/3*(b^2*d - 2*a*c*d - a*b*e)*arctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqr
t(-b^2 + 4*a*c)*a^2) + 1/3*(b*d*x^3 - a*x^3*e - a*d)/(a^2*x^3)
```

3.14 $\int \frac{x^4(d+ex^3)}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=723

$$\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\,2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right.}{}$$

```
[Out] (e*x^2)/(2*c) - ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*
ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/
(2^(2/3)*Sqrt[3]*c^(5/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)) - ((c*d - b*e + (b*
c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)
/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*c^(5/3)*(b + Sqr
t[b^2 - 4*a*c])^(1/3)) - ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 -
 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(2/3)
*c^(5/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)) - ((c*d - b*e + (b*c*d - b^2*e + 2*
a*c*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/
3)*x])/(3*2^(2/3)*c^(5/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)) + ((c*d - b*e - (b
*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3
) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])
/(6*2^(2/3)*c^(5/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + ((c*d - b*e + (b*c*d -
 b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^
(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^
(2/3)*c^(5/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3))
```

Rubi [A] time = 1.81328, antiderivative size = 723, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {1502, 1510, 292, 31, 634, 617, 204, 628}

$$\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\,2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] (e*x^2)/(2*c) - ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*
ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/
(2^(2/3)*Sqrt[3]*c^(5/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)) - ((c*d - b*e + (b*
c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)
/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*c^(5/3)*(b + Sqr
t[b^2 - 4*a*c])^(1/3)) - ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 -
 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(2/3)
*c^(5/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)) - ((c*d - b*e + (b*c*d - b^2*e + 2*
a*c*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/
3)*x])/(3*2^(2/3)*c^(5/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)) + ((c*d - b*e - (b
*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3
) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])
/(6*2^(2/3)*c^(5/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + ((c*d - b*e + (b*c*d -
 b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^
(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^
(2/3)*c^(5/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3))
```

<u>Rule 1502</u>

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(e*f^(n - 1)*(f*x)^(m - n + 1)*(a
+ b*x^n + c*x^(2*n))^(p + 1))/(c*(m + n*(2*p + 1) + 1)), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

<u>Rule 1510</u>

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

<u>Rule 292</u>

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

<u>Rule 31</u>

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^4\left(d+ex^3\right)}{a+bx^3+cx^6}\,dx = \frac{ex^2}{2c} - \frac{\int \frac{x\left(2ae-2(cd-be)x^3\right)}{a+bx^3+cx^6}\,dx}{2c}$$

$$= \frac{ex^2}{2c} + \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{x}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2c} + \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{x}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2c}$$

$$= \frac{ex^2}{2c} - \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}\,dx}{3\,2^{2/3}c^{4/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{\ldots}}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2\ldots}}}{\sqrt[3]{2}}\ldots}}{3\,2^{2/3}c^{4/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}$$

$$= \frac{ex^2}{2c} - \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\,2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} - \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\ldots}\right.}{3\,2^{2/3}c^{5/3}\sqrt[3]{b+\sqrt{\ldots}}}$$

$$= \frac{ex^2}{2c} - \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\,2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} - \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\ldots}\right.}{3\,2^{2/3}c^{5/3}\sqrt[3]{b+\sqrt{\ldots}}}$$

$$= \frac{ex^2}{2c} - \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} - \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{5/3}\sqrt[3]{b+\sqrt{b^2-4ac}}} \ldots$$

Mathematica [C] time = 0.0489129, size = 88, normalized size = 0.12

$$\frac{3ex^2-2\text{RootSum}\left[\#1^3b+\#1^6c+a\&,\frac{\#1^3be\log(x-\#1)+\#1^3(-c)d\log(x-\#1)+ae\log(x-\#1)}{2\#1^4c+\#1b}\&\right]}{6c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] (3*e*x^2 - 2*RootSum[a + b*#1^3 + c*#1^6 & , (a*e*Log[x - #1] - c*d*Log[x -
 #1]*#1^3 + b*e*Log[x - #1]*#1^3)/(b*#1 + 2*c*#1^4) & ])/(6*c)
```

Maple [C] time = 0.007, size = 70, normalized size = 0.1

$$\frac{e x^2}{2c} - \frac{1}{3c} \sum_{\_R=\mathrm{RootOf}\left(\_Z^6 c+\_Z^3 b+a\right)} \frac{\left((be-cd)\_R^4 + \_R\, ae\right)\ln\left(x-\_R\right)}{2\_R^5 c + \_R^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x^3+d)/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/2*e*x^2/c-1/3/c*sum(((b*e-c*d)*_R^4+_R*a*e)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R
=RootOf(_Z^6*c+_Z^3*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{e x^2}{2c} - \frac{-\int \frac{(cd-be)x^4-aex}{cx^6+bx^3+a}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/2*e*x^2/c - integrate(-((c*d - b*e)*x^4 - a*e*x)/(c*x^6 + b*x^3 + a), x)/
c
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x**3+d)/(c*x**6+b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.15 $\int \frac{x^3\left(d+ex^3\right)}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=718

$$-\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+c\right.}{}$$

```
[Out] (e*x)/c - ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan
[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/
3)*Sqrt[3]*c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - ((c*d - b*e + (b*c*d -
b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b +
Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(4/3)*(b + Sqrt[b^2
- 4*a*c])^(2/3)) + ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c
])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/3)*c^(4/
3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + ((c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)
/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])
/(3*2^(1/3)*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) - ((c*d - b*e - (b*c*d -
 b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^
(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^
(1/3)*c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - ((c*d - b*e + (b*c*d - b^2*e
 + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*
c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*
c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3))
```

Rubi [A] time = 1.45742, antiderivative size = 718, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {1502, 1422, 200, 31, 634, 617, 204, 628}

$$-\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] (e*x)/c - ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan
[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/
3)*Sqrt[3]*c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - ((c*d - b*e + (b*c*d -
b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b +
Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(4/3)*(b + Sqrt[b^2
- 4*a*c])^(2/3)) + ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c
])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/3)*c^(4/
3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + ((c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)
/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])
/(3*2^(1/3)*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) - ((c*d - b*e - (b*c*d -
 b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^
(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^
(1/3)*c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - ((c*d - b*e + (b*c*d - b^2*e
 + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*
c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*
c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3))
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(e*f^(n - 1)*(f*x)^(m - n + 1)*(a
+ b*x^n + c*x^(2*n))^(p + 1))/(c*(m + n*(2*p + 1) + 1)), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^3\left(d+ex^3\right)}{a+bx^3+cx^6}\,dx = \frac{ex}{c} - \frac{\int \frac{ae-(cd-be)x^3}{a+bx^3+cx^6}\,dx}{c}$$

$$= \frac{ex}{c} + \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2c} + \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2c}$$

$$= \frac{ex}{c} + \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}\,dx}{3\sqrt[3]{2}c\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{2^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}-}{\frac{\left(b-\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b-\sqrt{b^2-}}}{\sqrt[3]{2}}}}{3\sqrt[3]{2}c\left(b-\sqrt{b^2-4ac}\right)^{2/3}}$$

$$= \frac{ex}{c} + \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+}\right.}{3\sqrt[3]{2}c^{4/3}\left(b+\sqrt{b^2-}\right.}$$

$$= \frac{ex}{c} + \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+}\right.}{3\sqrt[3]{2}c^{4/3}\left(b+\sqrt{b^2-}\right.}$$

$$= \frac{ex}{c} - \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2/3}} +$$

Mathematica [C] time = 0.0520284, size = 88, normalized size = 0.12

$$\frac{ex}{c} - \frac{\text{RootSum}\left[\#1^3b+\#1^6c+a\&,\frac{\#1^3be\log(x-\#1)+\#1^3(-c)d\log(x-\#1)+ae\log(x-\#1)}{\#1^2b+2\#1^5c}\&\right]}{3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] (e*x)/c - RootSum[a + b*#1^3 + c*#1^6 & , (a*e*Log[x - #1] - c*d*Log[x - #1
]*#1^3 + b*e*Log[x - #1]*#1^3)/(b*#1^2 + 2*c*#1^5) & ]/(3*c)
```

Maple [C] time = 0.006, size = 67, normalized size = 0.1

$$\frac{ex}{c}+\frac{1}{3c}\sum_{\_R=\mathrm{RootOf}\left(\_Z^6c+\_Z^3b+a\right)}\frac{\left((-be+cd)\,\_R^3-ae\right)\ln\left(x-\_R\right)}{2\,\_R^5c+\_R^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^3+d)/(c*x^6+b*x^3+a),x)
```

```
[Out] e*x/c+1/3/c*sum(((-b*e+c*d)*_R^3-a*e)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(
_Z^6*c+_Z^3*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{ex}{c}-\frac{-\int\frac{(cd-be)x^3-ae}{cx^6+bx^3+a}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] e*x/c - integrate(-((c*d - b*e)*x^3 - a*e)/(c*x^6 + b*x^3 + a), x)/c
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**3+d)/(c*x**6+b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [F(-2)]   time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.16 $\int \frac{x\left(d+ex^3\right)}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=634

$$\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4a}\right.}{6\ 2^{2/3}c^{2/3}\sqrt[3]{}}$$

```
[Out] -(((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/
(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*c^(2/3)*(b - Sqrt
[b^2 - 4*a*c])^(1/3))) - ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 -
 (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sq
rt[3]*c^(2/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)) - ((e + (2*c*d - b*e)/Sqrt[b^2
 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(2/
3)*c^(2/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)) - ((e - (2*c*d - b*e)/Sqrt[b^2 -
4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(2/3)*
c^(2/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)) + ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a
*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*
a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*c^(2/3)*(b - Sqrt[b^2 - 4*
a*c])^(1/3)) + ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4
*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^
(2/3)*x^2])/(6*2^(2/3)*c^(2/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3))
```

Rubi [A] time = 0.727749, antiderivative size = 634, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {1510, 292, 31, 634, 617, 204, 628}

$$\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4a}\right.}{6\ 2^{2/3}c^{2/3}\sqrt[3]{}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] -(((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/
(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*c^(2/3)*(b - Sqrt
```

```
[b^2 - 4*a*c])^(1/3))) - ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 -
 (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sq
rt[3]*c^(2/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)) - ((e + (2*c*d - b*e)/Sqrt[b^2
 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(2/
3)*c^(2/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)) - ((e - (2*c*d - b*e)/Sqrt[b^2 -
4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(2/3)*
c^(2/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)) + ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a
*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*
a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*c^(2/3)*(b - Sqrt[b^2 - 4*
a*c])^(1/3)) + ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4
*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^
(2/3)*x^2])/(6*2^(2/3)*c^(2/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3))
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x\left(d+ex^3\right)}{a+bx^3+cx^6}\,dx = \frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{x}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx + \frac{1}{2}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{x}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,d$$

$$= -\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}\,dx}{3\;2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}} + \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}\,dx}{3\;2^{2/3}\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}} - \frac{\left(e+\cdots\right.}{3\cdots}$$

$$= -\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\;2^{2/3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} - \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\;2^{2/3}c^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

$$= -\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\;2^{2/3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} - \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\;2^{2/3}c^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

$$= -\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} - \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}c^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}} - \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{\cdots}\right.}{3\;2^{2/3}c^{2/3}\cdots}$$

Mathematica [C] time = 0.0312528, size = 59, normalized size = 0.09

$$\frac{1}{3}\text{RootSum}\left[\#1^3b+\#1^6c+a\&,\frac{\#1^3e\log(x-\#1)+d\log(x-\#1)}{2\#1^4c+\#1b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] RootSum[a + b*#1^3 + c*#1^6 & , (d*Log[x - #1] + e*Log[x - #1]*#1^3)/(b*#1
+ 2*c*#1^4) & ]/3
```

Maple [C] time = 0.002, size = 49, normalized size = 0.1

$$\frac{1}{3}\sum_{\_R=\text{RootOf}\left(\_Z^6c+\_Z^3b+a\right)}\frac{\left(\_R^4e+\_R\,d\right)\ln\left(x-\_R\right)}{2\_R^5c+\_R^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^3+d)/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/3*sum((_R^4*e+_R*d)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3*b+a)
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(ex^3+d\right)x}{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^3 + d)*x/(c*x^6 + b*x^3 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="fricas")

[Out] Timed out
```

Sympy [A] time = 133.311, size = 920, normalized size = 1.45

$$\mathrm{RootSum}\left(t^6\left(46656a^4c^5-34992a^3b^2c^4+8748a^2b^4c^3-729ab^6c^2\right)+t^3\left(-432a^3bc^2e^3+1296a^3c^3de^2+216a^2b^3ce^3-6\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**3+d)/(c*x**6+b*x**3+a),x)

[Out] RootSum(_t**6*(46656*a**4*c**5 - 34992*a**3*b**2*c**4 + 8748*a**2*b**4*c**3
 - 729*a*b**6*c**2) + _t**3*(-432*a**3*b*c**2*e**3 + 1296*a**3*c**3*d*e**2
+ 216*a**2*b**3*c*e**3 - 648*a**2*b**2*c**2*d*e**2 - 432*a**2*c**4*d**3 - 2
7*a*b**5*e**3 + 81*a*b**4*c*d*e**2 + 216*a*b**2*c**3*d**3 - 27*b**4*c**2*d*
*3) + a**3*e**6 - 3*a**2*b*d*e**5 + 3*a**2*c*d**2*e**4 + 3*a*b**2*d**2*e**4
 - 6*a*b*c*d**3*e**3 + 3*a*c**2*d**4*e**2 - b**3*d**3*e**3 + 3*b**2*c*d**4*
e**2 - 3*b*c**2*d**5*e + c**3*d**6, Lambda(_t, _t*log(x + (15552*_t**5*a**5
*c**5*e**2 - 11664*_t**5*a**4*b**2*c**4*e**2 - 15552*_t**5*a**4*c**6*d**2 +
 2916*_t**5*a**3*b**4*c**3*e**2 + 11664*_t**5*a**3*b**2*c**5*d**2 - 243*_t*
*5*a**2*b**6*c**2*e**2 - 2916*_t**5*a**2*b**4*c**4*d**2 + 243*_t**5*a*b**6*
c**3*d**2 - 108*_t**2*a**4*b*c**2*e**5 + 360*_t**2*a**4*c**3*d*e**4 + 63*_t
**2*a**3*b**3*c*e**5 - 270*_t**2*a**3*b**2*c**2*d*e**4 + 360*_t**2*a**3*b*c
**3*d**2*e**3 - 720*_t**2*a**3*c**4*d**3*e**2 - 9*_t**2*a**2*b**5*e**5 + 45
*_t**2*a**2*b**4*c*d*e**4 - 90*_t**2*a**2*b**3*c**2*d**2*e**3 + 180*_t**2*a
**2*b**2*c**3*d**3*e**2 + 180*_t**2*a**2*b*c**4*d**4*e + 72*_t**2*a**2*c**5
*d**5 - 45*_t**2*a*b**3*c**3*d**4*e - 54*_t**2*a*b**2*c**4*d**5 + 9*_t**2*b
**4*c**3*d**5)/(2*a**4*c*e**7 - a**3*b**2*e**7 - a**3*b*c*d*e**6 - 2*a**3*c
**2*d**2*e**5 + 2*a**2*b**3*d*e**6 - 6*a**2*b**2*c*d**2*e**5 + 15*a**2*b*c*
*2*d**3*e**4 - 10*a**2*c**3*d**4*e**3 - a*b**4*d**2*e**5 + 5*a*b**3*c*d**3*
e**4 - 15*a*b**2*c**2*d**4*e**3 + 17*a*b*c**3*d**5*e**2 - 6*a*c**4*d**6*e +
 b**3*c**2*d**5*e**2 - 2*b**2*c**3*d**6*e + b*c**4*d**7))))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^3 + d)x}{cx^6 + bx^3 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="giac")

[Out] integrate((e*x^3 + d)*x/(c*x^6 + b*x^3 + a), x)
```

3.17 $\int \frac{d+ex^3}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=634

$$-\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-}}\right.}{6\sqrt[3]{2}\sqrt[3]{c}\left(\sqrt{}\right.}$$

```
[Out] -(((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/
(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(1/3)*(b - Sqrt
[b^2 - 4*a*c])^(2/3))) - ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 -
 (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sq
rt[3]*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) + ((e + (2*c*d - b*e)/Sqrt[b^2
 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/
3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + ((e - (2*c*d - b*e)/Sqrt[b^2 -
4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/3)*
c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) - ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a
*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*
a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*
a*c])^(2/3)) - ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4
*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^
(2/3)*x^2])/(6*2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3))
```

Rubi [A] time = 0.653755, antiderivative size = 634, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1422, 200, 31, 634, 617, 204, 628}

$$-\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-}}\right.}{6\sqrt[3]{2}\sqrt[3]{c}\left(\sqrt{}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^3)/(a + b*x^3 + c*x^6),x]
```


```
[Out] -(((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/
(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(1/3)*(b - Sqrt
```

```
[b^2 - 4*a*c])^(2/3))) - ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 -
 (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sq
rt[3]*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) + ((e + (2*c*d - b*e)/Sqrt[b^2
 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/
3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + ((e - (2*c*d - b*e)/Sqrt[b^2 -
4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/3)*
c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) - ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a
*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*
a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*
a*c])^(2/3)) - ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4
*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^
(2/3)*x^2])/(6*2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3))
```

<u>Rule 1422</u>

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

<u>Rule 200</u>

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

<u>Rule 31</u>

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

<u>Rule 634</u>

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

<u>Rule 617</u>

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{d+ex^3}{a+bx^3+cx^6}\,dx = \frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx + \frac{1}{2}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx$$

$$= \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}\,dx}{3\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{2^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}-\sqrt[3]{c}x}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4ac}}x}{\sqrt[3]{2}}+c^{2/3}x^2}\,dx}{3\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(e+\frac{2c\ldots}{\sqrt{b^{\ldots}}}\right.\ldots}{3\sqrt[3]{\ldots}}$$

$$= \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\left(b+\sqrt{b^2-4ac}\right)^{2/3}} \ldots$$

$$= \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\left(b+\sqrt{b^2-4ac}\right)^{2/3}} \ldots$$

$$= -\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\left(b+\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{\ldots}\right.}{3\sqrt[3]{2}\sqrt[3]{c}\left(b\ldots\right.}$$

Mathematica [C] time = 0.0310151, size = 61, normalized size = 0.1

$$\frac{1}{3}\text{RootSum}\left[\#1^3b+\#1^6c+a\&,\frac{\#1^3e\log(x-\#1)+d\log(x-\#1)}{\#1^2b+2\#1^5c}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^3)/(a + b*x^3 + c*x^6),x]
```

```
[Out] RootSum[a + b*#1^3 + c*#1^6 & , (d*Log[x - #1] + e*Log[x - #1]*#1^3)/(b*#1^
2 + 2*c*#1^5) & ]/3
```

Maple [C] time = 0.003, size = 47, normalized size = 0.1

$$\frac{1}{3}\sum_{\_R=\text{RootOf}\left(\_Z^6c+\_Z^3b+a\right)}\frac{\left(\_R^3e+d\right)\ln\left(x-\_R\right)}{2\_R^5c+\_R^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^3+d)/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/3*sum((_R^3*e+d)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{ex^3+d}{cx^6+bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^3 + d)/(c*x^6 + b*x^3 + a), x)
```

Fricas [B] time = 108.462, size = 28045, normalized size = 44.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(3)*(1/2)^(1/3)*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 + (a^2*b^2*c - 4
*a^3*c^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 +
4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a
^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a
^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^
5)))/(a^2*b^2*c - 4*a^3*c^2))^(1/3)*arctan(-1/6*(2*(1/2)^(2/3)*(sqrt(3)*((a
^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)*d^2 - (a^3*b^6*c
 - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4)*e^2)*x*sqrt(-(12*a^4*b*c*d
*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2
 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*
c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2
 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)) - sqrt(3)*((b^5*c^2 - 6*a
*b^3*c^3 + 8*a^2*b*c^4)*d^5 - (7*a*b^4*c^2 - 36*a^2*b^2*c^3 + 32*a^3*c^4)*d
^4*e + (a*b^5*c + 12*a^2*b^3*c^2 - 64*a^3*b*c^3)*d^3*e^2 - 4*(a^2*b^4*c + 2
*a^3*b^2*c^2 - 24*a^4*c^3)*d^2*e^3 + 10*(a^3*b^3*c - 4*a^4*b*c^2)*d*e^4 - (
a^3*b^4 - 4*a^4*b^2*c)*e^5)*x)*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 + (a^2*b^2
*c - 4*a^3*c^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*
c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2
 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c
 + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*
a^7*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^(2/3) - (1/2)^(1/6)*(sqrt(3)*((a^2*b^6*
c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)*d^2 - (a^3*b^6*c - 12*a
^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4)*e^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^
4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*
b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^
3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5
*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)) - sqrt(3)*((b^5*c^2 - 6*a*b^3*c^3
+ 8*a^2*b*c^4)*d^5 - (7*a*b^4*c^2 - 36*a^2*b^2*c^3 + 32*a^3*c^4)*d^4*e + (a
*b^5*c + 12*a^2*b^3*c^2 - 64*a^3*b*c^3)*d^3*e^2 - 4*(a^2*b^4*c + 2*a^3*b^2*
c^2 - 24*a^4*c^3)*d^2*e^3 + 10*(a^3*b^3*c - 4*a^4*b*c^2)*d*e^4 - (a^3*b^4 -
 4*a^4*b^2*c)*e^5))*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 + (a^2*b^2*c - 4*a^3*
c^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2
*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^
3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^
2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))/
(a^2*b^2*c - 4*a^3*c^2))^(2/3)*sqrt((2*(a^4*b*e^7 - (b^2*c^3 - 2*a*c^4)*d^7
 + (2*b^3*c^2 - a*b*c^3)*d^6*e - (b^4*c + 6*a*b^2*c^2 + 2*a^2*c^3)*d^5*e^2
+ 5*(a*b^3*c + 3*a^2*b*c^2)*d^4*e^3 - 5*(3*a^2*b^2*c + 2*a^3*c^2)*d^3*e^4 +
 (a^2*b^3 + 17*a^3*b*c)*d^2*e^5 - 2*(a^3*b^2 + 3*a^4*c)*d*e^6)*x^2 - (1/2)^
(2/3)*((b^6*c - 8*a*b^4*c^2 + 20*a^2*b^2*c^3 - 16*a^3*c^4)*d^5 - 5*(a*b^5*c
```

- 6*a^2*b^3*c^2 + 8*a^3*b*c^3)*d^4*e + 2*(7*a^2*b^4*c - 36*a^3*b^2*c^2 + 32*a^4*c^3)*d^3*e^2 - (a^2*b^5 + 12*a^3*b^3*c - 64*a^4*b*c^2)*d^2*e^3 + 2*(a^3*b^4 + 2*a^4*b^2*c - 24*a^5*c^2)*d*e^4 - 2*(a^4*b^3 - 4*a^5*b*c)*e^5 - ((a^2*b^7*c - 12*a^3*b^5*c^2 + 48*a^4*b^3*c^3 - 64*a^5*b*c^4)*d^2 - 2*(a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4)*d*e)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 + (a^2*b^2*c - 4*a^3*c^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^(2/3) + (1/2)^(1/3)*(((a^2*b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*d^3 - (a^2*b^6*c - 6*a^3*b^4*c^2 + 32*a^5*c^4)*d^2*e + 3*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d*e^2 - 2*(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*e^3)*x*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)) - ((b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^6 - (b^5*c - 3*a*b^3*c^2 - 4*a^2*b*c^3)*d^5*e + 4*(a*b^4*c - 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^4*e^2 - 10*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e^3 + (a^2*b^4 + 2*a^3*b^2*c - 24*a^4*c^2)*d^2*e^4 - (a^3*b^3 - 4*a^4*b*c)*d*e^5)*x)*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 + (a^2*b^2*c - 4*a^3*c^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^(1/3))/(a^4*b*e^7 - (b^2*c^3 - 2*a*c^4)*d^7 + (2*b^3*c^2 - a*b*c^3)*d^6*e - (b^4*c + 6*a*b^2*c^2 + 2*a^2*c^3)*d^5*e^2 + 5*(a*b^3*c + 3*a^2*b*c^2)*d^4*e^3 - 5*(3*a^2*b^2*c + 2*a^3*c^2)*d^3*e^4 + (a^2*b^3 + 17*a^3*b*c)*d^2*e^5 - 2*(a^3*b^2 + 3*a^4*c)*d*e^6)) - 2*sqrt(3)*(a^4*b*e^7 - (b^2*c^3 - 2*a*c^4)*d^7 + (2*b^3*c^2 - a*b*c^3)*d^6*e - (b^4*c + 6*a*b^2*c^2 + 2*a^2*c^3)*d^5*e^2 + 5*(a*b^3*c + 3*a^2*b*c^2)*d^4*e^3 - 5*(3*a^2*b^2*c + 2*a^3*c^2)*d^3*e^4 + (a^2*b^3 + 17*a^3*b*c)*d^2*e^5 - 2*(a^3*b^2 + 3*a^4*c)*d*e^6))/(a^4*b*e^7 - (b^2*c^3 - 2*a*c^4)*d^7 + (2*b^3*c^2 - a*b*c^3)*d^6*e - (b^4*c + 6*a*b^2*c^2 + 2*a^2*c^3)*d^5*e^2 + 5*(a*b^3*c + 3*a^2*b*c^2)*d^4*e^3 - 5*(3*a^2*b^2*c + 2*a^3*c^2)*d^3*e^4 + (a^2*b^3 + 17*a^3*b*c)*d^2*e^5 - 2*(a^3*b^2 + 3*a^4*c)*d*e^6)) + 2/3*sqrt(3)*(1/2)^(1/3)*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 - (a^2*b^2*c - 4*a^3*c^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^(1/3)*arctan

```
(-1/6*(2*(1/2)^(2/3)*(sqrt(3)*((a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c
^4 - 64*a^5*c^5)*d^2 - (a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^
6*c^4)*e^2)*x*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^
3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 -
 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c +
 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^
7*c^5)) + sqrt(3)*((b^5*c^2 - 6*a*b^3*c^3 + 8*a^2*b*c^4)*d^5 - (7*a*b^4*c^2
 - 36*a^2*b^2*c^3 + 32*a^3*c^4)*d^4*e + (a*b^5*c + 12*a^2*b^3*c^2 - 64*a^3*
b*c^3)*d^3*e^2 - 4*(a^2*b^4*c + 2*a^3*b^2*c^2 - 24*a^4*c^3)*d^2*e^3 + 10*(a
^3*b^3*c - 4*a^4*b*c^2)*d*e^4 - (a^3*b^4 - 4*a^4*b^2*c)*e^5)*x)*((b*c*d^3 -
 3*a*c*d^2*e + a^2*e^3 - (a^2*b^2*c - 4*a^3*c^2)*sqrt(-(12*a^4*b*c*d*e^5 -
a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^
2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*
a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a
^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^(2/3)
- (1/2)^(1/6)*(sqrt(3)*((a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64
*a^5*c^5)*d^2 - (a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4)*
e^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2
*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^
3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^
2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)) +
 sqrt(3)*((b^5*c^2 - 6*a*b^3*c^3 + 8*a^2*b*c^4)*d^5 - (7*a*b^4*c^2 - 36*a^2
*b^2*c^3 + 32*a^3*c^4)*d^4*e + (a*b^5*c + 12*a^2*b^3*c^2 - 64*a^3*b*c^3)*d^
3*e^2 - 4*(a^2*b^4*c + 2*a^3*b^2*c^2 - 24*a^4*c^3)*d^2*e^3 + 10*(a^3*b^3*c
- 4*a^4*b*c^2)*d*e^4 - (a^3*b^4 - 4*a^4*b^2*c)*e^5))*((b*c*d^3 - 3*a*c*d^2*
e + a^2*e^3 - (a^2*b^2*c - 4*a^3*c^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6
 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^
5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*
d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3
+ 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^(2/3)*sqrt((2*(a^
4*b*e^7 - (b^2*c^3 - 2*a*c^4)*d^7 + (2*b^3*c^2 - a*b*c^3)*d^6*e - (b^4*c +
6*a*b^2*c^2 + 2*a^2*c^3)*d^5*e^2 + 5*(a*b^3*c + 3*a^2*b*c^2)*d^4*e^3 - 5*(3
*a^2*b^2*c + 2*a^3*c^2)*d^3*e^4 + (a^2*b^3 + 17*a^3*b*c)*d^2*e^5 - 2*(a^3*b
^2 + 3*a^4*c)*d*e^6)*x^2 - (1/2)^(2/3)*((b^6*c - 8*a*b^4*c^2 + 20*a^2*b^2*c
^3 - 16*a^3*c^4)*d^5 - 5*(a*b^5*c - 6*a^2*b^3*c^2 + 8*a^3*b*c^3)*d^4*e + 2*
(7*a^2*b^4*c - 36*a^3*b^2*c^2 + 32*a^4*c^3)*d^3*e^2 - (a^2*b^5 + 12*a^3*b^3
*c - 64*a^4*b*c^2)*d^2*e^3 + 2*(a^3*b^4 + 2*a^4*b^2*c - 24*a^5*c^2)*d*e^4 -
 2*(a^4*b^3 - 4*a^5*b*c)*e^5 + ((a^2*b^7*c - 12*a^3*b^5*c^2 + 48*a^4*b^3*c^
3 - 64*a^5*b*c^4)*d^2 - 2*(a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64
*a^6*c^4)*d*e)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c
^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2
- 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c
+ 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a
^7*c^5)))*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 - (a^2*b^2*c - 4*a^3*c^2)*sqrt(
-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6
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+ 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2
 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4
)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c
 - 4*a^3*c^2))^(2/3) - (1/2)^(1/3)*(((a^2*b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4*
b*c^4)*d^3 - (a^2*b^6*c - 6*a^3*b^4*c^2 + 32*a^5*c^4)*d^2*e + 3*(a^3*b^5*c
- 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d*e^2 - 2*(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a
^6*c^3)*e^3)*x*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c
^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2
- 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c
+ 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a
^7*c^5)) + ((b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^6 - (b^5*c - 3*a*b^3*c^2
- 4*a^2*b*c^3)*d^5*e + 4*(a*b^4*c - 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^4*e^2 - 10
*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e^3 + (a^2*b^4 + 2*a^3*b^2*c - 24*a^4*c^2)*d
^2*e^4 - (a^3*b^3 - 4*a^4*b*c)*d*e^5)*x)*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3
- (a^2*b^2*c - 4*a^3*c^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2
- 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a
^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*
(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2
*c^4 - 64*a^7*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^(1/3))/(a^4*b*e^7 - (b^2*c^3
- 2*a*c^4)*d^7 + (2*b^3*c^2 - a*b*c^3)*d^6*e - (b^4*c + 6*a*b^2*c^2 + 2*a^2
*c^3)*d^5*e^2 + 5*(a*b^3*c + 3*a^2*b*c^2)*d^4*e^3 - 5*(3*a^2*b^2*c + 2*a^3*
c^2)*d^3*e^4 + (a^2*b^3 + 17*a^3*b*c)*d^2*e^5 - 2*(a^3*b^2 + 3*a^4*c)*d*e^6
)) + 2*sqrt(3)*(a^4*b*e^7 - (b^2*c^3 - 2*a*c^4)*d^7 + (2*b^3*c^2 - a*b*c^3)
*d^6*e - (b^4*c + 6*a*b^2*c^2 + 2*a^2*c^3)*d^5*e^2 + 5*(a*b^3*c + 3*a^2*b*c
^2)*d^4*e^3 - 5*(3*a^2*b^2*c + 2*a^3*c^2)*d^3*e^4 + (a^2*b^3 + 17*a^3*b*c)*
d^2*e^5 - 2*(a^3*b^2 + 3*a^4*c)*d*e^6))/(a^4*b*e^7 - (b^2*c^3 - 2*a*c^4)*d^
7 + (2*b^3*c^2 - a*b*c^3)*d^6*e - (b^4*c + 6*a*b^2*c^2 + 2*a^2*c^3)*d^5*e^2
 + 5*(a*b^3*c + 3*a^2*b*c^2)*d^4*e^3 - 5*(3*a^2*b^2*c + 2*a^3*c^2)*d^3*e^4
+ (a^2*b^3 + 17*a^3*b*c)*d^2*e^5 - 2*(a^3*b^2 + 3*a^4*c)*d*e^6)) - 1/6*(1/2
)^(1/3)*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 + (a^2*b^2*c - 4*a^3*c^2)*sqrt(-(
12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 +
6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 +
 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/
(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c -
 4*a^3*c^2))^(1/3)*log(2*(a^4*b*e^7 - (b^2*c^3 - 2*a*c^4)*d^7 + (2*b^3*c^2
- a*b*c^3)*d^6*e - (b^4*c + 6*a*b^2*c^2 + 2*a^2*c^3)*d^5*e^2 + 5*(a*b^3*c +
 3*a^2*b*c^2)*d^4*e^3 - 5*(3*a^2*b^2*c + 2*a^3*c^2)*d^3*e^4 + (a^2*b^3 + 17
*a^3*b*c)*d^2*e^5 - 2*(a^3*b^2 + 3*a^4*c)*d*e^6)*x^2 - (1/2)^(2/3)*((b^6*c
- 8*a*b^4*c^2 + 20*a^2*b^2*c^3 - 16*a^3*c^4)*d^5 - 5*(a*b^5*c - 6*a^2*b^3*c
^2 + 8*a^3*b*c^3)*d^4*e + 2*(7*a^2*b^4*c - 36*a^3*b^2*c^2 + 32*a^4*c^3)*d^3
*e^2 - (a^2*b^5 + 12*a^3*b^3*c - 64*a^4*b*c^2)*d^2*e^3 + 2*(a^3*b^4 + 2*a^4
*b^2*c - 24*a^5*c^2)*d*e^4 - 2*(a^4*b^3 - 4*a^5*b*c)*e^5 - ((a^2*b^7*c - 12
*a^3*b^5*c^2 + 48*a^4*b^3*c^3 - 64*a^5*b*c^4)*d^2 - 2*(a^3*b^6*c - 12*a^4*b
^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4)*d*e)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^
2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^
```

```
3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*
c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4
*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 + (
a^2*b^2*c - 4*a^3*c^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4
*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*
b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^
3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^
4 - 64*a^7*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^(2/3) + (1/2)^(1/3)*(((a^2*b^5*c
^2 - 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*d^3 - (a^2*b^6*c - 6*a^3*b^4*c^2 + 32*a^
5*c^4)*d^2*e + 3*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d*e^2 - 2*(a^4*
b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*e^3)*x*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^
2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^
3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*
c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4
*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)) - ((b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)
*d^6 - (b^5*c - 3*a*b^3*c^2 - 4*a^2*b*c^3)*d^5*e + 4*(a*b^4*c - 3*a^2*b^2*c
^2 - 4*a^3*c^3)*d^4*e^2 - 10*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e^3 + (a^2*b^4 +
 2*a^3*b^2*c - 24*a^4*c^2)*d^2*e^4 - (a^3*b^3 - 4*a^4*b*c)*d*e^5)*x)*((b*c*
d^3 - 3*a*c*d^2*e + a^2*e^3 + (a^2*b^2*c - 4*a^3*c^2)*sqrt(-(12*a^4*b*c*d*e
^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 -
 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c
+ 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 -
 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^(
1/3)) - 1/6*(1/2)^(1/3)*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 - (a^2*b^2*c - 4*
a^3*c^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4
*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^
3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^
4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5
)))/(a^2*b^2*c - 4*a^3*c^2))^(1/3)*log(2*(a^4*b*e^7 - (b^2*c^3 - 2*a*c^4)*d
^7 + (2*b^3*c^2 - a*b*c^3)*d^6*e - (b^4*c + 6*a*b^2*c^2 + 2*a^2*c^3)*d^5*e^
2 + 5*(a*b^3*c + 3*a^2*b*c^2)*d^4*e^3 - 5*(3*a^2*b^2*c + 2*a^3*c^2)*d^3*e^4
 + (a^2*b^3 + 17*a^3*b*c)*d^2*e^5 - 2*(a^3*b^2 + 3*a^4*c)*d*e^6)*x^2 - (1/2
)^(2/3)*((b^6*c - 8*a*b^4*c^2 + 20*a^2*b^2*c^3 - 16*a^3*c^4)*d^5 - 5*(a*b^5
*c - 6*a^2*b^3*c^2 + 8*a^3*b*c^3)*d^4*e + 2*(7*a^2*b^4*c - 36*a^3*b^2*c^2 +
 32*a^4*c^3)*d^3*e^2 - (a^2*b^5 + 12*a^3*b^3*c - 64*a^4*b*c^2)*d^2*e^3 + 2*
(a^3*b^4 + 2*a^4*b^2*c - 24*a^5*c^2)*d*e^4 - 2*(a^4*b^3 - 4*a^5*b*c)*e^5 +
((a^2*b^7*c - 12*a^3*b^5*c^2 + 48*a^4*b^3*c^3 - 64*a^5*b*c^4)*d^2 - 2*(a^3*
b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4)*d*e)*sqrt(-(12*a^4*b*
c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*
c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b
^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*
c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))*((b*c*d^3 - 3*a*c*d^2
*e + a^2*e^3 - (a^2*b^2*c - 4*a^3*c^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^
6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d
^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)
```

*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^(2/3) - (1/2)^(1/3)*(((a^2*b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*d^3 - (a^2*b^6*c - 6*a^3*b^4*c^2 + 32*a^5*c^4)*d^2*e + 3*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d*e^2 - 2*(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*e^3)*x*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)) + ((b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^6 - (b^5*c - 3*a*b^3*c^2 - 4*a^2*b*c^3)*d^5*e + 4*(a*b^4*c - 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^4*e^2 - 10*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e^3 + (a^2*b^4 + 2*a^3*b^2*c - 24*a^4*c^2)*d^2*e^4 - (a^3*b^3 - 4*a^4*b*c)*d*e^5)*x)*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 - (a^2*b^2*c - 4*a^3*c^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^(1/3)) + 1/3*(1/2)^(1/3)*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 + (a^2*b^2*c - 4*a^3*c^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^(1/3)*log(2*(10*a^2*b*c*d^2*e^3 + a^3*b*e^5 - (b^2*c^2 - 2*a*c^3)*d^5 + (b^3*c + a*b*c^2)*d^4*e - 4*(a*b^2*c + a^2*c^2)*d^3*e^2 - (a^2*b^2 + 6*a^3*c)*d*e^4)*x + (1/2)^(1/3)*((b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*d^4 - 3*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + 6*(a^2*b^2*c - 4*a^3*c^2)*d^2*e^2 - (a^2*b^3 - 4*a^3*b*c)*d*e^3 - ((a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*d - 2*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*e)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 + (a^2*b^2*c - 4*a^3*c^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^(1/3)) + 1/3*(1/2)^(1/3)*((b*c*d^3 - 3*a*c*d^2*e + a^2*e^3 - (a^2*b^2*c - 4*a^3*c^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^(1/3)*log(2*(10*a^2*b*c*d^2*e^3 + a^3*b*e^5 - (b^2*c^2 - 2*a*c^3)*d^5 + (b^3*c + a*b*c^2)*d^4*e - 4*(a*b^2*c + a^2*c^2)*d^3*e^2 - (a^2*b^2 + 6*a^3*c)*d*e^4)*x + (1/2)^(1/3)*((b^4*c - 6*a*b^2*c

```
^2 + 8*a^2*c^3)*d^4 - 3*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + 6*(a^2*b^2*c - 4*a^
3*c^2)*d^2*e^2 - (a^2*b^3 - 4*a^3*b*c)*d*e^3 + ((a^2*b^5*c - 8*a^3*b^3*c^2
+ 16*a^4*b*c^3)*d - 2*(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*e)*sqrt(-(12
*a^4*b*c*d*e^5 - a^4*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*
(a*b^3*c^2 - 2*a^2*b*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2
*(a^2*b^3*c + 16*a^3*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a
^4*b^6*c^2 - 12*a^5*b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))*((b*c*d^3 - 3*
a*c*d^2*e + a^2*e^3 - (a^2*b^2*c - 4*a^3*c^2)*sqrt(-(12*a^4*b*c*d*e^5 - a^4
*b^2*e^6 - (b^4*c^2 - 4*a*b^2*c^3 + 4*a^2*c^4)*d^6 + 6*(a*b^3*c^2 - 2*a^2*b
*c^3)*d^5*e - 3*(7*a^2*b^2*c^2 - 8*a^3*c^3)*d^4*e^2 + 2*(a^2*b^3*c + 16*a^3
*b*c^2)*d^3*e^3 - 6*(a^3*b^2*c + 6*a^4*c^2)*d^2*e^4)/(a^4*b^6*c^2 - 12*a^5*
b^4*c^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)))/(a^2*b^2*c - 4*a^3*c^2))^(1/3))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**3+d)/(c*x**6+b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^3 + d}{cx^6 + bx^3 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((e*x^3 + d)/(c*x^6 + b*x^3 + a), x)
```

3.18 $\int \frac{d+ex^3}{x^2\left(a+bx^3+cx^6\right)}\,dx$

Optimal. Leaf size=653

$$-\frac{\sqrt[3]{c}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\,2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\sqrt[3]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{}}\right.}{6\,2^{2/3}}$$

```
[Out] -(d/(a*x)) + (c^(1/3)*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*
2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3
]*a*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + (c^(1/3)*(d - (b*d - 2*a*e)/Sqrt[b^2 -
 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/S
qrt[3]])/(2^(2/3)*Sqrt[3]*a*(b + Sqrt[b^2 - 4*a*c])^(1/3)) + (c^(1/3)*(d +
(b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3
)*c^(1/3)*x])/(3*2^(2/3)*a*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + (c^(1/3)*(d - (
b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)
*c^(1/3)*x])/(3*2^(2/3)*a*(b + Sqrt[b^2 - 4*a*c])^(1/3)) - (c^(1/3)*(d + (b
*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*
c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*
a*(b - Sqrt[b^2 - 4*a*c])^(1/3)) - (c^(1/3)*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4
*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 -
4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*a*(b + Sqrt[b^2 - 4*a*c]
)^(1/3))
```

Rubi [A] time = 1.17526, antiderivative size = 653, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {1504, 1510, 292, 31, 634, 617, 204, 628}

$$-\frac{\sqrt[3]{c}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\,2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\sqrt[3]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{}}\right.}{6\,2^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^3)/(x^2*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -(d/(a*x)) + (c^(1/3)*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*
2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3
]*a*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + (c^(1/3)*(d - (b*d - 2*a*e)/Sqrt[b^2 -
 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/S
qrt[3]])/(2^(2/3)*Sqrt[3]*a*(b + Sqrt[b^2 - 4*a*c])^(1/3)) + (c^(1/3)*(d +
(b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3
)*c^(1/3)*x])/(3*2^(2/3)*a*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + (c^(1/3)*(d - (
b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)
*c^(1/3)*x])/(3*2^(2/3)*a*(b + Sqrt[b^2 - 4*a*c])^(1/3)) - (c^(1/3)*(d + (b
*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*
c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*
a*(b - Sqrt[b^2 - 4*a*c])^(1/3)) - (c^(1/3)*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4
*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 -
4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*a*(b + Sqrt[b^2 - 4*a*c]
)^(1/3))
```

Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{d+ex^3}{x^2\left(a+bx^3+cx^6\right)}\,dx = -\frac{d}{ax} - \frac{\int \frac{x\left(bd-ae+cdx^3\right)}{a+bx^3+cx^6}\,dx}{a}$$

$$= -\frac{d}{ax} - \frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{x}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2a} - \frac{\left(c\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{x}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2a}$$

$$= -\frac{d}{ax} + \frac{\left(c^{2/3}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}\,dx}{3\,2^{2/3}a\sqrt[3]{b+\sqrt{b^2-4ac}}} - \frac{\left(c^{2/3}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\cdots}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b+\cdots}}{\sqrt[3]{2}}\cdots}}{3\,2^{2/3}a\sqrt[3]{b+\sqrt{b^2-4ac}}}\cdots$$

$$= -\frac{d}{ax} + \frac{\sqrt[3]{c}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\,2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\sqrt[3]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2\cdots}}\right.}{3\,2^{2/3}a\sqrt[3]{b+\sqrt{b^2-\cdots}}}\cdots$$

$$= -\frac{d}{ax} + \frac{\sqrt[3]{c}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\,2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\sqrt[3]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+\sqrt{b^2\cdots}}\right.}{3\,2^{2/3}a\sqrt[3]{b+\sqrt{b^2-\cdots}}}\cdots$$

$$= -\frac{d}{ax} + \frac{\sqrt[3]{c}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\sqrt[3]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a\sqrt[3]{b+\sqrt{b^2-4ac}}} + \cdots$$

Mathematica [C] time = 0.0487376, size = 85, normalized size = 0.13

$$-\frac{\text{RootSum}\left[\#1^3b+\#1^6c+a\&,\frac{\#1^3cd\log(x-\#1)-ae\log(x-\#1)+bd\log(x-\#1)}{2\#1^4c+\#1b}\&\right]}{3a}-\frac{d}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^3)/(x^2*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -(d/(a*x)) - RootSum[a + b*#1^3 + c*#1^6 & , (b*d*Log[x - #1] - a*e*Log[x -
 #1] + c*d*Log[x - #1]*#1^3)/(b*#1 + 2*c*#1^4) & ]/(3*a)
```

Maple [C] time = 0.006, size = 70, normalized size = 0.1

$$-\frac{d}{ax}-\frac{1}{3a}\sum_{\_R=\mathrm{RootOf}\left(\_Z^6c+\_Z^3b+a\right)}\frac{\left(cd\_R^4+(-ae+bd)\_R\right)\ln\left(x-\_R\right)}{2\_R^5c+\_R^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^3+d)/x^2/(c*x^6+b*x^3+a),x)
```

```
[Out] -d/a/x-1/3/a*sum((c*d*_R^4+(-a*e+b*d)*_R)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=Roo
tOf(_Z^6*c+_Z^3*b+a))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/x^2/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/x^2/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**3+d)/x**2/(c*x**6+b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/x^2/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.19 $\int \frac{d+ex^3}{x^3\left(a+bx^3+cx^6\right)}\,dx$

Optimal. Leaf size=655

$$\frac{c^{2/3}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{c^{2/3}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{\ldots}}\right.}{6\sqrt[3]{2}a\,(\ldots}$$

```
[Out] -d/(2*a*x^2) + (c^(2/3)*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (
2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt
[3]*a*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + (c^(2/3)*(d - (b*d - 2*a*e)/Sqrt[b^2
 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))
/Sqrt[3]])/(2^(1/3)*Sqrt[3]*a*(b + Sqrt[b^2 - 4*a*c])^(2/3)) - (c^(2/3)*(d
+ (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1
/3)*c^(1/3)*x])/(3*2^(1/3)*a*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - (c^(2/3)*(d -
 (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/
3)*c^(1/3)*x])/(3*2^(1/3)*a*(b + Sqrt[b^2 - 4*a*c])^(2/3)) + (c^(2/3)*(d +
(b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3
)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3
)*a*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + (c^(2/3)*(d - (b*d - 2*a*e)/Sqrt[b^2 -
 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2
- 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*a*(b + Sqrt[b^2 - 4*a*
c])^(2/3))
```

Rubi [A] time = 1.11005, antiderivative size = 655, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {1504, 1422, 200, 31, 634, 617, 204, 628}

$$\frac{c^{2/3}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{c^{2/3}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{\ldots}}\right.}{6\sqrt[3]{2}a\,(\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^3)/(x^3*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -d/(2*a*x^2) + (c^(2/3)*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (
2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt
[3]*a*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + (c^(2/3)*(d - (b*d - 2*a*e)/Sqrt[b^2
 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))
/Sqrt[3]])/(2^(1/3)*Sqrt[3]*a*(b + Sqrt[b^2 - 4*a*c])^(2/3)) - (c^(2/3)*(d
+ (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1
/3)*c^(1/3)*x])/(3*2^(1/3)*a*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - (c^(2/3)*(d -
 (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/
3)*c^(1/3)*x])/(3*2^(1/3)*a*(b + Sqrt[b^2 - 4*a*c])^(2/3)) + (c^(2/3)*(d +
(b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3
)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3
)*a*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + (c^(2/3)*(d - (b*d - 2*a*e)/Sqrt[b^2 -
 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2
- 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*a*(b + Sqrt[b^2 - 4*a*
c])^(2/3))
```

Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
```

```
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{d+ex^3}{x^3\left(a+bx^3+cx^6\right)}\,dx = -\frac{d}{2ax^2}-\frac{\int \frac{2(bd-ae)+2cdx^3}{a+bx^3+cx^6}\,dx}{2a}$$

$$= -\frac{d}{2ax^2}-\frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2a}-\frac{\left(c\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2a}$$

$$= -\frac{d}{2ax^2}-\frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}x}\,dx}{3\sqrt[3]{2}a\left(b+\sqrt{b^2-4ac}\right)^{2/3}}-\frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{2^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}-\sqrt[3]{c}}{\frac{\left(b+\sqrt{b^2-4ac}\right)^{2/3}}{2^{2/3}}-\frac{\sqrt[3]{c}\sqrt[3]{b+\sqrt{b^2-4a}}}{\sqrt[3]{2}}}}{3\sqrt[3]{2}a\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

$$= -\frac{d}{2ax^2}-\frac{c^{2/3}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{c^{2/3}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+}\right.}{3\sqrt[3]{2}a\left(b+\sqrt{b^2-}\right.}$$

$$= -\frac{d}{2ax^2}-\frac{c^{2/3}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{c^{2/3}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b+}\right.}{3\sqrt[3]{2}a\left(b+\sqrt{b^2-}\right.}$$

$$= -\frac{d}{2ax^2}+\frac{c^{2/3}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{c^{2/3}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}a\left(b+\sqrt{b^2-4ac}\right)^{2/3}}-$$

Mathematica [C] time = 0.0483937, size = 89, normalized size = 0.14

$$-\frac{\text{RootSum}\left[\#1^3 b+\#1^6 c+a\&,\frac{\#1^3 cd\log(x-\#1)-ae\log(x-\#1)+bd\log(x-\#1)}{\#1^2 b+2\#1^5 c}\&\right]}{3a}-\frac{d}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^3)/(x^3*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -d/(2*a*x^2) - RootSum[a + b*#1^3 + c*#1^6 & , (b*d*Log[x - #1] - a*e*Log[x
 - #1] + c*d*Log[x - #1]*#1^3)/(b*#1^2 + 2*c*#1^5) & ]/(3*a)
```

Maple [C] time = 0.006, size = 68, normalized size = 0.1

$$\frac{\sum_{\_R=\mathit{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)}\frac{\left(-\_R^{3}cd+ae-bd\right)\ln\left(x-\_R\right)}{2\_R^{5}c+\_R^{2}b}}{3a}-\frac{d}{2ax^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^3+d)/x^3/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/3/a*sum((-_R^3*c*d+a*e-b*d)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_
Z^3*b+a))-1/2*d/a/x^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/x^3/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/x^3/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**3+d)/x**3/(c*x**6+b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/x^3/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.20 $\int \frac{x^8(1-x^3)}{1-x^3+x^6}\,dx$

Optimal. Leaf size=46

$$-\frac{x^6}{6}+\frac{1}{6}\log\left(x^6-x^3+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] -x^6/6 - ArcTan[(1 - 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) + Log[1 - x^3 + x^6]/6
```

Rubi [A] time = 0.0578566, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {1474, 800, 634, 618, 204, 628}

$$-\frac{x^6}{6}+\frac{1}{6}\log\left(x^6-x^3+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] -x^6/6 - ArcTan[(1 - 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) + Log[1 - x^3 + x^6]/6
```

Rule 1474

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^8\left(1-x^3\right)}{1-x^3+x^6}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{(1-x)x^2}{1-x+x^2}\,dx, x, x^3\right)\\
&= \frac{1}{3}\operatorname{Subst}\left(\int \left(-x+\frac{x}{1-x+x^2}\right)dx, x, x^3\right)\\
&= -\frac{x^6}{6}+\frac{1}{3}\operatorname{Subst}\left(\int \frac{x}{1-x+x^2}\,dx, x, x^3\right)\\
&= -\frac{x^6}{6}+\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^3\right)+\frac{1}{6}\operatorname{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, x^3\right)\\
&= -\frac{x^6}{6}+\frac{1}{6}\log\left(1-x^3+x^6\right)-\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^3\right)\\
&= -\frac{x^6}{6}-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{6}\log\left(1-x^3+x^6\right)
\end{aligned}
$$

Mathematica [A] time = 0.0158543, size = 46, normalized size = 1.

$$
-\frac{x^6}{6}+\frac{1}{6}\log\left(x^6-x^3+1\right)+\frac{\tan^{-1}\left(\frac{2x^3-1}{\sqrt{3}}\right)}{3\sqrt{3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] -x^6/6 + ArcTan[(-1 + 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) + Log[1 - x^3 + x^6]/6
```

Maple [A] time = 0.003, size = 38, normalized size = 0.8

$$-\frac{x^6}{6}+\frac{\ln\left(x^6-x^3+1\right)}{6}+\frac{\sqrt{3}}{9}\arctan\left(\frac{\left(2x^3-1\right)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(-x^3+1)/(x^6-x^3+1),x)
```

```
[Out] -1/6*x^6+1/6*ln(x^6-x^3+1)+1/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

Maxima [A] time = 1.49105, size = 50, normalized size = 1.09

$$-\frac{1}{6}x^6+\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)+\frac{1}{6}\log\left(x^6-x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -1/6*x^6 + 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3
+ 1)
```

Fricas [A] time = 1.77774, size = 109, normalized size = 2.37

$$-\frac{1}{6}x^6+\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)+\frac{1}{6}\log\left(x^6-x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] -1/6*x^6 + 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3
+ 1)
```

Sympy [A] time = 0.149908, size = 42, normalized size = 0.91

$$-\frac{x^6}{6}+\frac{\log\left(x^6-x^3+1\right)}{6}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^3}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(-x**3+1)/(x**6-x**3+1),x)
```

```
[Out] -x**6/6 + log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/
3)/9
```

Giac [A] time = 1.11026, size = 50, normalized size = 1.09

$$-\frac{1}{6}x^6+\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)+\frac{1}{6}\log\left(x^6-x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(-x^3+1)/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] -1/6*x^6 + 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3
+ 1)
```

3.21 $\int \frac{x^5(1-x^3)}{1-x^3+x^6} \, dx$

Optimal. Leaf size=31

$$-\frac{x^3}{3}-\frac{2\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] -x^3/3 - (2*ArcTan[(1 - 2*x^3)/Sqrt[3]])/(3*Sqrt[3])
```

Rubi [A] time = 0.0354362, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1474, 773, 618, 204}

$$-\frac{x^3}{3}-\frac{2\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] -x^3/3 - (2*ArcTan[(1 - 2*x^3)/Sqrt[3]])/(3*Sqrt[3])
```

Rule 1474

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^5\left(1-x^3\right)}{1-x^3+x^6}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{(1-x)x}{1-x+x^2}\,dx,x,x^3\right)\\ &= -\frac{x^3}{3}+\frac{1}{3}\,\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^3\right)\\ &= -\frac{x^3}{3}-\frac{2}{3}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x^3\right)\\ &= -\frac{x^3}{3}-\frac{2\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.0077299, size = 31, normalized size = 1.

$$\frac{2\tan^{-1}\left(\frac{2x^3-1}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] -x^3/3 + (2*ArcTan[(-1 + 2*x^3)/Sqrt[3]])/(3*Sqrt[3])
```

Maple [A] time = 0.003, size = 25, normalized size = 0.8

$$-\frac{x^3}{3}+\frac{2\sqrt{3}}{9}\arctan\left(\frac{\left(2x^3-1\right)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(-x^3+1)/(x^6-x^3+1),x)
```

```
[Out] -1/3*x^3+2/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

Maxima [A] time = 1.45651, size = 32, normalized size = 1.03

$$-\frac{1}{3}\,x^3+\frac{2}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^3-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -1/3*x^3 + 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1))
```

Fricas [A] time = 1.80033, size = 76, normalized size = 2.45

$$-\frac{1}{3}\,x^3+\frac{2}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^3-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] -1/3*x^3 + 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1))
```

Sympy [A] time = 0.126467, size = 32, normalized size = 1.03

$$-\frac{x^3}{3}+\frac{2\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}x^3}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(-x**3+1)/(x**6-x**3+1),x)
```

```
[Out] -x**3/3 + 2*sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3)/9
```

<u>Giac</u> [A] time = 1.16666, size = 32, normalized size = 1.03

$$-\frac{1}{3}x^3 + \frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3 - 1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-x^3+1)/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] -1/3*x^3 + 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1))
```

3.22 $\int \frac{x^2(1-x^3)}{1-x^3+x^6}\,dx$

Optimal. Leaf size=39

$$-\frac{1}{6}\log\left(x^6-x^3+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] -ArcTan[(1 - 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) - Log[1 - x^3 + x^6]/6
```

Rubi [A] time = 0.0395079, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1468, 634, 618, 204, 628}

$$-\frac{1}{6}\log\left(x^6-x^3+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] -ArcTan[(1 - 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) - Log[1 - x^3 + x^6]/6
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\left(1-x^3\right)}{1-x^3+x^6}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1-x}{1-x+x^2}\,dx, x, x^3\right)\\
&= \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^3\right) - \frac{1}{6}\,\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, x^3\right)\\
&= -\frac{1}{6}\log\left(1-x^3+x^6\right) - \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^3\right)\\
&= -\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{1}{6}\log\left(1-x^3+x^6\right)
\end{aligned}$$

Mathematica [A] time = 0.0085705, size = 39, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{2x^3-1}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{1}{6}\log\left(x^6-x^3+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] ArcTan[(-1 + 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) - Log[1 - x^3 + x^6]/6
```

Maple [A] time = 0.002, size = 33, normalized size = 0.9

$$-\frac{\ln\left(x^{6}-x^{3}+1\right)}{6}+\frac{\sqrt{3}}{9}\arctan\left(\frac{\left(2x^{3}-1\right)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-x^3+1)/(x^6-x^3+1),x)
```

```
[Out] -1/6*ln(x^6-x^3+1)+1/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

Maxima [A] time = 1.50671, size = 43, normalized size = 1.1

$$\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{3}-1\right)\right)-\frac{1}{6}\log\left(x^{6}-x^{3}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1)
```

Fricas [A] time = 1.68099, size = 95, normalized size = 2.44

$$\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{3}-1\right)\right)-\frac{1}{6}\log\left(x^{6}-x^{3}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1)
```

Sympy [A] time = 0.140165, size = 37, normalized size = 0.95

$$-\frac{\log\left(x^{6}-x^{3}+1\right)}{6}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^{3}}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-x**3+1)/(x**6-x**3+1),x)
```

```
[Out] -log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3)/9
```

Giac [A] time = 1.14617, size = 43, normalized size = 1.1

$$\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)-\frac{1}{6}\log\left(x^6-x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^3+1)/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1)
```

3.23 $\int \frac{1-x^3}{x(1-x^3+x^6)}\,dx$

Optimal. Leaf size=41

$$-\frac{1}{6}\log\left(x^6-x^3+1\right)+\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}+\log(x)$$

```
[Out] ArcTan[(1 - 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) + Log[x] - Log[1 - x^3 + x^6]/6
```

Rubi [A] time = 0.05475, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {1474, 800, 634, 618, 204, 628}

$$-\frac{1}{6}\log\left(x^6-x^3+1\right)+\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)/(x*(1 - x^3 + x^6)),x]
```

```
[Out] ArcTan[(1 - 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) + Log[x] - Log[1 - x^3 + x^6]/6
```

Rule 1474

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1-x^3}{x\left(1-x^3+x^6\right)}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1-x}{x\left(1-x+x^2\right)}\,dx, x, x^3\right)\\
&= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{1}{x}-\frac{x}{1-x+x^2}\right)dx, x, x^3\right)\\
&= \log(x)-\frac{1}{3}\operatorname{Subst}\left(\int \frac{x}{1-x+x^2}\,dx, x, x^3\right)\\
&= \log(x)-\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^3\right)-\frac{1}{6}\operatorname{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, x^3\right)\\
&= \log(x)-\frac{1}{6}\log\left(1-x^3+x^6\right)+\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^3\right)\\
&= \frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}+\log(x)-\frac{1}{6}\log\left(1-x^3+x^6\right)
\end{aligned}
$$

Mathematica [C] time = 0.0129708, size = 44, normalized size = 1.07

$$\log(x)-\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1^3\log(x-\#1)}{2\#1^3-1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^3)/(x*(1 - x^3 + x^6)),x]
```

```
[Out] Log[x] - RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1^3)/(-1 + 2*#1^3) & ]/3
```

Maple [A] time = 0.005, size = 35, normalized size = 0.9

$$\ln(x) - \frac{\ln\left(x^6 - x^3 + 1\right)}{6} - \frac{\sqrt{3}}{9}\arctan\left(\frac{\left(2x^3 - 1\right)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)/x/(x^6-x^3+1),x)
```

```
[Out] ln(x)-1/6*ln(x^6-x^3+1)-1/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

Maxima [A] time = 1.45781, size = 51, normalized size = 1.24

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3 - 1\right)\right) - \frac{1}{6}\log\left(x^6 - x^3 + 1\right) + \frac{1}{3}\log\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + 1/3
*log(x^3)
```

Fricas [A] time = 1.76562, size = 108, normalized size = 2.63

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3 - 1\right)\right) - \frac{1}{6}\log\left(x^6 - x^3 + 1\right) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log
(x)
```

Sympy [A] time = 0.154134, size = 41, normalized size = 1.

$$\log(x) - \frac{\log\left(x^6 - x^3 + 1\right)}{6} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^3}{3} - \frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)/x/(x**6-x**3+1),x)
```

```
[Out] log(x) - log(x**6 - x**3 + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3
)/9
```

Giac [A] time = 1.10961, size = 47, normalized size = 1.15

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^3 - 1\right)\right) - \frac{1}{6}\log\left(x^6 - x^3 + 1\right) + \log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log
(abs(x))
```

3.24 $\int \frac{1-x^3}{x^4\left(1-x^3+x^6\right)}\,dx$

Optimal. Leaf size=31

$$\frac{2\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{3x^3}$$

```
[Out] -1/(3*x^3) + (2*ArcTan[(1 - 2*x^3)/Sqrt[3]])/(3*Sqrt[3])
```

Rubi [A] time = 0.0451354, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1474, 800, 618, 204}

$$\frac{2\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)/(x^4*(1 - x^3 + x^6)),x]
```

```
[Out] -1/(3*x^3) + (2*ArcTan[(1 - 2*x^3)/Sqrt[3]])/(3*Sqrt[3])
```

Rule 1474

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1-x^3}{x^4\left(1-x^3+x^6\right)}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1-x}{x^2\left(1-x+x^2\right)}\,dx, x, x^3\right)\\
&= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{1}{x^2}+\frac{1}{-1+x-x^2}\right)dx, x, x^3\right)\\
&= -\frac{1}{3x^3}+\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{-1+x-x^2}\,dx, x, x^3\right)\\
&= -\frac{1}{3x^3}-\frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-2x^3\right)\\
&= -\frac{1}{3x^3}+\frac{2\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}
\end{aligned}$$

Mathematica [C] time = 0.0128113, size = 45, normalized size = 1.45

$$-\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\log(x-\#1)}{2\#1^3-1}\&\right]-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^3)/(x^4*(1 - x^3 + x^6)),x]
```

```
[Out] -1/(3*x^3) - RootSum[1 - #1^3 + #1^6 & , Log[x - #1]/(-1 + 2*#1^3) & ]/3
```

Maple [A] time = 0.004, size = 25, normalized size = 0.8

$$-\frac{2\sqrt{3}}{9}\arctan\left(\frac{\left(2x^3-1\right)\sqrt{3}}{3}\right)-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)/x^4/(x^6-x^3+1),x)
```

```
[Out] -2/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))-1/3/x^3
```

Maxima [A] time = 1.52609, size = 32, normalized size = 1.03

$$-\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x^4/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/3/x^3
```

Fricas [A] time = 1.35235, size = 84, normalized size = 2.71

$$-\frac{2\sqrt{3}x^3\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)+3}{9x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x^4/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] -1/9*(2*sqrt(3)*x^3*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 3)/x^3
```

Sympy [A] time = 0.158256, size = 36, normalized size = 1.16

$$-\frac{2\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^3}{3}-\frac{\sqrt{3}}{3}\right)}{9}-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)/x**4/(x**6-x**3+1),x)
```

```
[Out] -2*sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3)/9 - 1/(3*x**3)
```

Giac [A] time = 1.10625, size = 32, normalized size = 1.03

$$-\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x^4/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] -2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/3/x^3
```

3.25 $\int \frac{x^6(1-x^3)}{1-x^3+x^6}\,dx$

Optimal. Leaf size=418

$$-\frac{x^4}{4}-\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+$$

```
[Out] -x^4/4 - ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3
]])/(3*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) + ((I - Sqrt[3])*ArcTan[(1 + (2*x)/((
1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(1/3)*(1 + I*Sqrt[3])^(2/3)) + ((3
+ I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(1 - I*Sqrt
[3])^(2/3)) + ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2
^(1/3)*(1 + I*Sqrt[3])^(2/3)) - ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3)
+ (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 - I*Sqrt[3])^(
2/3)) - ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(1
/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 + I*Sqrt[3])^(2/3))
```

Rubi [A] time = 0.537566, antiderivative size = 418, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {1502, 12, 1374, 200, 31, 634, 617, 204, 628}

$$-\frac{x^4}{4}-\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(1 - x^3))/(1 - x^3 + x^6),x]
```


```
[Out] -x^4/4 - ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3
]])/(3*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) + ((I - Sqrt[3])*ArcTan[(1 + (2*x)/((
1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(1/3)*(1 + I*Sqrt[3])^(2/3)) + ((3
+ I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(1 - I*Sqrt
[3])^(2/3)) + ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2
^(1/3)*(1 + I*Sqrt[3])^(2/3)) - ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3)
```

```
+ (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 - I*Sqrt[3])^(
2/3)) - ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(1
/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 + I*Sqrt[3])^(2/3))
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(e*f^(n - 1)*(f*x)^(m - n + 1)*(a
+ b*x^n + c*x^(2*n))^(p + 1))/(c*(m + n*(2*p + 1) + 1)), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^6\left(1-x^3\right)}{1-x^3+x^6}\,dx = -\frac{x^4}{4}-\frac{1}{4}\int -\frac{4x^3}{1-x^3+x^6}\,dx$$

$$= -\frac{x^4}{4}+\int \frac{x^3}{1-x^3+x^6}\,dx$$

$$= -\frac{x^4}{4}-\frac{1}{6}\left(-3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx+\frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx$$

$$= -\frac{x^4}{4}+\frac{\left(3-i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+x}\,dx}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+\frac{\left(3-i\sqrt{3}\right)\int \frac{-2^{2/3}\sqrt[3]{1+i\sqrt{3}}-x}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+x^2}\,dx}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+\frac{\left(3+i\sqrt{3}\right)\int \cdots}{9\sqrt[3]{2}\left(1-\cdots\right.}$$

$$= -\frac{x^4}{4}+\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\int \cdots}{18\sqrt[3]{2}\left(\cdots\right.}$$

$$= -\frac{x^4}{4}+\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{\left(3+i\sqrt{3}\right)\log\left(\left(1-\cdots\right.\right.}{1\cdots}$$

$$= -\frac{x^4}{4}-\frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\cdots\right.}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}$$

Mathematica [C] time = 0.0113329, size = 47, normalized size = 0.11

$$\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1\log(x-\#1)}{2\#1^3-1}\&\right]-\frac{x^4}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] -x^4/4 + RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1)/(-1 + 2*#1^3) & ]/3
```

Maple [C] time = 0.006, size = 46, normalized size = 0.1

$$-\frac{x^4}{4}+\frac{1}{3}\sum_{\_R=\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\_R^3\ln\left(x-\_R\right)}{2\_R^5-\_R^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(-x^3+1)/(x^6-x^3+1),x)
```

```
[Out] -1/4*x^4+1/3*sum(_R^3/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{4}x^4+\int\frac{x^3}{x^6-x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -1/4*x^4 + integrate(x^3/(x^6 - x^3 + 1), x)
```

Fricas [B] time = 1.60718, size = 3918, normalized size = 9.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] -1/4*x^4 + 1/54*18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) + 2))*log(2*18^(2/
3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) + 3*18^(1/3)*12^(1/3)*co
s(2/3*arctan(sqrt(3) + 2))^2 + 3*18^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) +
 2))^2 + 18*x^2) + 2/27*18^(2/3)*12^(1/6)*arctan(1/216*(18^(1/3)*12^(5/6)*s
qrt(3)*sqrt(2)*sqrt(2*18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) +
2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 3*18^(1/3)*12^(1
/3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18*x^2) - 6*18^(1/3)*12^(5/6)*sqrt(3)*
x - 216*sin(2/3*arctan(sqrt(3) + 2)))/cos(2/3*arctan(sqrt(3) + 2)))*sin(2/3
*arctan(sqrt(3) + 2)) - 1/27*(18^(2/3)*12^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt
```

```
(3) + 2)) - 18^(2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3) + 2)))*arctan(-1/108*(
6*18^(1/3)*12^(5/6)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) + 2)) + 108*sqrt(3)*co
s(2/3*arctan(sqrt(3) + 2))^2 + 108*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2))^2 -
 18*(18^(1/3)*12^(5/6)*x + 24*cos(2/3*arctan(sqrt(3) + 2)))*sin(2/3*arctan(
sqrt(3) + 2)) - sqrt(-18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) +
2)) + 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) + 2)) + 3*18^(1/3)*12^(1
/3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 3*18^(1/3)*12^(1/3)*sin(2/3*arctan(sqr
t(3) + 2))^2 + 18*x^2)*(18^(1/3)*12^(5/6)*sqrt(3)*sqrt(2)*cos(2/3*arctan(sq
rt(3) + 2)) - 3*18^(1/3)*12^(5/6)*sqrt(2)*sin(2/3*arctan(sqrt(3) + 2))))/(c
os(2/3*arctan(sqrt(3) + 2))^2 - 3*sin(2/3*arctan(sqrt(3) + 2))^2)) - 1/27*(
18^(2/3)*12^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2)) + 18^(2/3)*12^(1/6)*
sin(2/3*arctan(sqrt(3) + 2)))*arctan(1/108*(6*18^(1/3)*12^(5/6)*sqrt(3)*x*c
os(2/3*arctan(sqrt(3) + 2)) - 108*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^2 -
108*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18*(18^(1/3)*12^(5/6)*x - 24*c
os(2/3*arctan(sqrt(3) + 2)))*sin(2/3*arctan(sqrt(3) + 2)) - sqrt(-18^(2/3)*
12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) - 3*18^(2/3)*12^(1/6)*x*cos
(2/3*arctan(sqrt(3) + 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) + 2)
)^2 + 3*18^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18*x^2)*(18^(1/3
)*12^(5/6)*sqrt(3)*sqrt(2)*cos(2/3*arctan(sqrt(3) + 2)) + 3*18^(1/3)*12^(5/
6)*sqrt(2)*sin(2/3*arctan(sqrt(3) + 2))))/(cos(2/3*arctan(sqrt(3) + 2))^2 -
 3*sin(2/3*arctan(sqrt(3) + 2))^2)) - 1/108*(18^(2/3)*12^(1/6)*sqrt(3)*sin(
2/3*arctan(sqrt(3) + 2)) + 18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) + 2)))*
log(-18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) + 3*18^(2/3)*
12^(1/6)*x*cos(2/3*arctan(sqrt(3) + 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arcta
n(sqrt(3) + 2))^2 + 3*18^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18
*x^2) + 1/108*(18^(2/3)*12^(1/6)*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2)) - 18^
(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) + 2)))*log(-18^(2/3)*12^(1/6)*sqrt(3)
*x*sin(2/3*arctan(sqrt(3) + 2)) - 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt
(3) + 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 3*18^(1/3)
*12^(1/3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18*x^2)
```

Sympy [A] time = 0.176743, size = 31, normalized size = 0.07

$$-\frac{x^4}{4} - \mathrm{RootSum}\left(19683t^6 - 243t^3 + 1, \left(t \mapsto t\log\left(-1458t^4 + 9t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(-x**3+1)/(x**6-x**3+1),x)
```

```
[Out] -x**4/4 - RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(-1458*_t**
4 + 9*_t + x)))
```

Giac [B] time = 1.1675, size = 867, normalized size = 2.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(-x^3+1)/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] -1/4*x^4 - 1/9*(2*sqrt(3)*cos(4/9*pi)^4 - 12*sqrt(3)*cos(4/9*pi)^2*sin(4/9*
pi)^2 + 2*sqrt(3)*sin(4/9*pi)^4 + 8*cos(4/9*pi)^3*sin(4/9*pi) - 8*cos(4/9*p
i)*sin(4/9*pi)^3 + sqrt(3)*cos(4/9*pi) + sin(4/9*pi))*arctan(-((sqrt(3)*i +
 1)*cos(4/9*pi) - 2*x)/((sqrt(3)*i + 1)*sin(4/9*pi))) - 1/9*(2*sqrt(3)*cos(
2/9*pi)^4 - 12*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^2 + 2*sqrt(3)*sin(2/9*pi)^
4 + 8*cos(2/9*pi)^3*sin(2/9*pi) - 8*cos(2/9*pi)*sin(2/9*pi)^3 + sqrt(3)*cos
(2/9*pi) + sin(2/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(2/9*pi) - 2*x)/((sqrt(
3)*i + 1)*sin(2/9*pi))) - 1/9*(2*sqrt(3)*cos(1/9*pi)^4 - 12*sqrt(3)*cos(1/9
*pi)^2*sin(1/9*pi)^2 + 2*sqrt(3)*sin(1/9*pi)^4 - 8*cos(1/9*pi)^3*sin(1/9*pi
) + 8*cos(1/9*pi)*sin(1/9*pi)^3 - sqrt(3)*cos(1/9*pi) + sin(1/9*pi))*arctan
(((sqrt(3)*i + 1)*cos(1/9*pi) + 2*x)/((sqrt(3)*i + 1)*sin(1/9*pi))) - 1/18*
(8*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi) - 8*sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^3
- 2*cos(4/9*pi)^4 + 12*cos(4/9*pi)^2*sin(4/9*pi)^2 - 2*sin(4/9*pi)^4 + sqrt
(3)*sin(4/9*pi) - cos(4/9*pi))*log(-(sqrt(3)*i*cos(4/9*pi) + cos(4/9*pi))*x
 + x^2 + 1) - 1/18*(8*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi) - 8*sqrt(3)*cos(2/9
*pi)*sin(2/9*pi)^3 - 2*cos(2/9*pi)^4 + 12*cos(2/9*pi)^2*sin(2/9*pi)^2 - 2*s
in(2/9*pi)^4 + sqrt(3)*sin(2/9*pi) - cos(2/9*pi))*log(-(sqrt(3)*i*cos(2/9*p
i) + cos(2/9*pi))*x + x^2 + 1) + 1/18*(8*sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi)
- 8*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^3 + 2*cos(1/9*pi)^4 - 12*cos(1/9*pi)^2*
sin(1/9*pi)^2 + 2*sin(1/9*pi)^4 - sqrt(3)*sin(1/9*pi) - cos(1/9*pi))*log((s
qrt(3)*i*cos(1/9*pi) + cos(1/9*pi))*x + x^2 + 1)
```

3.26 $\int \frac{x^4(1-x^3)}{1-x^3+x^6}\,dx$

Optimal. Leaf size=382

$$-\frac{x^2}{2}-\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{3\ 2^{2/3}\sqrt{3}\sqrt[3]{1-i\sqrt{3}}}+\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{3\ 2^{2/3}\sqrt{3}\sqrt[3]{1+i\sqrt{3}}}+\frac{i\log\left(-\sqrt[3]{2}x+\right.}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1\right.}}$$

```
[Out] -x^2/2 + ((I/3)*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/((1
- I*Sqrt[3])/2)^(1/3) - ((I/3)*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))
/Sqrt[3]])/((1 + I*Sqrt[3])/2)^(1/3) + ((I/3)*Log[(1 - I*Sqrt[3])^(1/3) - 2
^(1/3)*x])/(Sqrt[3]*((1 - I*Sqrt[3])/2)^(1/3)) - ((I/3)*Log[(1 + I*Sqrt[3])
^(1/3) - 2^(1/3)*x])/(Sqrt[3]*((1 + I*Sqrt[3])/2)^(1/3)) - ((I/3)*Log[(1 -
I*Sqrt[3])^(2/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(2^(2/3)*Sqr
t[3]*(1 - I*Sqrt[3])^(1/3)) + ((I/3)*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*
Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(2^(2/3)*Sqrt[3]*(1 + I*Sqrt[3])^(1/3))
```

Rubi [A] time = 0.327587, antiderivative size = 382, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {1502, 12, 1375, 292, 31, 634, 617, 204, 628}

$$-\frac{x^2}{2}-\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{3\ 2^{2/3}\sqrt{3}\sqrt[3]{1-i\sqrt{3}}}+\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{3\ 2^{2/3}\sqrt{3}\sqrt[3]{1+i\sqrt{3}}}+\frac{i\log\left(-\sqrt[3]{2}x+\right.}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] -x^2/2 + ((I/3)*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/((1
- I*Sqrt[3])/2)^(1/3) - ((I/3)*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))
/Sqrt[3]])/((1 + I*Sqrt[3])/2)^(1/3) + ((I/3)*Log[(1 - I*Sqrt[3])^(1/3) - 2
^(1/3)*x])/(Sqrt[3]*((1 - I*Sqrt[3])/2)^(1/3)) - ((I/3)*Log[(1 + I*Sqrt[3])
^(1/3) - 2^(1/3)*x])/(Sqrt[3]*((1 + I*Sqrt[3])/2)^(1/3)) - ((I/3)*Log[(1 -
I*Sqrt[3])^(2/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(2^(2/3)*Sqr
t[3]*(1 - I*Sqrt[3])^(1/3)) + ((I/3)*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*
```

```
Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(2^(2/3)*Sqrt[3]*(1 + I*Sqrt[3])^(1/3))
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(e*f^(n - 1)*(f*x)^(m - n + 1)*(a
+ b*x^n + c*x^(2*n))^(p + 1))/(c*(m + n*(2*p + 1) + 1)), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1375

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4\left(1-x^3\right)}{1-x^3+x^6}\,dx &= -\frac{x^2}{2}-\frac{1}{2}\int -\frac{2x}{1-x^3+x^6}\,dx\\
&= -\frac{x^2}{2}+\int \frac{x}{1-x^3+x^6}\,dx\\
&= -\frac{x^2}{2}-\frac{i\int \frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx}{\sqrt{3}}+\frac{i\int \frac{x}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx}{\sqrt{3}}\\
&= -\frac{x^2}{2}+\frac{i\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}\,dx}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}-\frac{i\int \frac{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,dx}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}-\frac{i\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+x}\,dx}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+\cdots\\
&= -\frac{x^2}{2}+\frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}-\frac{i\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+\frac{i\int \frac{1}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,dx}{2\sqrt{3}}-\cdots\\
&= -\frac{x^2}{2}+\frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}-\frac{i\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}-\frac{i\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2\cdots\right)}{3\,2^{2/3}\sqrt{3}\sqrt[3]{1-i\sqrt{3}}}\cdots\\
&= -\frac{x^2}{2}+\frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}-\frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+\frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}-\frac{i\log\left(\sqrt[3]{1+i\sqrt{3}}-\cdots\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1+i\cdots\right)}}
\end{aligned}
$$

Mathematica [C] time = 0.0135111, size = 48, normalized size = 0.13

$$\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\log(x-\#1)}{2\#1^4-\#1}\&\right]-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] -x^2/2 + RootSum[1 - #1^3 + #1^6 & , Log[x - #1]/(-#1 + 2*#1^4) & ]/3
```

Maple [C] time = 0.004, size = 44, normalized size = 0.1

$$-\frac{x^2}{2}+\frac{1}{3}\sum_{\_R=\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\_R\ln\left(x-\_R\right)}{2\_R^5-\_R^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-x^3+1)/(x^6-x^3+1),x)
```

```
[Out] -1/2*x^2+1/3*sum(_R/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}x^2+\int\frac{x}{x^6-x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -1/2*x^2 + integrate(x/(x^6 - x^3 + 1), x)
```

Fricas [B] time = 1.94217, size = 5949, normalized size = 15.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/54*18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) - 2))*log(18^(2/3)*12^(2/3)*c
os(2/3*arctan(sqrt(3) - 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) -
2))^4 + 12*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3
*arctan(sqrt(3) - 2)) + 6*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2))^
2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 3*18^(1/3)*12^(1/
3)*x)*sin(2/3*arctan(sqrt(3) - 2))^2 + 36*x^2) - 2/27*18^(2/3)*12^(1/6)*arc
tan(1/108*(6*18^(2/3)*12^(2/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 1
08*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^4 + 108*sqrt(3)*sin(2/3*arctan(sqrt
(3) - 2))^4 - 864*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2))
```

```
^3 - 6*(18^(2/3)*12^(2/3)*sqrt(3)*x - 36*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2
))^2)*sin(2/3*arctan(sqrt(3) - 2))^2 + 12*(18^(2/3)*12^(2/3)*x*cos(2/3*arct
an(sqrt(3) - 2)) + 72*cos(2/3*arctan(sqrt(3) - 2))^3)*sin(2/3*arctan(sqrt(3
) - 2)) - sqrt(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^4 + 18^(2/3)*
12^(2/3)*sin(2/3*arctan(sqrt(3) - 2))^4 + 12*18^(1/3)*12^(1/3)*sqrt(3)*x*co
s(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2)) + 6*18^(1/3)*12^(1/
3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(s
qrt(3) - 2))^2 - 3*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) - 2))^2 + 36
*x^2)*(18^(2/3)*12^(2/3)*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 18^(2/3)*
12^(2/3)*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 2*18^(2/3)*12^(2/3)*cos(2
/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2))))/(3*cos(2/3*arctan(sq
rt(3) - 2))^4 - 10*cos(2/3*arctan(sqrt(3) - 2))^2*sin(2/3*arctan(sqrt(3) -
2))^2 + 3*sin(2/3*arctan(sqrt(3) - 2))^4))*sin(2/3*arctan(sqrt(3) - 2)) - 1
/2*x^2 - 1/27*(18^(2/3)*12^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2)) + 18^
(2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3) - 2)))*arctan(1/108*(6*18^(2/3)*12^(2
/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 108*sqrt(3)*cos(2/3*arctan(s
qrt(3) - 2))^4 + 108*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2))^4 + 864*cos(2/3*a
rctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2))^3 - 6*(18^(2/3)*12^(2/3)*s
qrt(3)*x - 36*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2)*sin(2/3*arctan(sqrt(3
) - 2))^2 - 12*(18^(2/3)*12^(2/3)*x*cos(2/3*arctan(sqrt(3) - 2)) + 72*cos(2
/3*arctan(sqrt(3) - 2))^3)*sin(2/3*arctan(sqrt(3) - 2)) - sqrt(18^(2/3)*12^
(2/3)*cos(2/3*arctan(sqrt(3) - 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqr
t(3) - 2))^4 - 12*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))*
sin(2/3*arctan(sqrt(3) - 2)) + 6*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3)
 - 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 3*18^(1/3)
*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) - 2))^2 + 36*x^2)*(18^(2/3)*12^(2/3)*sq
rt(3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 18^(2/3)*12^(2/3)*sqrt(3)*sin(2/3*ar
ctan(sqrt(3) - 2))^2 - 2*18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))*sin
(2/3*arctan(sqrt(3) - 2))))/(3*cos(2/3*arctan(sqrt(3) - 2))^4 - 10*cos(2/3*
arctan(sqrt(3) - 2))^2*sin(2/3*arctan(sqrt(3) - 2))^2 + 3*sin(2/3*arctan(sq
rt(3) - 2))^4)) + 1/27*(18^(2/3)*12^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) -
2)) - 18^(2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3) - 2)))*arctan(-1/432*(6*18^(
2/3)*12^(2/3)*x - 216*cos(2/3*arctan(sqrt(3) - 2))^2 + 216*sin(2/3*arctan(s
qrt(3) - 2))^2 - 18^(2/3)*12^(2/3)*sqrt(18^(2/3)*12^(2/3)*cos(2/3*arctan(sq
rt(3) - 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) - 2))^4 - 12*18^(1
/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/
3*arctan(sqrt(3) - 2))^2 + 6*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) -
2))^2 + 36*x^2))/(cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2))
)) + 1/108*(18^(2/3)*12^(1/6)*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2)) - 18^(2/
3)*12^(1/6)*cos(2/3*arctan(sqrt(3) - 2)))*log(18^(2/3)*12^(2/3)*cos(2/3*arc
tan(sqrt(3) - 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) - 2))^4 - 12
*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sq
rt(3) - 2)) + 6*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 2*(18^
(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 3*18^(1/3)*12^(1/3)*x)*sin(
2/3*arctan(sqrt(3) - 2))^2 + 36*x^2) - 1/108*(18^(2/3)*12^(1/6)*sqrt(3)*sin
```

```
(2/3*arctan(sqrt(3) - 2)) + 18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) - 2)))
*log(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^4 + 18^(2/3)*12^(2/3)*s
in(2/3*arctan(sqrt(3) - 2))^4 - 12*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(
3) - 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 6*18^(1/
3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) - 2))^2 + 36*x^2)
```

Sympy [A] time = 0.193264, size = 32, normalized size = 0.08

$$-\frac{x^2}{2} - \text{RootSum}\left(19683t^6 + 243t^3 + 1, \left(t \mapsto t\log\left(-6561t^5 - 27t^2 + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-x**3+1)/(x**6-x**3+1),x)
```

```
[Out] -x**2/2 - RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(-6561*_t**
5 - 27*_t**2 + x)))
```

Giac [B] time = 1.15957, size = 1103, normalized size = 2.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-x^3+1)/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] -1/2*x^2 - 1/9*(sqrt(3)*cos(4/9*pi)^5 - 10*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi
)^2 + 5*sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^4 - 5*cos(4/9*pi)^4*sin(4/9*pi) + 1
0*cos(4/9*pi)^2*sin(4/9*pi)^3 - sin(4/9*pi)^5 - sqrt(3)*cos(4/9*pi)^2 + sqr
t(3)*sin(4/9*pi)^2 + 2*cos(4/9*pi)*sin(4/9*pi))*arctan(-((sqrt(3)*i + 1)*co
s(4/9*pi) - 2*x)/((sqrt(3)*i + 1)*sin(4/9*pi))) - 1/9*(sqrt(3)*cos(2/9*pi)^
5 - 10*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi)^2 + 5*sqrt(3)*cos(2/9*pi)*sin(2/9*
pi)^4 - 5*cos(2/9*pi)^4*sin(2/9*pi) + 10*cos(2/9*pi)^2*sin(2/9*pi)^3 - sin(
2/9*pi)^5 - sqrt(3)*cos(2/9*pi)^2 + sqrt(3)*sin(2/9*pi)^2 + 2*cos(2/9*pi)*s
in(2/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(2/9*pi) - 2*x)/((sqrt(3)*i + 1)*si
n(2/9*pi))) + 1/9*(sqrt(3)*cos(1/9*pi)^5 - 10*sqrt(3)*cos(1/9*pi)^3*sin(1/9
*pi)^2 + 5*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^4 + 5*cos(1/9*pi)^4*sin(1/9*pi)
- 10*cos(1/9*pi)^2*sin(1/9*pi)^3 + sin(1/9*pi)^5 + sqrt(3)*cos(1/9*pi)^2 -
sqrt(3)*sin(1/9*pi)^2 + 2*cos(1/9*pi)*sin(1/9*pi))*arctan(((sqrt(3)*i + 1)*
cos(1/9*pi) + 2*x)/((sqrt(3)*i + 1)*sin(1/9*pi))) - 1/18*(5*sqrt(3)*cos(4/9
```

```
*pi)^4*sin(4/9*pi) - 10*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^3 + sqrt(3)*sin(4
/9*pi)^5 + cos(4/9*pi)^5 - 10*cos(4/9*pi)^3*sin(4/9*pi)^2 + 5*cos(4/9*pi)*s
in(4/9*pi)^4 - 2*sqrt(3)*cos(4/9*pi)*sin(4/9*pi) - cos(4/9*pi)^2 + sin(4/9*
pi)^2)*log(-(sqrt(3)*i*cos(4/9*pi) + cos(4/9*pi))*x + x^2 + 1) - 1/18*(5*sq
rt(3)*cos(2/9*pi)^4*sin(2/9*pi) - 10*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^3 +
sqrt(3)*sin(2/9*pi)^5 + cos(2/9*pi)^5 - 10*cos(2/9*pi)^3*sin(2/9*pi)^2 + 5*
cos(2/9*pi)*sin(2/9*pi)^4 - 2*sqrt(3)*cos(2/9*pi)*sin(2/9*pi) - cos(2/9*pi)
^2 + sin(2/9*pi)^2)*log(-(sqrt(3)*i*cos(2/9*pi) + cos(2/9*pi))*x + x^2 + 1)
 - 1/18*(5*sqrt(3)*cos(1/9*pi)^4*sin(1/9*pi) - 10*sqrt(3)*cos(1/9*pi)^2*sin
(1/9*pi)^3 + sqrt(3)*sin(1/9*pi)^5 - cos(1/9*pi)^5 + 10*cos(1/9*pi)^3*sin(1
/9*pi)^2 - 5*cos(1/9*pi)*sin(1/9*pi)^4 + 2*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)
- cos(1/9*pi)^2 + sin(1/9*pi)^2)*log((sqrt(3)*i*cos(1/9*pi) + cos(1/9*pi))*
x + x^2 + 1)
```

3.27 $\int \frac{x^3(1-x^3)}{1-x^3+x^6}\,dx$

Optimal. Leaf size=378

$$-\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{3\sqrt[3]{2}\sqrt{3}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{3\sqrt[3]{2}\sqrt{3}\left(1+i\sqrt{3}\right)^{2/3}}-x+\frac{i\log\left(-\sqrt[3]{2}x+\sqrt[3]{\cdots}\right.}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\cdots\right.\right.}$$

```
[Out] -x - ((I/3)*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/((1 - I*
Sqrt[3])/2)^(2/3) + ((I/3)*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqr
t[3]])/((1 + I*Sqrt[3])/2)^(2/3) + ((I/3)*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/
3)*x])/(Sqrt[3]*((1 - I*Sqrt[3])/2)^(2/3)) - ((I/3)*Log[(1 + I*Sqrt[3])^(1/
3) - 2^(1/3)*x])/(Sqrt[3]*((1 + I*Sqrt[3])/2)^(2/3)) - ((I/3)*Log[(1 - I*Sq
rt[3])^(2/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(2^(1/3)*Sqrt[3]
*(1 - I*Sqrt[3])^(2/3)) + ((I/3)*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt
[3]))^(1/3)*x + 2^(2/3)*x^2])/(2^(1/3)*Sqrt[3]*(1 + I*Sqrt[3])^(2/3))
```

Rubi [A] time = 0.25865, antiderivative size = 378, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {1502, 1347, 200, 31, 634, 617, 204, 628}

$$-\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{3\sqrt[3]{2}\sqrt{3}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{3\sqrt[3]{2}\sqrt{3}\left(1+i\sqrt{3}\right)^{2/3}}-x+\frac{i\log\left(-\sqrt[3]{2}x+\sqrt[3]{\cdots}\right.}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\cdots\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] -x - ((I/3)*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/((1 - I*
Sqrt[3])/2)^(2/3) + ((I/3)*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqr
t[3]])/((1 + I*Sqrt[3])/2)^(2/3) + ((I/3)*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/
3)*x])/(Sqrt[3]*((1 - I*Sqrt[3])/2)^(2/3)) - ((I/3)*Log[(1 + I*Sqrt[3])^(1/
3) - 2^(1/3)*x])/(Sqrt[3]*((1 + I*Sqrt[3])/2)^(2/3)) - ((I/3)*Log[(1 - I*Sq
rt[3])^(2/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(2^(1/3)*Sqrt[3]
*(1 - I*Sqrt[3])^(2/3)) + ((I/3)*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt
```

```
[3]))^(1/3)*x + 2^(2/3)*x^2])/(2^(1/3)*Sqrt[3]*(1 + I*Sqrt[3])^(2/3))
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(e*f^(n - 1)*(f*x)^(m - n + 1)*(a
+ b*x^n + c*x^(2*n))^(p + 1))/(c*(m + n*(2*p + 1) + 1)), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rule 1347

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^3\left(1-x^3\right)}{1-x^3+x^6}\,dx = -x+\int \frac{1}{1-x^3+x^6}\,dx$$

$$= -x-\frac{i\int \frac{1}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx}{\sqrt{3}}+\frac{i\int \frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx}{\sqrt{3}}$$

$$= -x+\frac{i\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}\,dx}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}+\frac{i\int \frac{-2^{2/3}\sqrt[3]{1-i\sqrt{3}}-x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,dx}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{i\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+x}\,dx}{3\sqrt{3}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{i\int \ldots}{3\sqrt{\ldots}}$$

$$= -x+\frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{i\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{i\int \frac{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+2x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,dx}{3\sqrt[3]{2}\sqrt{3}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{i\int \ldots}{\ldots}$$

$$= -x+\frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{i\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{i\log\left(\left(1-i\sqrt{3}\right)^{2/3}+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+2\ldots\right.}{3\sqrt[3]{2}\sqrt{3}\left(1-i\sqrt{3}\right)^{2/3}}$$

$$= -x-\frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}+\frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}+\frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{i\log\left(\sqrt[3]{1+i\sqrt{3}}-\ldots\right.}{3\sqrt{3}\left(\frac{1}{2}\left(1+i\sqrt{\ldots}\right.\right.}$$

Mathematica [C] time = 0.0127722, size = 46, normalized size = 0.12

$$\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\log(x-\#1)}{2\#1^5-\#1^2}\&\right]-x$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] -x + RootSum[1 - #1^3 + #1^6 & , Log[x - #1]/(-#1^2 + 2*#1^5) & ]/3
```

Maple [C] time = 0.004, size = 41, normalized size = 0.1

$$-x+\frac{1}{3}\sum_{\_R=\text{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\ln(x-\_R)}{2\_R^5-\_R^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-x^3+1)/(x^6-x^3+1),x)
```

```
[Out] -x+1/3*sum(1/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-x+\int\frac{1}{x^6-x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -x + integrate(1/(x^6 - x^3 + 1), x)
```

Fricas [B] time = 1.67323, size = 3906, normalized size = 10.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/54*18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) - 2))*log(18^(2/3)*12^(1/6)*s
qrt(3)*x*sin(2/3*arctan(sqrt(3) - 2)) + 3*18^(2/3)*12^(1/6)*x*cos(2/3*arcta
n(sqrt(3) - 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18
^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2) - 2/27*18^(2/3)*12
^(1/6)*arctan(1/108*(6*18^(1/3)*12^(5/6)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) -
 2)) + 108*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 108*sqrt(3)*sin(2/3*arc
tan(sqrt(3) - 2))^2 + 18*(18^(1/3)*12^(5/6)*x + 24*cos(2/3*arctan(sqrt(3) -
 2)))*sin(2/3*arctan(sqrt(3) - 2)) - sqrt(18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2
/3*arctan(sqrt(3) - 2)) + 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) - 2)
) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3)*12^(1/3
)*sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2)*(18^(1/3)*12^(5/6)*sqrt(3)*sqrt(
2)*cos(2/3*arctan(sqrt(3) - 2)) + 3*18^(1/3)*12^(5/6)*sqrt(2)*sin(2/3*arcta
n(sqrt(3) - 2))))/(cos(2/3*arctan(sqrt(3) - 2))^2 - 3*sin(2/3*arctan(sqrt(3
) - 2))^2))*sin(2/3*arctan(sqrt(3) - 2)) - 1/27*(18^(2/3)*12^(1/6)*sqrt(3)*
cos(2/3*arctan(sqrt(3) - 2)) + 18^(2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3) - 2
)))*arctan(-1/108*(6*18^(1/3)*12^(5/6)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2
)) - 108*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 108*sqrt(3)*sin(2/3*arcta
n(sqrt(3) - 2))^2 - 18*(18^(1/3)*12^(5/6)*x - 24*cos(2/3*arctan(sqrt(3) - 2
)))*sin(2/3*arctan(sqrt(3) - 2)) - sqrt(18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3
*arctan(sqrt(3) - 2)) - 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) - 2))
+ 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3)*12^(1/3)*
sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2)*(18^(1/3)*12^(5/6)*sqrt(3)*sqrt(2)
*cos(2/3*arctan(sqrt(3) - 2)) - 3*18^(1/3)*12^(5/6)*sqrt(2)*sin(2/3*arctan(
sqrt(3) - 2))))/(cos(2/3*arctan(sqrt(3) - 2))^2 - 3*sin(2/3*arctan(sqrt(3)
- 2))^2)) - 1/27*(18^(2/3)*12^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2)) -
18^(2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3) - 2)))*arctan(1/216*(18^(1/3)*12^(
5/6)*sqrt(3)*sqrt(2)*sqrt(-2*18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqr
t(3) - 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3
)*12^(1/3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2) - 6*18^(1/3)*12^(5/6)*s
qrt(3)*x + 216*sin(2/3*arctan(sqrt(3) - 2)))/cos(2/3*arctan(sqrt(3) - 2)))
+ 1/108*(18^(2/3)*12^(1/6)*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2)) - 18^(2/3)*
12^(1/6)*cos(2/3*arctan(sqrt(3) - 2)))*log(18^(2/3)*12^(1/6)*sqrt(3)*x*sin(
2/3*arctan(sqrt(3) - 2)) - 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) - 2
)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3)*12^(1/
3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2) - 1/108*(18^(2/3)*12^(1/6)*sqrt
(3)*sin(2/3*arctan(sqrt(3) - 2)) + 18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3)
 - 2)))*log(-2*18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) - 2)) + 3
*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3)*12^(1/3)*sin
(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2) - x
```

Sympy [A] time = 0.17863, size = 24, normalized size = 0.06

$$-x - \text{RootSum}\left(19683t^6 + 243t^3 + 1, \left(t \mapsto t \log\left(729t^4 + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-x**3+1)/(x**6-x**3+1),x)
```

```
[Out] -x - RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(729*_t**4 + x))
)
```

Giac [B] time = 1.17756, size = 853, normalized size = 2.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-x^3+1)/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] -1/9*(sqrt(3)*cos(4/9*pi)^4 - 6*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^2 + sqrt(
3)*sin(4/9*pi)^4 + 4*cos(4/9*pi)^3*sin(4/9*pi) - 4*cos(4/9*pi)*sin(4/9*pi)^
3 - sqrt(3)*cos(4/9*pi) - sin(4/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(4/9*pi)
 - 2*x)/((sqrt(3)*i + 1)*sin(4/9*pi))) - 1/9*(sqrt(3)*cos(2/9*pi)^4 - 6*sqr
t(3)*cos(2/9*pi)^2*sin(2/9*pi)^2 + sqrt(3)*sin(2/9*pi)^4 + 4*cos(2/9*pi)^3*
sin(2/9*pi) - 4*cos(2/9*pi)*sin(2/9*pi)^3 - sqrt(3)*cos(2/9*pi) - sin(2/9*p
i))*arctan(-((sqrt(3)*i + 1)*cos(2/9*pi) - 2*x)/((sqrt(3)*i + 1)*sin(2/9*pi
))) - 1/9*(sqrt(3)*cos(1/9*pi)^4 - 6*sqrt(3)*cos(1/9*pi)^2*sin(1/9*pi)^2 +
sqrt(3)*sin(1/9*pi)^4 - 4*cos(1/9*pi)^3*sin(1/9*pi) + 4*cos(1/9*pi)*sin(1/9
*pi)^3 + sqrt(3)*cos(1/9*pi) - sin(1/9*pi))*arctan(((sqrt(3)*i + 1)*cos(1/9
*pi) + 2*x)/((sqrt(3)*i + 1)*sin(1/9*pi))) - 1/18*(4*sqrt(3)*cos(4/9*pi)^3*
sin(4/9*pi) - 4*sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^3 - cos(4/9*pi)^4 + 6*cos(4
/9*pi)^2*sin(4/9*pi)^2 - sin(4/9*pi)^4 - sqrt(3)*sin(4/9*pi) + cos(4/9*pi))
*log(-(sqrt(3)*i*cos(4/9*pi) + cos(4/9*pi))*x + x^2 + 1) - 1/18*(4*sqrt(3)*
cos(2/9*pi)^3*sin(2/9*pi) - 4*sqrt(3)*cos(2/9*pi)*sin(2/9*pi)^3 - cos(2/9*p
i)^4 + 6*cos(2/9*pi)^2*sin(2/9*pi)^2 - sin(2/9*pi)^4 - sqrt(3)*sin(2/9*pi)
+ cos(2/9*pi))*log(-(sqrt(3)*i*cos(2/9*pi) + cos(2/9*pi))*x + x^2 + 1) + 1/
18*(4*sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi) - 4*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)
^3 + cos(1/9*pi)^4 - 6*cos(1/9*pi)^2*sin(1/9*pi)^2 + sin(1/9*pi)^4 + sqrt(3
)*sin(1/9*pi) + cos(1/9*pi))*log((sqrt(3)*i*cos(1/9*pi) + cos(1/9*pi))*x +
x^2 + 1) - x
```

3.28 $\int \frac{x(1-x^3)}{1-x^3+x^6}\,dx$

Optimal. Leaf size=411

$$\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}-\frac{(3-}{}$$

```
[Out] ((I - Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^
(2/3)*(1 - I*Sqrt[3])^(1/3)) - ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 + I*Sqr
t[3])/2)^(1/3))/Sqrt[3]])/(3*2^(2/3)*(1 + I*Sqrt[3])^(1/3)) - ((3 - I*Sqrt[
3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(2/3)*(1 - I*Sqrt[3])^(1/3
)) - ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(2/3)*(1
 + I*Sqrt[3])^(1/3)) + ((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3) + (2*(1 -
 I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 - I*Sqrt[3])^(1/3)) + (
(3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(1/3)*x + 2
^(2/3)*x^2])/(18*2^(2/3)*(1 + I*Sqrt[3])^(1/3))
```

Rubi [A] time = 0.276176, antiderivative size = 411, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1510, 292, 31, 634, 617, 204, 628}

$$\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}-\frac{(3-}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] ((I - Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^
(2/3)*(1 - I*Sqrt[3])^(1/3)) - ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 + I*Sqr
t[3])/2)^(1/3))/Sqrt[3]])/(3*2^(2/3)*(1 + I*Sqrt[3])^(1/3)) - ((3 - I*Sqrt[
3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(2/3)*(1 - I*Sqrt[3])^(1/3
)) - ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(2/3)*(1
 + I*Sqrt[3])^(1/3)) + ((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3) + (2*(1 -
```

```
 I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 - I*Sqrt[3])^(1/3)) + (
(3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(1/3)*x + 2
^(2/3)*x^2])/(18*2^(2/3)*(1 + I*Sqrt[3])^(1/3))
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x\left(1-x^3\right)}{1-x^3+x^6}\,dx = \frac{1}{6}\left(-3+i\sqrt{3}\right)\int \frac{x}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx - \frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx$$

$$= -\frac{\left(3-i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}\,dx}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} + \frac{\left(3-i\sqrt{3}\right)\int \frac{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,dx}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{\left(3+i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1+i\right.}}}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{\ }}}$$

$$= -\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}} + \frac{\left(3-i\sqrt{3}\right)\int \frac{\sqrt[3]{\frac{1}{2}\left(1-\right.}}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+}}{18\ 2^{2/3}\sqrt[3]{1-}}$$

$$= -\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}} + \frac{\left(3-i\sqrt{3}\right)\log\left(\left(1-i\sqrt{3}\right)\right.}{18\ 2^{2/}}$$

$$= \frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}} - \frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}} -$$

Mathematica [C] time = 0.0121177, size = 55, normalized size = 0.13

$$-\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1^3\log(x-\#1)-\log(x-\#1)}{2\#1^4-\#1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] -RootSum[1 - #1^3 + #1^6 & , (-Log[x - #1] + Log[x - #1]*#1^3)/(-#1 + 2*#1^
4) & ]/3
```

Maple [C] time = 0.006, size = 44, normalized size = 0.1

$$-\frac{1}{3}\sum_{\_R=\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\left(\_R^4-\_R\right)\ln\left(x-\_R\right)}{2\_R^5-\_R^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^3+1)/(x^6-x^3+1),x)
```

```
[Out] -1/3*sum((_R^4-_R)/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\left(x^3-1\right)x}{x^6-x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^3 - 1)*x/(x^6 - x^3 + 1), x)
```

Fricas [B] time = 2.01332, size = 5936, normalized size = 14.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/54*18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) + 2))*log(18^(2/3)*12^(2/3)*c
os(2/3*arctan(sqrt(3) + 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) +
2))^4 - 12*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 2*(18^(2/3)
```

```
*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 6*18^(1/3)*12^(1/3)*x)*sin(2/3*a
rctan(sqrt(3) + 2))^2 + 36*x^2) + 2/27*18^(2/3)*12^(1/6)*arctan(-1/432*(6*1
8^(2/3)*12^(2/3)*x - 216*cos(2/3*arctan(sqrt(3) + 2))^2 + 216*sin(2/3*arcta
n(sqrt(3) + 2))^2 - 18^(2/3)*12^(2/3)*sqrt(18^(2/3)*12^(2/3)*cos(2/3*arctan
(sqrt(3) + 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) + 2))^4 - 12*18
^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 2*(18^(2/3)*12^(2/3)*cos
(2/3*arctan(sqrt(3) + 2))^2 + 6*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3)
 + 2))^2 + 36*x^2))/(cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) +
2))))*sin(2/3*arctan(sqrt(3) + 2)) + 1/27*(18^(2/3)*12^(1/6)*sqrt(3)*cos(2/
3*arctan(sqrt(3) + 2)) - 18^(2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3) + 2)))*ar
ctan(1/108*(6*18^(2/3)*12^(2/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 +
108*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^4 + 108*sqrt(3)*sin(2/3*arctan(sqr
t(3) + 2))^4 + 864*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2)
)^3 - 6*(18^(2/3)*12^(2/3)*sqrt(3)*x - 36*sqrt(3)*cos(2/3*arctan(sqrt(3) +
2))^2)*sin(2/3*arctan(sqrt(3) + 2))^2 - 12*(18^(2/3)*12^(2/3)*x*cos(2/3*arc
tan(sqrt(3) + 2)) + 72*cos(2/3*arctan(sqrt(3) + 2))^3)*sin(2/3*arctan(sqrt(
3) + 2)) - sqrt(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))^4 + 18^(2/3)
*12^(2/3)*sin(2/3*arctan(sqrt(3) + 2))^4 - 12*18^(1/3)*12^(1/3)*sqrt(3)*x*c
os(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2)) + 6*18^(1/3)*12^(1
/3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(
sqrt(3) + 2))^2 - 3*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) + 2))^2 + 3
6*x^2)*(18^(2/3)*12^(2/3)*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^2 - 18^(2/3)
*12^(2/3)*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2))^2 - 2*18^(2/3)*12^(2/3)*cos(
2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2))))/(3*cos(2/3*arctan(s
qrt(3) + 2))^4 - 10*cos(2/3*arctan(sqrt(3) + 2))^2*sin(2/3*arctan(sqrt(3) +
 2))^2 + 3*sin(2/3*arctan(sqrt(3) + 2))^4)) + 1/27*(18^(2/3)*12^(1/6)*sqrt(
3)*cos(2/3*arctan(sqrt(3) + 2)) + 18^(2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3)
+ 2)))*arctan(1/108*(6*18^(2/3)*12^(2/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) +
 2))^2 + 108*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^4 + 108*sqrt(3)*sin(2/3*a
rctan(sqrt(3) + 2))^4 - 864*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqr
t(3) + 2))^3 - 6*(18^(2/3)*12^(2/3)*sqrt(3)*x - 36*sqrt(3)*cos(2/3*arctan(s
qrt(3) + 2))^2)*sin(2/3*arctan(sqrt(3) + 2))^2 + 12*(18^(2/3)*12^(2/3)*x*co
s(2/3*arctan(sqrt(3) + 2)) + 72*cos(2/3*arctan(sqrt(3) + 2))^3)*sin(2/3*arc
tan(sqrt(3) + 2)) - sqrt(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))^4 +
 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) + 2))^4 + 12*18^(1/3)*12^(1/3)*sq
rt(3)*x*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2)) + 6*18^(1
/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/
3*arctan(sqrt(3) + 2))^2 - 3*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) +
2))^2 + 36*x^2)*(18^(2/3)*12^(2/3)*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^2 -
 18^(2/3)*12^(2/3)*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 2*18^(2/3)*12^(
2/3)*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2))))/(3*cos(2/3
*arctan(sqrt(3) + 2))^4 - 10*cos(2/3*arctan(sqrt(3) + 2))^2*sin(2/3*arctan(
sqrt(3) + 2))^2 + 3*sin(2/3*arctan(sqrt(3) + 2))^4)) + 1/108*(18^(2/3)*12^(
1/6)*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2)) - 18^(2/3)*12^(1/6)*cos(2/3*arcta
n(sqrt(3) + 2)))*log(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))^4 + 18^
```

```
(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) + 2))^4 + 12*18^(1/3)*12^(1/3)*sqrt(3
)*x*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2)) + 6*18^(1/3)*
12^(1/3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*ar
ctan(sqrt(3) + 2))^2 - 3*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) + 2))^
2 + 36*x^2) - 1/108*(18^(2/3)*12^(1/6)*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2))
 + 18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) + 2)))*log(18^(2/3)*12^(2/3)*co
s(2/3*arctan(sqrt(3) + 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) + 2
))^4 - 12*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*
arctan(sqrt(3) + 2)) + 6*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) + 2))^2
 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))^2 - 3*18^(1/3)*12^(1/3
)*x)*sin(2/3*arctan(sqrt(3) + 2))^2 + 36*x^2)
```

Sympy [A] time = 0.183276, size = 22, normalized size = 0.05

$$-\operatorname{RootSum}\left(19683t^6 - 243t^3 + 1, \left(t \mapsto t\log\left(-27t^2 + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**3+1)/(x**6-x**3+1),x)
```

```
[Out] -RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(-27*_t**2 + x)))
```

Giac [B] time = 1.19578, size = 1108, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^3+1)/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] 1/9*(sqrt(3)*cos(4/9*pi)^5 - 10*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi)^2 + 5*sqr
t(3)*cos(4/9*pi)*sin(4/9*pi)^4 - 5*cos(4/9*pi)^4*sin(4/9*pi) + 10*cos(4/9*p
i)^2*sin(4/9*pi)^3 - sin(4/9*pi)^5 + 2*sqrt(3)*cos(4/9*pi)^2 - 2*sqrt(3)*si
n(4/9*pi)^2 - 4*cos(4/9*pi)*sin(4/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(4/9*p
i) - 2*x)/((sqrt(3)*i + 1)*sin(4/9*pi))) + 1/9*(sqrt(3)*cos(2/9*pi)^5 - 10*
sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi)^2 + 5*sqrt(3)*cos(2/9*pi)*sin(2/9*pi)^4 -
 5*cos(2/9*pi)^4*sin(2/9*pi) + 10*cos(2/9*pi)^2*sin(2/9*pi)^3 - sin(2/9*pi)
^5 + 2*sqrt(3)*cos(2/9*pi)^2 - 2*sqrt(3)*sin(2/9*pi)^2 - 4*cos(2/9*pi)*sin(
2/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(2/9*pi) - 2*x)/((sqrt(3)*i + 1)*sin(2
```

```
/9*pi))) - 1/9*(sqrt(3)*cos(1/9*pi)^5 - 10*sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi
)^2 + 5*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^4 + 5*cos(1/9*pi)^4*sin(1/9*pi) - 1
0*cos(1/9*pi)^2*sin(1/9*pi)^3 + sin(1/9*pi)^5 - 2*sqrt(3)*cos(1/9*pi)^2 + 2
*sqrt(3)*sin(1/9*pi)^2 - 4*cos(1/9*pi)*sin(1/9*pi))*arctan(((sqrt(3)*i + 1)
*cos(1/9*pi) + 2*x)/((sqrt(3)*i + 1)*sin(1/9*pi))) + 1/18*(5*sqrt(3)*cos(4/
9*pi)^4*sin(4/9*pi) - 10*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^3 + sqrt(3)*sin(
4/9*pi)^5 + cos(4/9*pi)^5 - 10*cos(4/9*pi)^3*sin(4/9*pi)^2 + 5*cos(4/9*pi)*
sin(4/9*pi)^4 + 4*sqrt(3)*cos(4/9*pi)*sin(4/9*pi) + 2*cos(4/9*pi)^2 - 2*sin
(4/9*pi)^2)*log(-(sqrt(3)*i*cos(4/9*pi) + cos(4/9*pi))*x + x^2 + 1) + 1/18*
(5*sqrt(3)*cos(2/9*pi)^4*sin(2/9*pi) - 10*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)
^3 + sqrt(3)*sin(2/9*pi)^5 + cos(2/9*pi)^5 - 10*cos(2/9*pi)^3*sin(2/9*pi)^2
 + 5*cos(2/9*pi)*sin(2/9*pi)^4 + 4*sqrt(3)*cos(2/9*pi)*sin(2/9*pi) + 2*cos(
2/9*pi)^2 - 2*sin(2/9*pi)^2)*log(-(sqrt(3)*i*cos(2/9*pi) + cos(2/9*pi))*x +
 x^2 + 1) + 1/18*(5*sqrt(3)*cos(1/9*pi)^4*sin(1/9*pi) - 10*sqrt(3)*cos(1/9*
pi)^2*sin(1/9*pi)^3 + sqrt(3)*sin(1/9*pi)^5 - cos(1/9*pi)^5 + 10*cos(1/9*pi
)^3*sin(1/9*pi)^2 - 5*cos(1/9*pi)*sin(1/9*pi)^4 - 4*sqrt(3)*cos(1/9*pi)*sin
(1/9*pi) + 2*cos(1/9*pi)^2 - 2*sin(1/9*pi)^2)*log((sqrt(3)*i*cos(1/9*pi) +
cos(1/9*pi))*x + x^2 + 1)
```

3.29 $\int \frac{1-x^3}{1-x^3+x^6}\,dx$

Optimal. Leaf size=411

$$\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{(3-i}{}$$

```
[Out] -((I - Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2
^(1/3)*(1 - I*Sqrt[3])^(2/3)) + ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 + I*Sq
rt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(1/3)*(1 + I*Sqrt[3])^(2/3)) - ((3 - I*Sqrt
[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(1 - I*Sqrt[3])^(2/
3)) - ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(
1 + I*Sqrt[3])^(2/3)) + ((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3) + (2*(1
- I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) +
((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(1/3)*x +
2^(2/3)*x^2])/(18*2^(1/3)*(1 + I*Sqrt[3])^(2/3))
```

Rubi [A] time = 0.276748, antiderivative size = 411, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {1422, 200, 31, 634, 617, 204, 628}

$$\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{(3-i}{}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)/(1 - x^3 + x^6),x]
```


```
[Out] -((I - Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2
^(1/3)*(1 - I*Sqrt[3])^(2/3)) + ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 + I*Sq
rt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(1/3)*(1 + I*Sqrt[3])^(2/3)) - ((3 - I*Sqrt
[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(1 - I*Sqrt[3])^(2/
3)) - ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(
1 + I*Sqrt[3])^(2/3)) + ((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3) + (2*(1
- I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) +
```

```
((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(1/3)*x +
2^(2/3)*x^2])/(18*2^(1/3)*(1 + I*Sqrt[3])^(2/3))
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1-x^3}{1-x^3+x^6}\,dx = \frac{1}{6}\left(-3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx - \frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx$$

$$= -\frac{\left(3-i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}\,dx}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} - \frac{\left(3-i\sqrt{3}\right)\int \frac{-2^{2/3}\sqrt[3]{1-i\sqrt{3}}-x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,dx}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} - \frac{\left(3+i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

$$= -\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} - \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} + \frac{\left(3-i\sqrt{3}\right)\int \frac{\sqrt[3]{\frac{1}{2}\left(1-i\right.}}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{}}}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right.}$$

$$= -\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} - \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} + \frac{\left(3-i\sqrt{3}\right)\log\left(\left(1-i\sqrt{3}\right)^{2/}\right.}{18\sqrt[3]{2}\left(1\right.}$$

$$= -\frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} + \frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} - \frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} -$$

Mathematica [C] time = 0.0116653, size = 57, normalized size = 0.14

$$-\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1^3\log(x-\#1)-\log(x-\#1)}{2\#1^5-\#1^2}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^3)/(1 - x^3 + x^6),x]
```

```
[Out] -RootSum[1 - #1^3 + #1^6 & , (-Log[x - #1] + Log[x - #1]*#1^3)/(-#1^2 + 2*#
1^5) & ]/3
```

Maple [C] time = 0.004, size = 44, normalized size = 0.1

$$\frac{1}{3}\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)}\frac{\left(-\_R^{3}+1\right)\ln\left(x-\_R\right)}{2\_R^{5}-\_R^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)/(x^6-x^3+1),x)
```

```
[Out] 1/3*sum((-_R^3+1)/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x^{3}-1}{x^{6}-x^{3}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^3 - 1)/(x^6 - x^3 + 1), x)
```

Fricas [B] time = 1.639, size = 3903, normalized size = 9.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/54*18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) + 2))*log(-18^(2/3)*12^(1/6)*
sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) + 3*18^(2/3)*12^(1/6)*x*cos(2/3*arct
an(sqrt(3) + 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 3*1
8^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18*x^2) - 2/27*18^(2/3)*1
2^(1/6)*arctan(-1/108*(6*18^(1/3)*12^(5/6)*sqrt(3)*x*cos(2/3*arctan(sqrt(3)
 + 2)) + 108*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 108*sqrt(3)*sin(2/3*a
rctan(sqrt(3) + 2))^2 - 18*(18^(1/3)*12^(5/6)*x + 24*cos(2/3*arctan(sqrt(3)
```

```
 + 2)))*sin(2/3*arctan(sqrt(3) + 2)) - sqrt(-18^(2/3)*12^(1/6)*sqrt(3)*x*si
n(2/3*arctan(sqrt(3) + 2)) + 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) +
 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 3*18^(1/3)*12^(
1/3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18*x^2)*(18^(1/3)*12^(5/6)*sqrt(3)*sq
rt(2)*cos(2/3*arctan(sqrt(3) + 2)) - 3*18^(1/3)*12^(5/6)*sqrt(2)*sin(2/3*ar
ctan(sqrt(3) + 2))))/(cos(2/3*arctan(sqrt(3) + 2))^2 - 3*sin(2/3*arctan(sqr
t(3) + 2))^2))*sin(2/3*arctan(sqrt(3) + 2)) + 1/27*(18^(2/3)*12^(1/6)*sqrt(
3)*cos(2/3*arctan(sqrt(3) + 2)) - 18^(2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3)
+ 2)))*arctan(1/108*(6*18^(1/3)*12^(5/6)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) +
 2)) - 108*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^2 - 108*sqrt(3)*sin(2/3*arc
tan(sqrt(3) + 2))^2 + 18*(18^(1/3)*12^(5/6)*x - 24*cos(2/3*arctan(sqrt(3) +
 2)))*sin(2/3*arctan(sqrt(3) + 2)) - sqrt(-18^(2/3)*12^(1/6)*sqrt(3)*x*sin(
2/3*arctan(sqrt(3) + 2)) - 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) + 2
)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 3*18^(1/3)*12^(1/
3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18*x^2)*(18^(1/3)*12^(5/6)*sqrt(3)*sqrt
(2)*cos(2/3*arctan(sqrt(3) + 2)) + 3*18^(1/3)*12^(5/6)*sqrt(2)*sin(2/3*arct
an(sqrt(3) + 2))))/(cos(2/3*arctan(sqrt(3) + 2))^2 - 3*sin(2/3*arctan(sqrt(
3) + 2))^2)) - 1/27*(18^(2/3)*12^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))
 + 18^(2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3) + 2)))*arctan(1/216*(18^(1/3)*1
2^(5/6)*sqrt(3)*sqrt(2)*sqrt(2*18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(s
qrt(3) + 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 3*18^(1
/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18*x^2) - 6*18^(1/3)*12^(5/6)
*sqrt(3)*x - 216*sin(2/3*arctan(sqrt(3) + 2)))/cos(2/3*arctan(sqrt(3) + 2))
) + 1/108*(18^(2/3)*12^(1/6)*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2)) - 18^(2/3
)*12^(1/6)*cos(2/3*arctan(sqrt(3) + 2)))*log(2*18^(2/3)*12^(1/6)*sqrt(3)*x*
sin(2/3*arctan(sqrt(3) + 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) +
 2))^2 + 3*18^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) + 2))^2 + 18*x^2) - 1/1
08*(18^(2/3)*12^(1/6)*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2)) + 18^(2/3)*12^(1
/6)*cos(2/3*arctan(sqrt(3) + 2)))*log(-18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*
arctan(sqrt(3) + 2)) - 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) + 2)) +
 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 3*18^(1/3)*12^(1/3)*s
in(2/3*arctan(sqrt(3) + 2))^2 + 18*x^2)
```

Sympy [A] time = 0.184936, size = 26, normalized size = 0.06

$$-\operatorname{RootSum}\left(19683t^6-243t^3+1,\left(t\mapsto t\log\left(729t^4-9t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)/(x**6-x**3+1),x)
```

```
[Out] -RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(729*_t**4 - 9*_t +
```

```
x)))
```

Giac [B] time = 1.19457, size = 860, normalized size = 2.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] 1/9*(sqrt(3)*cos(4/9*pi)^4 - 6*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^2 + sqrt(3
)*sin(4/9*pi)^4 + 4*cos(4/9*pi)^3*sin(4/9*pi) - 4*cos(4/9*pi)*sin(4/9*pi)^3
 + 2*sqrt(3)*cos(4/9*pi) + 2*sin(4/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(4/9*
pi) - 2*x)/((sqrt(3)*i + 1)*sin(4/9*pi))) + 1/9*(sqrt(3)*cos(2/9*pi)^4 - 6*
sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^2 + sqrt(3)*sin(2/9*pi)^4 + 4*cos(2/9*pi)
^3*sin(2/9*pi) - 4*cos(2/9*pi)*sin(2/9*pi)^3 + 2*sqrt(3)*cos(2/9*pi) + 2*si
n(2/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(2/9*pi) - 2*x)/((sqrt(3)*i + 1)*sin
(2/9*pi))) + 1/9*(sqrt(3)*cos(1/9*pi)^4 - 6*sqrt(3)*cos(1/9*pi)^2*sin(1/9*p
i)^2 + sqrt(3)*sin(1/9*pi)^4 - 4*cos(1/9*pi)^3*sin(1/9*pi) + 4*cos(1/9*pi)*
sin(1/9*pi)^3 - 2*sqrt(3)*cos(1/9*pi) + 2*sin(1/9*pi))*arctan(((sqrt(3)*i +
 1)*cos(1/9*pi) + 2*x)/((sqrt(3)*i + 1)*sin(1/9*pi))) + 1/18*(4*sqrt(3)*cos
(4/9*pi)^3*sin(4/9*pi) - 4*sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^3 - cos(4/9*pi)^
4 + 6*cos(4/9*pi)^2*sin(4/9*pi)^2 - sin(4/9*pi)^4 + 2*sqrt(3)*sin(4/9*pi) -
 2*cos(4/9*pi))*log(-(sqrt(3)*i*cos(4/9*pi) + cos(4/9*pi))*x + x^2 + 1) + 1
/18*(4*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi) - 4*sqrt(3)*cos(2/9*pi)*sin(2/9*pi
)^3 - cos(2/9*pi)^4 + 6*cos(2/9*pi)^2*sin(2/9*pi)^2 - sin(2/9*pi)^4 + 2*sqr
t(3)*sin(2/9*pi) - 2*cos(2/9*pi))*log(-(sqrt(3)*i*cos(2/9*pi) + cos(2/9*pi)
)*x + x^2 + 1) - 1/18*(4*sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi) - 4*sqrt(3)*cos(
1/9*pi)*sin(1/9*pi)^3 + cos(1/9*pi)^4 - 6*cos(1/9*pi)^2*sin(1/9*pi)^2 + sin
(1/9*pi)^4 - 2*sqrt(3)*sin(1/9*pi) - 2*cos(1/9*pi))*log((sqrt(3)*i*cos(1/9*
pi) + cos(1/9*pi))*x + x^2 + 1)
```

3.30 $\int \frac{1-x^3}{x^2\left(1-x^3+x^6\right)}\,dx$

Optimal. Leaf size=416

$$\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}-\frac{1}{x}-\cdots$$

```
[Out] -x^(-1) - ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[
3]])/(3*2^(2/3)*(1 - I*Sqrt[3])^(1/3)) + ((I - Sqrt[3])*ArcTan[(1 + (2*x)/(
(1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(2/3)*(1 + I*Sqrt[3])^(1/3)) - ((3
 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(2/3)*(1 - I*Sqr
t[3])^(1/3)) - ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*
2^(2/3)*(1 + I*Sqrt[3])^(1/3)) + ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3)
 + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 - I*Sqrt[3])^
(1/3)) + ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(
1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 + I*Sqrt[3])^(1/3))
```

Rubi [A] time = 0.275499, antiderivative size = 416, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {1504, 1374, 292, 31, 634, 617, 204, 628}

$$\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}-\frac{1}{x}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)/(x^2*(1 - x^3 + x^6)),x]
```

```
[Out] -x^(-1) - ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[
3]])/(3*2^(2/3)*(1 - I*Sqrt[3])^(1/3)) + ((I - Sqrt[3])*ArcTan[(1 + (2*x)/(
(1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(2/3)*(1 + I*Sqrt[3])^(1/3)) - ((3
 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(2/3)*(1 - I*Sqr
t[3])^(1/3)) - ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*
2^(2/3)*(1 + I*Sqrt[3])^(1/3)) + ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3)
```

```
 + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 - I*Sqrt[3])^
(1/3)) + ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(
1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 + I*Sqrt[3])^(1/3))
```

Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1-x^3}{x^2\left(1-x^3+x^6\right)}\,dx = -\frac{1}{x}-\int \frac{x^4}{1-x^3+x^6}\,dx$$

$$= -\frac{1}{x}+\frac{1}{6}\left(-3+i\sqrt{3}\right)\int \frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx-\frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{x}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx$$

$$= -\frac{1}{x}+-\frac{\left(-3-i\sqrt{3}\right)\int \frac{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,dx}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(3-i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+x}\,dx}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}+\frac{\left(3-i\sqrt{3}\right)}{}$$

$$= -\frac{1}{x}-\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}+-\frac{\left(-3-i\sqrt{3}\right)\int}{18}$$

$$= -\frac{1}{x}-\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}+\frac{\left(3+i\sqrt{3}\right)\log\left(\left(1\right.\right.}{}$$

$$= -\frac{1}{x}-\frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}-\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}\right.}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}$$

Mathematica [C] time = 0.0136518, size = 47, normalized size = 0.11

$$-\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1^2\log(x-\#1)}{2\#1^3-1}\&\right]-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^3)/(x^2*(1 - x^3 + x^6)),x]
```

```
[Out] -x^(-1) - RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1^2)/(-1 + 2*#1^3) & ]/
3
```

Maple [C] time = 0.006, size = 46, normalized size = 0.1

$$-\frac{1}{3}\sum_{\_R=\text{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\_R^4\ln(x-\_R)}{2\_R^5-\_R^2}-x^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)/x^2/(x^6-x^3+1),x)
```

```
[Out] -1/3*sum(_R^4/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))-1/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{x}-\int\frac{x^4}{x^6-x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x^2/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -1/x - integrate(x^4/(x^6 - x^3 + 1), x)
```

Fricas [B] time = 1.93695, size = 5952, normalized size = 14.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x^2/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/108*(2*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) - 2))*log(18^(2/3)*12^(
2/3)*cos(2/3*arctan(sqrt(3) - 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt
(3) - 2))^4 - 12*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))*s
in(2/3*arctan(sqrt(3) - 2)) + 6*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3)
- 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 3*18^(1/3)*
12^(1/3)*x)*sin(2/3*arctan(sqrt(3) - 2))^2 + 36*x^2) + 8*18^(2/3)*12^(1/6)*
x*arctan(1/108*(6*18^(2/3)*12^(2/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))^
2 + 108*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^4 + 108*sqrt(3)*sin(2/3*arctan
(sqrt(3) - 2))^4 + 864*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3)
- 2))^3 - 6*(18^(2/3)*12^(2/3)*sqrt(3)*x - 36*sqrt(3)*cos(2/3*arctan(sqrt(3
) - 2))^2)*sin(2/3*arctan(sqrt(3) - 2))^2 - 12*(18^(2/3)*12^(2/3)*x*cos(2/3
*arctan(sqrt(3) - 2)) + 72*cos(2/3*arctan(sqrt(3) - 2))^3)*sin(2/3*arctan(s
qrt(3) - 2)) - sqrt(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^4 + 18^(
2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) - 2))^4 - 12*18^(1/3)*12^(1/3)*sqrt(3)
*x*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2)) + 6*18^(1/3)*1
2^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arc
tan(sqrt(3) - 2))^2 - 3*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) - 2))^2
 + 36*x^2)*(18^(2/3)*12^(2/3)*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 18^(
2/3)*12^(2/3)*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2))^2 - 2*18^(2/3)*12^(2/3)*
cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2))))/(3*cos(2/3*arct
an(sqrt(3) - 2))^4 - 10*cos(2/3*arctan(sqrt(3) - 2))^2*sin(2/3*arctan(sqrt(
3) - 2))^2 + 3*sin(2/3*arctan(sqrt(3) - 2))^4))*sin(2/3*arctan(sqrt(3) - 2)
) - 4*(18^(2/3)*12^(1/6)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2)) - 18^(2/3)*
12^(1/6)*x*sin(2/3*arctan(sqrt(3) - 2)))*arctan(1/108*(6*18^(2/3)*12^(2/3)*
sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 108*sqrt(3)*cos(2/3*arctan(sqrt(
3) - 2))^4 + 108*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2))^4 - 864*cos(2/3*arcta
n(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2))^3 - 6*(18^(2/3)*12^(2/3)*sqrt(
3)*x - 36*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2)*sin(2/3*arctan(sqrt(3) -
2))^2 + 12*(18^(2/3)*12^(2/3)*x*cos(2/3*arctan(sqrt(3) - 2)) + 72*cos(2/3*a
rctan(sqrt(3) - 2))^3)*sin(2/3*arctan(sqrt(3) - 2)) - sqrt(18^(2/3)*12^(2/3
)*cos(2/3*arctan(sqrt(3) - 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3)
 - 2))^4 + 12*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))*sin(
2/3*arctan(sqrt(3) - 2)) + 6*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2
))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 3*18^(1/3)*12^
(1/3)*x)*sin(2/3*arctan(sqrt(3) - 2))^2 + 36*x^2)*(18^(2/3)*12^(2/3)*sqrt(3
)*cos(2/3*arctan(sqrt(3) - 2))^2 - 18^(2/3)*12^(2/3)*sqrt(3)*sin(2/3*arctan
(sqrt(3) - 2))^2 + 2*18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3
*arctan(sqrt(3) - 2))))/(3*cos(2/3*arctan(sqrt(3) - 2))^4 - 10*cos(2/3*arct
an(sqrt(3) - 2))^2*sin(2/3*arctan(sqrt(3) - 2))^2 + 3*sin(2/3*arctan(sqrt(3
) - 2))^4)) - 4*(18^(2/3)*12^(1/6)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2)) +
```

```
 18^(2/3)*12^(1/6)*x*sin(2/3*arctan(sqrt(3) - 2)))*arctan(-1/432*(6*18^(2/3
)*12^(2/3)*x - 216*cos(2/3*arctan(sqrt(3) - 2))^2 + 216*sin(2/3*arctan(sqrt
(3) - 2))^2 - 18^(2/3)*12^(2/3)*sqrt(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(
3) - 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) - 2))^4 - 12*18^(1/3)
*12^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*a
rctan(sqrt(3) - 2))^2 + 6*18^(1/3)*12^(1/3)*x)*sin(2/3*arctan(sqrt(3) - 2))
^2 + 36*x^2))/(cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2))))
- (18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) - 2)) + 18^(2/3)*12^(
1/6)*x*cos(2/3*arctan(sqrt(3) - 2)))*log(18^(2/3)*12^(2/3)*cos(2/3*arctan(s
qrt(3) - 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*arctan(sqrt(3) - 2))^4 + 12*18^(
1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3)
 - 2)) + 6*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 2*(18^(2/3)
*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 3*18^(1/3)*12^(1/3)*x)*sin(2/3*a
rctan(sqrt(3) - 2))^2 + 36*x^2) + (18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arct
an(sqrt(3) - 2)) - 18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) - 2)))*log(18
^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^4 + 18^(2/3)*12^(2/3)*sin(2/3*
arctan(sqrt(3) - 2))^4 - 12*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2)
)^2 + 2*(18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 6*18^(1/3)*12^(
1/3)*x)*sin(2/3*arctan(sqrt(3) - 2))^2 + 36*x^2) - 108)/x
```

Sympy [A] time = 0.18953, size = 31, normalized size = 0.07

$$-\operatorname{RootSum}\left(19683t^6+243t^3+1,\left(t\mapsto t\log\left(6561t^5+54t^2+x\right)\right)\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)/x**2/(x**6-x**3+1),x)
```

```
[Out] -RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(6561*_t**5 + 54*_t*
*2 + x))) - 1/x
```

Giac [B] time = 1.15749, size = 1119, normalized size = 2.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x^2/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] 1/9*(2*sqrt(3)*cos(4/9*pi)^5 - 20*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi)^2 + 10*
sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^4 - 10*cos(4/9*pi)^4*sin(4/9*pi) + 20*cos(4
/9*pi)^2*sin(4/9*pi)^3 - 2*sin(4/9*pi)^5 + sqrt(3)*cos(4/9*pi)^2 - sqrt(3)*
sin(4/9*pi)^2 - 2*cos(4/9*pi)*sin(4/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(4/9
*pi) - 2*x)/((sqrt(3)*i + 1)*sin(4/9*pi))) + 1/9*(2*sqrt(3)*cos(2/9*pi)^5 -
 20*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi)^2 + 10*sqrt(3)*cos(2/9*pi)*sin(2/9*pi
)^4 - 10*cos(2/9*pi)^4*sin(2/9*pi) + 20*cos(2/9*pi)^2*sin(2/9*pi)^3 - 2*sin
(2/9*pi)^5 + sqrt(3)*cos(2/9*pi)^2 - sqrt(3)*sin(2/9*pi)^2 - 2*cos(2/9*pi)*
sin(2/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(2/9*pi) - 2*x)/((sqrt(3)*i + 1)*s
in(2/9*pi))) - 1/9*(2*sqrt(3)*cos(1/9*pi)^5 - 20*sqrt(3)*cos(1/9*pi)^3*sin(
1/9*pi)^2 + 10*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^4 + 10*cos(1/9*pi)^4*sin(1/9
*pi) - 20*cos(1/9*pi)^2*sin(1/9*pi)^3 + 2*sin(1/9*pi)^5 - sqrt(3)*cos(1/9*p
i)^2 + sqrt(3)*sin(1/9*pi)^2 - 2*cos(1/9*pi)*sin(1/9*pi))*arctan(((sqrt(3)*
i + 1)*cos(1/9*pi) + 2*x)/((sqrt(3)*i + 1)*sin(1/9*pi))) + 1/18*(10*sqrt(3)
*cos(4/9*pi)^4*sin(4/9*pi) - 20*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^3 + 2*sqr
t(3)*sin(4/9*pi)^5 + 2*cos(4/9*pi)^5 - 20*cos(4/9*pi)^3*sin(4/9*pi)^2 + 10*
cos(4/9*pi)*sin(4/9*pi)^4 + 2*sqrt(3)*cos(4/9*pi)*sin(4/9*pi) + cos(4/9*pi)
^2 - sin(4/9*pi)^2)*log(-(sqrt(3)*i*cos(4/9*pi) + cos(4/9*pi))*x + x^2 + 1)
 + 1/18*(10*sqrt(3)*cos(2/9*pi)^4*sin(2/9*pi) - 20*sqrt(3)*cos(2/9*pi)^2*si
n(2/9*pi)^3 + 2*sqrt(3)*sin(2/9*pi)^5 + 2*cos(2/9*pi)^5 - 20*cos(2/9*pi)^3*
sin(2/9*pi)^2 + 10*cos(2/9*pi)*sin(2/9*pi)^4 + 2*sqrt(3)*cos(2/9*pi)*sin(2/
9*pi) + cos(2/9*pi)^2 - sin(2/9*pi)^2)*log(-(sqrt(3)*i*cos(2/9*pi) + cos(2/
9*pi))*x + x^2 + 1) + 1/18*(10*sqrt(3)*cos(1/9*pi)^4*sin(1/9*pi) - 20*sqrt(
3)*cos(1/9*pi)^2*sin(1/9*pi)^3 + 2*sqrt(3)*sin(1/9*pi)^5 - 2*cos(1/9*pi)^5
+ 20*cos(1/9*pi)^3*sin(1/9*pi)^2 - 10*cos(1/9*pi)*sin(1/9*pi)^4 - 2*sqrt(3)
*cos(1/9*pi)*sin(1/9*pi) + cos(1/9*pi)^2 - sin(1/9*pi)^2)*log((sqrt(3)*i*co
s(1/9*pi) + cos(1/9*pi))*x + x^2 + 1) - 1/x
```

3.31 $\int \frac{1-x^3}{x^3\left(1-x^3+x^6\right)}\, dx$

Optimal. Leaf size=418

$$-\frac{1}{2x^2}+\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

```
[Out] -1/(2*x^2) + ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sq
rt[3]])/(3*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) - ((I - Sqrt[3])*ArcTan[(1 + (2*x
)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(1/3)*(1 + I*Sqrt[3])^(2/3)) -
((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(1 - I*
Sqrt[3])^(2/3)) - ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/
(9*2^(1/3)*(1 + I*Sqrt[3])^(2/3)) + ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2
/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 - I*Sqrt[3
])^(2/3)) + ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3])
)^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 + I*Sqrt[3])^(2/3))
```

Rubi [A] time = 0.358926, antiderivative size = 418, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {1504, 12, 1374, 200, 31, 634, 617, 204, 628}

$$-\frac{1}{2x^2}+\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)/(x^3*(1 - x^3 + x^6)),x]
```

```
[Out] -1/(2*x^2) + ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sq
rt[3]])/(3*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) - ((I - Sqrt[3])*ArcTan[(1 + (2*x
)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(1/3)*(1 + I*Sqrt[3])^(2/3)) -
((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(1 - I*
Sqrt[3])^(2/3)) - ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/
(9*2^(1/3)*(1 + I*Sqrt[3])^(2/3)) + ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2
```

```
/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 - I*Sqrt[3
])^(2/3)) + ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3])
)^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 + I*Sqrt[3])^(2/3))
```

Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1374

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n*(b/q + 1))/2, Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n*(b/q - 1))/2, Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1-x^3}{x^3\left(1-x^3+x^6\right)}\,dx = -\frac{1}{2x^2}-\frac{1}{2}\int \frac{2x^3}{1-x^3+x^6}\,dx$$
$$= -\frac{1}{2x^2}-\int \frac{x^3}{1-x^3+x^6}\,dx$$
$$= -\frac{1}{2x^2}+\frac{1}{6}\left(-3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx-\frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx$$
$$= -\frac{1}{2x^2}-\frac{\left(3-i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+x}\,dx}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\int \frac{-2^{2/3}\sqrt[3]{1+i\sqrt{3}}-x}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}x+x^2}\,dx}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{\left(3+i\sqrt{3}\right)\int \cdots}{9\sqrt[3]{2}\left(\cdots\right.}$$
$$= -\frac{1}{2x^2}-\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+\frac{\left(3-i\sqrt{3}\right)\int \cdots}{18\cdots}$$
$$= -\frac{1}{2x^2}-\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+\frac{\left(3+i\sqrt{3}\right)\log\left(\cdots\right.}{\cdots}$$
$$= -\frac{1}{2x^2}+\frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\cdots}\right.}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2\cdots}}$$

Mathematica [C] time = 0.011925, size = 47, normalized size = 0.11

$$-\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1\log(x-\#1)}{2\#1^3-1}\&\right]-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^3)/(x^3*(1 - x^3 + x^6)),x]
```

```
[Out] -1/(2*x^2) - RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1)/(-1 + 2*#1^3) & ]
/3
```

Maple [C] time = 0.006, size = 46, normalized size = 0.1

$$-\frac{1}{3}\sum_{\_R=\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\_R^3\ln\left(x-\_R\right)}{2\_R^5-\_R^2}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)/x^3/(x^6-x^3+1),x)
```

```
[Out] -1/3*sum(_R^3/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))-1/2/x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2x^2}-\int\frac{x^3}{x^6-x^3+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x^3/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -1/2/x^2 - integrate(x^3/(x^6 - x^3 + 1), x)
```

Fricas [B] time = 1.62593, size = 3945, normalized size = 9.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x^3/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/108*(2*18^(2/3)*12^(1/6)*x^2*cos(2/3*arctan(sqrt(3) - 2))*log(-2*18^(2/3)
*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) - 2)) + 3*18^(1/3)*12^(1/3)*cos(
2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) - 2
))^2 + 18*x^2) - 8*18^(2/3)*12^(1/6)*x^2*arctan(1/216*(18^(1/3)*12^(5/6)*sq
rt(3)*sqrt(2)*sqrt(-2*18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) -
2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3)*12^(1
/3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2) - 6*18^(1/3)*12^(5/6)*sqrt(3)*
x + 216*sin(2/3*arctan(sqrt(3) - 2)))/cos(2/3*arctan(sqrt(3) - 2)))*sin(2/3
*arctan(sqrt(3) - 2)) + 4*(18^(2/3)*12^(1/6)*sqrt(3)*x^2*cos(2/3*arctan(sqr
```

```
t(3) - 2)) + 18^(2/3)*12^(1/6)*x^2*sin(2/3*arctan(sqrt(3) - 2)))*arctan(1/1
08*(6*18^(1/3)*12^(5/6)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2)) + 108*sqrt(3
)*cos(2/3*arctan(sqrt(3) - 2))^2 + 108*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2))
^2 + 18*(18^(1/3)*12^(5/6)*x + 24*cos(2/3*arctan(sqrt(3) - 2)))*sin(2/3*arc
tan(sqrt(3) - 2)) - sqrt(18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3)
 - 2)) + 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) - 2)) + 3*18^(1/3)*12
^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3)*12^(1/3)*sin(2/3*arctan(
sqrt(3) - 2))^2 + 18*x^2)*(18^(1/3)*12^(5/6)*sqrt(3)*sqrt(2)*cos(2/3*arctan
(sqrt(3) - 2)) + 3*18^(1/3)*12^(5/6)*sqrt(2)*sin(2/3*arctan(sqrt(3) - 2))))
/(cos(2/3*arctan(sqrt(3) - 2))^2 - 3*sin(2/3*arctan(sqrt(3) - 2))^2)) + 4*(
18^(2/3)*12^(1/6)*sqrt(3)*x^2*cos(2/3*arctan(sqrt(3) - 2)) - 18^(2/3)*12^(1
/6)*x^2*sin(2/3*arctan(sqrt(3) - 2)))*arctan(-1/108*(6*18^(1/3)*12^(5/6)*sq
rt(3)*x*cos(2/3*arctan(sqrt(3) - 2)) - 108*sqrt(3)*cos(2/3*arctan(sqrt(3) -
 2))^2 - 108*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2))^2 - 18*(18^(1/3)*12^(5/6)
*x - 24*cos(2/3*arctan(sqrt(3) - 2)))*sin(2/3*arctan(sqrt(3) - 2)) - sqrt(1
8^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) - 2)) - 3*18^(2/3)*12^(1/
6)*x*cos(2/3*arctan(sqrt(3) - 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt
(3) - 2))^2 + 3*18^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2)*
(18^(1/3)*12^(5/6)*sqrt(3)*sqrt(2)*cos(2/3*arctan(sqrt(3) - 2)) - 3*18^(1/3
)*12^(5/6)*sqrt(2)*sin(2/3*arctan(sqrt(3) - 2))))/(cos(2/3*arctan(sqrt(3) -
 2))^2 - 3*sin(2/3*arctan(sqrt(3) - 2))^2)) + (18^(2/3)*12^(1/6)*sqrt(3)*x^
2*sin(2/3*arctan(sqrt(3) - 2)) - 18^(2/3)*12^(1/6)*x^2*cos(2/3*arctan(sqrt(
3) - 2)))*log(18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) - 2)) + 3*
18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) - 2)) + 3*18^(1/3)*12^(1/3)*cos(
2/3*arctan(sqrt(3) - 2))^2 + 3*18^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) - 2
))^2 + 18*x^2) - (18^(2/3)*12^(1/6)*sqrt(3)*x^2*sin(2/3*arctan(sqrt(3) - 2)
) + 18^(2/3)*12^(1/6)*x^2*cos(2/3*arctan(sqrt(3) - 2)))*log(18^(2/3)*12^(1/
6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) - 2)) - 3*18^(2/3)*12^(1/6)*x*cos(2/3*a
rctan(sqrt(3) - 2)) + 3*18^(1/3)*12^(1/3)*cos(2/3*arctan(sqrt(3) - 2))^2 +
3*18^(1/3)*12^(1/3)*sin(2/3*arctan(sqrt(3) - 2))^2 + 18*x^2) - 54)/x^2
```

Sympy [A] time = 0.199761, size = 32, normalized size = 0.08

$$-\operatorname{RootSum}\left(19683t^6 + 243t^3 + 1, \left(t \mapsto t\log\left(-1458t^4 - 9t + x\right)\right)\right) - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)/x**3/(x**6-x**3+1),x)
```

```
[Out] -RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(-1458*_t**4 - 9*_t
+ x))) - 1/(2*x**2)
```

Giac [B] time = 1.16375, size = 867, normalized size = 2.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x^3/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] 1/9*(2*sqrt(3)*cos(4/9*pi)^4 - 12*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^2 + 2*s
qrt(3)*sin(4/9*pi)^4 + 8*cos(4/9*pi)^3*sin(4/9*pi) - 8*cos(4/9*pi)*sin(4/9*
pi)^3 + sqrt(3)*cos(4/9*pi) + sin(4/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(4/9
*pi) - 2*x)/((sqrt(3)*i + 1)*sin(4/9*pi))) + 1/9*(2*sqrt(3)*cos(2/9*pi)^4 -
 12*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^2 + 2*sqrt(3)*sin(2/9*pi)^4 + 8*cos(2
/9*pi)^3*sin(2/9*pi) - 8*cos(2/9*pi)*sin(2/9*pi)^3 + sqrt(3)*cos(2/9*pi) +
sin(2/9*pi))*arctan(-((sqrt(3)*i + 1)*cos(2/9*pi) - 2*x)/((sqrt(3)*i + 1)*s
in(2/9*pi))) + 1/9*(2*sqrt(3)*cos(1/9*pi)^4 - 12*sqrt(3)*cos(1/9*pi)^2*sin(
1/9*pi)^2 + 2*sqrt(3)*sin(1/9*pi)^4 - 8*cos(1/9*pi)^3*sin(1/9*pi) + 8*cos(1
/9*pi)*sin(1/9*pi)^3 - sqrt(3)*cos(1/9*pi) + sin(1/9*pi))*arctan(((sqrt(3)*
i + 1)*cos(1/9*pi) + 2*x)/((sqrt(3)*i + 1)*sin(1/9*pi))) + 1/18*(8*sqrt(3)*
cos(4/9*pi)^3*sin(4/9*pi) - 8*sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^3 - 2*cos(4/9
*pi)^4 + 12*cos(4/9*pi)^2*sin(4/9*pi)^2 - 2*sin(4/9*pi)^4 + sqrt(3)*sin(4/9
*pi) - cos(4/9*pi))*log(-(sqrt(3)*i*cos(4/9*pi) + cos(4/9*pi))*x + x^2 + 1)
 + 1/18*(8*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi) - 8*sqrt(3)*cos(2/9*pi)*sin(2/
9*pi)^3 - 2*cos(2/9*pi)^4 + 12*cos(2/9*pi)^2*sin(2/9*pi)^2 - 2*sin(2/9*pi)^
4 + sqrt(3)*sin(2/9*pi) - cos(2/9*pi))*log(-(sqrt(3)*i*cos(2/9*pi) + cos(2/
9*pi))*x + x^2 + 1) - 1/18*(8*sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi) - 8*sqrt(3)
*cos(1/9*pi)*sin(1/9*pi)^3 + 2*cos(1/9*pi)^4 - 12*cos(1/9*pi)^2*sin(1/9*pi)
^2 + 2*sin(1/9*pi)^4 - sqrt(3)*sin(1/9*pi) - cos(1/9*pi))*log((sqrt(3)*i*co
s(1/9*pi) + cos(1/9*pi))*x + x^2 + 1) - 1/2/x^2
```

3.32 $\int \frac{x^2\left(-2+x^3\right)}{1-x^3+x^6} \, dx$

Optimal. Leaf size=36

$$\frac{1}{6}\log\left(x^6-x^3+1\right)+\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] ArcTan[(1 - 2*x^3)/Sqrt[3]]/Sqrt[3] + Log[1 - x^3 + x^6]/6
```

Rubi [A] time = 0.0388101, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1468, 634, 618, 204, 628}

$$\frac{1}{6}\log\left(x^6-x^3+1\right)+\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(-2 + x^3))/(1 - x^3 + x^6),x]
```

```
[Out] ArcTan[(1 - 2*x^3)/Sqrt[3]]/Sqrt[3] + Log[1 - x^3 + x^6]/6
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\left(-2+x^3\right)}{1-x^3+x^6}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{-2+x}{1-x+x^2}\,dx,x,x^3\right)\\
&= \frac{1}{6}\,\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx,x,x^3\right)-\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^3\right)\\
&= \frac{1}{6}\log\left(1-x^3+x^6\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x^3\right)\\
&= -\frac{\tan^{-1}\left(\frac{-1+2x^3}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{6}\log\left(1-x^3+x^6\right)
\end{aligned}$$

Mathematica [A] time = 0.0095847, size = 37, normalized size = 1.03

$$\frac{1}{6}\log\left(x^6-x^3+1\right)-\frac{\tan^{-1}\left(\frac{2x^3-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(-2 + x^3))/(1 - x^3 + x^6),x]
```

```
[Out] -(ArcTan[(-1 + 2*x^3)/Sqrt[3]]/Sqrt[3]) + Log[1 - x^3 + x^6]/6
```

Maple [A] time = 0.003, size = 33, normalized size = 0.9

$$\frac{\ln\left(x^{6}-x^{3}+1\right)}{6}-\frac{\sqrt{3}}{3}\arctan\left(\frac{\left(2x^{3}-1\right)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^3-2)/(x^6-x^3+1),x)
```

```
[Out] 1/6*ln(x^6-x^3+1)-1/3*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

Maxima [A] time = 1.49955, size = 43, normalized size = 1.19

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{3}-1\right)\right)+\frac{1}{6}\log\left(x^{6}-x^{3}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3-2)/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3 + 1)
```

Fricas [A] time = 1.27571, size = 96, normalized size = 2.67

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{3}-1\right)\right)+\frac{1}{6}\log\left(x^{6}-x^{3}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3-2)/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3 + 1)
```

Sympy [A] time = 0.135508, size = 37, normalized size = 1.03

$$\frac{\log\left(x^{6}-x^{3}+1\right)}{6}-\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}x^{3}}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(x**3-2)/(x**6-x**3+1),x)
```

```
[Out] log(x**6 - x**3 + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3)/3
```

Giac [A] time = 1.17078, size = 43, normalized size = 1.19

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)+\frac{1}{6}\log\left(x^6-x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3-2)/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3 + 1)
```

3.33 $\int \frac{1+x^3}{x\left(1-x^3+x^6\right)} \, dx$

Optimal. Leaf size=39

$$-\frac{1}{6}\log\left(x^6-x^3+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}}+\log(x)$$

```
[Out] -(ArcTan[(1 - 2*x^3)/Sqrt[3]]/Sqrt[3]) + Log[x] - Log[1 - x^3 + x^6]/6
```

Rubi [A] time = 0.0563145, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1474, 800, 634, 618, 204, 628}

$$-\frac{1}{6}\log\left(x^6-x^3+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^3)/(x*(1 - x^3 + x^6)),x]
```

```
[Out] -(ArcTan[(1 - 2*x^3)/Sqrt[3]]/Sqrt[3]) + Log[x] - Log[1 - x^3 + x^6]/6
```

Rule 1474

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^3}{x\left(1-x^3+x^6\right)}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1+x}{x\left(1-x+x^2\right)}\,dx, x, x^3\right)\\
&= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{1}{x}+\frac{2-x}{1-x+x^2}\right)dx, x, x^3\right)\\
&= \log(x)+\frac{1}{3}\,\text{Subst}\left(\int \frac{2-x}{1-x+x^2}\,dx, x, x^3\right)\\
&= \log(x)-\frac{1}{6}\,\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, x^3\right)+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^3\right)\\
&= \log(x)-\frac{1}{6}\log\left(1-x^3+x^6\right)-\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^3\right)\\
&= \frac{\tan^{-1}\left(\frac{-1+2x^3}{\sqrt{3}}\right)}{\sqrt{3}}+\log(x)-\frac{1}{6}\log\left(1-x^3+x^6\right)
\end{aligned}$$

Mathematica [C] time = 0.0138819, size = 55, normalized size = 1.41

$$\log(x)-\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1^3\log(x-\#1)-2\log(x-\#1)}{2\#1^3-1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^3)/(x*(1 - x^3 + x^6)),x]
```

```
[Out] Log[x] - RootSum[1 - #1^3 + #1^6 & , (-2*Log[x - #1] + Log[x - #1]*#1^3)/(-
1 + 2*#1^3) & ]/3
```

Maple [A] time = 0.006, size = 35, normalized size = 0.9

$$\ln(x) - \frac{\ln(x^6 - x^3 + 1)}{6} + \frac{\sqrt{3}}{3}\arctan\left(\frac{(2x^3 - 1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)/x/(x^6-x^3+1),x)
```

```
[Out] ln(x)-1/6*ln(x^6-x^3+1)+1/3*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

Maxima [A] time = 1.4859, size = 51, normalized size = 1.31

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3 - 1)\right) - \frac{1}{6}\log(x^6 - x^3 + 1) + \frac{1}{3}\log(x^3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/x/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + 1/3*
log(x^3)
```

Fricas [A] time = 1.47567, size = 107, normalized size = 2.74

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x^3 - 1)\right) - \frac{1}{6}\log(x^6 - x^3 + 1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/x/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log(
x)
```

Sympy [A] time = 0.14287, size = 41, normalized size = 1.05

$$\log(x) - \frac{\log\left(x^6 - x^3 + 1\right)}{6} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^3}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)/x/(x**6-x**3+1),x)
```

```
[Out] log(x) - log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3
)/3
```

Giac [A] time = 1.12803, size = 47, normalized size = 1.21

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^3 - 1\right)\right) - \frac{1}{6}\log\left(x^6 - x^3 + 1\right) + \log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/x/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log(
abs(x))
```

3.34 $\int \frac{1+x^3}{x-x^4+x^7} \, dx$

Optimal. Leaf size=39

$$-\frac{1}{6}\log\left(x^6-x^3+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}}+\log(x)$$

```
[Out] -(ArcTan[(1 - 2*x^3)/Sqrt[3]]/Sqrt[3]) + Log[x] - Log[1 - x^3 + x^6]/6
```

Rubi [A] time = 0.0629689, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1594, 1474, 800, 634, 618, 204, 628}

$$-\frac{1}{6}\log\left(x^6-x^3+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^3)/(x - x^4 + x^7),x]
```

```
[Out] -(ArcTan[(1 - 2*x^3)/Sqrt[3]]/Sqrt[3]) + Log[x] - Log[1 - x^3 + x^6]/6
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1474

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
```

```
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^3}{x-x^4+x^7}\,dx &= \int \frac{1+x^3}{x\left(1-x^3+x^6\right)}\,dx \\
&= \frac{1}{3}\,\text{Subst}\left(\int \frac{1+x}{x\left(1-x+x^2\right)}\,dx, x, x^3\right) \\
&= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{1}{x}+\frac{2-x}{1-x+x^2}\right)dx, x, x^3\right) \\
&= \log(x)+\frac{1}{3}\,\text{Subst}\left(\int \frac{2-x}{1-x+x^2}\,dx, x, x^3\right) \\
&= \log(x)-\frac{1}{6}\,\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, x^3\right)+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^3\right) \\
&= \log(x)-\frac{1}{6}\log\left(1-x^3+x^6\right)-\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^3\right) \\
&= \frac{\tan^{-1}\left(\frac{-1+2x^3}{\sqrt{3}}\right)}{\sqrt{3}}+\log(x)-\frac{1}{6}\log\left(1-x^3+x^6\right)
\end{aligned}$$

Mathematica [C] time = 0.0104177, size = 55, normalized size = 1.41

$$\log(x)-\frac{1}{3}\text{RootSum}\left[\#1^6-\#1^3+1\&,\frac{\#1^3\log(x-\#1)-2\log(x-\#1)}{2\#1^3-1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^3)/(x - x^4 + x^7),x]
```

```
[Out] Log[x] - RootSum[1 - #1^3 + #1^6 & , (-2*Log[x - #1] + Log[x - #1]*#1^3)/(-
1 + 2*#1^3) & ]/3
```

Maple [A] time = 0.004, size = 35, normalized size = 0.9

$$\ln(x)-\frac{\ln\left(x^6-x^3+1\right)}{6}+\frac{\sqrt{3}}{3}\arctan\left(\frac{\left(2x^3-1\right)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)/(x^7-x^4+x),x)
```

```
[Out] ln(x)-1/6*ln(x^6-x^3+1)+1/3*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^5 - 2x^2}{x^6 - x^3 + 1}\,dx + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^7-x^4+x),x, algorithm="maxima")
```

```
[Out] -integrate((x^5 - 2*x^2)/(x^6 - x^3 + 1), x) + log(x)
```

Fricas [A] time = 1.49832, size = 107, normalized size = 2.74

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3 - 1\right)\right) - \frac{1}{6}\log\left(x^6 - x^3 + 1\right) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^7-x^4+x),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log(
x)
```

Sympy [A] time = 0.14817, size = 41, normalized size = 1.05

$$\log(x) - \frac{\log\left(x^6 - x^3 + 1\right)}{6} + \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^3}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)/(x**7-x**4+x),x)
```

```
[Out] log(x) - log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3
)/3
```

Giac [A] time = 1.09442, size = 47, normalized size = 1.21

$$\frac{1}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^3-1\right)\right)-\frac{1}{6}\,\log\left(x^6-x^3+1\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^7-x^4+x),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log(
abs(x))
```

3.35 $\int \left(d+ex^3\right)^{5/2} \left(a+bx^3+cx^6\right) dx$

Optimal. Leaf size=396

$$\frac{54\, 3^{3/4}\sqrt{2+\sqrt{3}}d^3\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\left(667ae^2-58bde+16cd^2\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x}{\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x}\right),-7-4\sqrt{}\right.}{124729e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}$$

```
[Out] (54*d^2*(16*c*d^2 - 58*b*d*e + 667*a*e^2)*x*Sqrt[d + e*x^3])/(124729*e^2) +
 (30*d*(16*c*d^2 - 58*b*d*e + 667*a*e^2)*x*(d + e*x^3)^(3/2))/(124729*e^2)
+ (2*(16*c*d^2 - 58*b*d*e + 667*a*e^2)*x*(d + e*x^3)^(5/2))/(11339*e^2) - (
2*(8*c*d - 29*b*e)*x*(d + e*x^3)^(7/2))/(667*e^2) + (2*c*x^4*(d + e*x^3)^(7
/2))/(29*e) + (54*3^(3/4)*Sqrt[2 + Sqrt[3]]*d^3*(16*c*d^2 - 58*b*d*e + 667*
a*e^2)*(d^(1/3) + e^(1/3)*x)*Sqrt[(d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^
2)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*d
^(1/3) + e^(1/3)*x)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)], -7 - 4*Sqrt[3]])/
(124729*e^(7/3)*Sqrt[(d^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + Sqrt[3])*d^(1/3)
 + e^(1/3)*x)^2]*Sqrt[d + e*x^3])
```

Rubi [A] time = 0.41577, antiderivative size = 396, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1411, 388, 195, 218}

$$\frac{2x\left(d+ex^3\right)^{5/2}\left(667ae^2-58bde+16cd^2\right)}{11339e^2}+\frac{30dx\left(d+ex^3\right)^{3/2}\left(667ae^2-58bde+16cd^2\right)}{124729e^2}+\frac{54d^2x\sqrt{d+ex^3}\left(667ae^2-58\right.}{124729e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^3)^(5/2)*(a + b*x^3 + c*x^6),x]
```


```
[Out] (54*d^2*(16*c*d^2 - 58*b*d*e + 667*a*e^2)*x*Sqrt[d + e*x^3])/(124729*e^2) +
 (30*d*(16*c*d^2 - 58*b*d*e + 667*a*e^2)*x*(d + e*x^3)^(3/2))/(124729*e^2)
+ (2*(16*c*d^2 - 58*b*d*e + 667*a*e^2)*x*(d + e*x^3)^(5/2))/(11339*e^2) - (
2*(8*c*d - 29*b*e)*x*(d + e*x^3)^(7/2))/(667*e^2) + (2*c*x^4*(d + e*x^3)^(7
/2))/(29*e) + (54*3^(3/4)*Sqrt[2 + Sqrt[3]]*d^3*(16*c*d^2 - 58*b*d*e + 667*
```

```
a*e^2)*(d^(1/3) + e^(1/3)*x)*Sqrt[(d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^
2)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*d
^(1/3) + e^(1/3)*x)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)], -7 - 4*Sqrt[3]])/
(124729*e^(7/3)*Sqrt[(d^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + Sqrt[3])*d^(1/3)
 + e^(1/3)*x)^2]*Sqrt[d + e*x^3])
```

Rule 1411

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[(c*x^(n + 1)*(d + e*x^n)^(q + 1))/(e*(n*(q + 2) + 1))
, x] + Dist[1/(e*(n*(q + 2) + 1)), Int[(d + e*x^n)^q*(a*e*(n*(q + 2) + 1) -
 (c*d*(n + 1) - b*e*(n*(q + 2) + 1))*x^n), x], x] /; FreeQ[{a, b, c, d, e,
n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e
^2, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \left(d+ex^3\right)^{5/2}\left(a+bx^3+cx^6\right)dx = \frac{2cx^4\left(d+ex^3\right)^{7/2}}{29e}+\frac{2\int \left(d+ex^3\right)^{5/2}\left(\frac{29ae}{2}-\left(4cd-\frac{29be}{2}\right)x^3\right)dx}{29e}$$
$$= -\frac{2(8cd-29be)x\left(d+ex^3\right)^{7/2}}{667e^2}+\frac{2cx^4\left(d+ex^3\right)^{7/2}}{29e}-\frac{1}{667}\left(-667a-\frac{2d(8cd-29be)}{e^2}\right.$$
$$= \frac{2\left(667a+\frac{2d(8cd-29be)}{e^2}\right)x\left(d+ex^3\right)^{5/2}}{11339}-\frac{2(8cd-29be)x\left(d+ex^3\right)^{7/2}}{667e^2}+\frac{2cx^4\left(d+ex^3\right.}{29e}$$
$$= \frac{30d\left(667a+\frac{2d(8cd-29be)}{e^2}\right)x\left(d+ex^3\right)^{3/2}}{124729}+\frac{2\left(667a+\frac{2d(8cd-29be)}{e^2}\right)x\left(d+ex^3\right)^{5/2}}{11339}-$$
$$= \frac{54d^2\left(667a+\frac{2d(8cd-29be)}{e^2}\right)x\sqrt{d+ex^3}}{124729}+\frac{30d\left(667a+\frac{2d(8cd-29be)}{e^2}\right)x\left(d+ex^3\right)^{3/2}}{124729}+$$
$$= \frac{54d^2\left(667a+\frac{2d(8cd-29be)}{e^2}\right)x\sqrt{d+ex^3}}{124729}+\frac{30d\left(667a+\frac{2d(8cd-29be)}{e^2}\right)x\left(d+ex^3\right)^{3/2}}{124729}+$$

Mathematica [C] time = 0.178911, size = 103, normalized size = 0.26

$$\frac{x\sqrt{d+ex^3}\left(\frac{{}_2F_1\left(-\frac{5}{2},\frac{1}{3};\frac{4}{3};-\frac{ex^3}{d}\right)\left(29d^2e(23ae-2bd)+16cd^4\right)}{\sqrt{\frac{ex^3}{d}+1}}-2\left(d+ex^3\right)^3\left(-29be+8cd-23cex^3\right)\right)}{667e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^3)^(5/2)*(a + b*x^3 + c*x^6),x]
```

```
[Out] (x*Sqrt[d + e*x^3]*(-2*(d + e*x^3)^3*(8*c*d - 29*b*e - 23*c*e*x^3) + ((16*c
*d^4 + 29*d^2*e*(-2*b*d + 23*a*e))*Hypergeometric2F1[-5/2, 1/3, 4/3, -((e*x
^3)/d)])/Sqrt[1 + (e*x^3)/d]))/(667*e^2)
```

Maple [B] time = 0.161, size = 1070, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^3+d)^(5/2)*(c*x^6+b*x^3+a),x)
```

```
[Out] c*(2/29*e^2*x^13*(e*x^3+d)^(1/2)+122/667*d*e*x^10*(e*x^3+d)^(1/2)+1562/1133
9*d^2*x^7*(e*x^3+d)^(1/2)+810/124729*d^3/e*x^4*(e*x^3+d)^(1/2)-1296/124729*
d^4/e^2*x*(e*x^3+d)^(1/2)-864/124729*I*d^5/e^3*3^(1/2)*(-d*e^2)^(1/3)*(I*(x
+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1
/3))^(1/2)*((x-1/e*(-d*e^2)^(1/3))/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(
-d*e^2)^(1/3)))^(1/2)*(-I*(x+1/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^
(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2
)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*
e^2)^(1/3))^(1/2),(I*3^(1/2)/e*(-d*e^2)^(1/3)/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*
3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)))+b*(2/23*e^2*x^10*(e*x^3+d)^(1/2)+98/391*
d*e*x^7*(e*x^3+d)^(1/2)+974/4301*d^2*x^4*(e*x^3+d)^(1/2)+162/4301*d^3/e*x*(
e*x^3+d)^(1/2)+108/4301*I*d^4/e^2*3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2/e*(-d*e^
2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)*((
x-1/e*(-d*e^2)^(1/3))/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)
))^(1/2)*(-I*(x+1/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2
)*e/(-d*e^2)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e
*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(
1/2),(I*3^(1/2)/e*(-d*e^2)^(1/3)/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d
*e^2)^(1/3)))^(1/2)))+a*(2/17*e^2*x^7*(e*x^3+d)^(1/2)+74/187*d*e*x^4*(e*x^3
+d)^(1/2)+106/187*d^2*x*(e*x^3+d)^(1/2)-54/187*I*d^3*3^(1/2)/e*(-d*e^2)^(1/
3)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d
*e^2)^(1/3))^(1/2)*((x-1/e*(-d*e^2)^(1/3))/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(
1/2)/e*(-d*e^2)^(1/3)))^(1/2)*(-I*(x+1/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(
-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/
3*3^(1/2)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2
)*e/(-d*e^2)^(1/3))^(1/2),(I*3^(1/2)/e*(-d*e^2)^(1/3)/(-3/2/e*(-d*e^2)^(1/3
)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^6 + bx^3 + a\right)\left(ex^3 + d\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^(5/2)*(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)*(e*x^3 + d)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(ce^2x^{12}+\left(2\,cde+be^2\right)x^9+\left(cd^2+2\,bde+ae^2\right)x^6+\left(bd^2+2\,ade\right)x^3+ad^2\right)\sqrt{ex^3+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^(5/2)*(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] integral((c*e^2*x^12 + (2*c*d*e + b*e^2)*x^9 + (c*d^2 + 2*b*d*e + a*e^2)*x^
6 + (b*d^2 + 2*a*d*e)*x^3 + a*d^2)*sqrt(e*x^3 + d), x)
```

Sympy [A] time = 10.0827, size = 400, normalized size = 1.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**3+d)**(5/2)*(c*x**6+b*x**3+a),x)
```

```
[Out] a*d**(5/2)*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), e*x**3*exp_polar(I*pi)/d
)/(3*gamma(4/3)) + 2*a*d**(3/2)*e*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,)
, e*x**3*exp_polar(I*pi)/d)/(3*gamma(7/3)) + a*sqrt(d)*e**2*x**7*gamma(7/3)
*hyper((-1/2, 7/3), (10/3,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(10/3)) + b*
d**(5/2)*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), e*x**3*exp_polar(I*pi)/
d)/(3*gamma(7/3)) + 2*b*d**(3/2)*e*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3
,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(10/3)) + b*sqrt(d)*e**2*x**10*gamma(
10/3)*hyper((-1/2, 10/3), (13/3,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(13/3)
) + c*d**(5/2)*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), e*x**3*exp_polar
(I*pi)/d)/(3*gamma(10/3)) + 2*c*d**(3/2)*e*x**10*gamma(10/3)*hyper((-1/2, 1
0/3), (13/3,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(13/3)) + c*sqrt(d)*e**2*x
**13*gamma(13/3)*hyper((-1/2, 13/3), (16/3,), e*x**3*exp_polar(I*pi)/d)/(3*
gamma(16/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(cx^6+bx^3+a\right)\left(ex^3+d\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^(5/2)*(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)*(e*x^3 + d)^(5/2), x)
```

3.36 $\int \left(d+ex^3\right)^{3/2} \left(a+bx^3+cx^6\right) dx$

Optimal. Leaf size=356

$$\frac{18\ 3^{3/4}\sqrt{2+\sqrt{3}}d^2\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\left(391ae^2-46bde+16cd^2\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x}{\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x}\right),-7-4\sqrt{}\right.}{21505e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}$$

```
[Out] (18*d*(16*c*d^2 - 46*b*d*e + 391*a*e^2)*x*Sqrt[d + e*x^3])/(21505*e^2) + (2
*(16*c*d^2 - 46*b*d*e + 391*a*e^2)*x*(d + e*x^3)^(3/2))/(4301*e^2) - (2*(8*
c*d - 23*b*e)*x*(d + e*x^3)^(5/2))/(391*e^2) + (2*c*x^4*(d + e*x^3)^(5/2))/
(23*e) + (18*3^(3/4)*Sqrt[2 + Sqrt[3]]*d^2*(16*c*d^2 - 46*b*d*e + 391*a*e^2
)*(d^(1/3) + e^(1/3)*x)*Sqrt[(d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2)/((
1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*d^(1/3
) + e^(1/3)*x)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)], -7 - 4*Sqrt[3]])/(2150
5*e^(7/3)*Sqrt[(d^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + Sqrt[3])*d^(1/3) + e^(
1/3)*x)^2]*Sqrt[d + e*x^3])
```

Rubi [A] time = 0.311484, antiderivative size = 356, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1411, 388, 195, 218}

$$\frac{2x\left(d+ex^3\right)^{3/2}\left(391ae^2-46bde+16cd^2\right)}{4301e^2}+\frac{18dx\sqrt{d+ex^3}\left(391ae^2-46bde+16cd^2\right)}{21505e^2}+\frac{18\ 3^{3/4}\sqrt{2+\sqrt{3}}d^2\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^3)^(3/2)*(a + b*x^3 + c*x^6),x]
```


```
[Out] (18*d*(16*c*d^2 - 46*b*d*e + 391*a*e^2)*x*Sqrt[d + e*x^3])/(21505*e^2) + (2
*(16*c*d^2 - 46*b*d*e + 391*a*e^2)*x*(d + e*x^3)^(3/2))/(4301*e^2) - (2*(8*
c*d - 23*b*e)*x*(d + e*x^3)^(5/2))/(391*e^2) + (2*c*x^4*(d + e*x^3)^(5/2))/
(23*e) + (18*3^(3/4)*Sqrt[2 + Sqrt[3]]*d^2*(16*c*d^2 - 46*b*d*e + 391*a*e^2
)*(d^(1/3) + e^(1/3)*x)*Sqrt[(d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2)/((
1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*d^(1/3
```

```
) + e^(1/3)*x)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)], -7 - 4*Sqrt[3]])/(2150
5*e^(7/3)*Sqrt[(d^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + Sqrt[3])*d^(1/3) + e^(
1/3)*x)^2]*Sqrt[d + e*x^3])
```

Rule 1411

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[(c*x^(n + 1)*(d + e*x^n)^(q + 1))/(e*(n*(q + 2) + 1))
, x] + Dist[1/(e*(n*(q + 2) + 1)), Int[(d + e*x^n)^q*(a*e*(n*(q + 2) + 1) -
 (c*d*(n + 1) - b*e*(n*(q + 2) + 1))*x^n), x], x] /; FreeQ[{a, b, c, d, e,
n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e
^2, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \left(d+ex^3\right)^{3/2}\left(a+bx^3+cx^6\right)dx = \frac{2cx^4\left(d+ex^3\right)^{5/2}}{23e}+\frac{2\int \left(d+ex^3\right)^{3/2}\left(\frac{23ae}{2}-\left(4cd-\frac{23be}{2}\right)x^3\right)dx}{23e}$$

$$= -\frac{2(8cd-23be)x\left(d+ex^3\right)^{5/2}}{391e^2}+\frac{2cx^4\left(d+ex^3\right)^{5/2}}{23e}-\frac{1}{391}\left(-391a-\frac{2d(8cd-23be)}{e^2}\right)$$

$$= \frac{2\left(391a+\frac{2d(8cd-23be)}{e^2}\right)x\left(d+ex^3\right)^{3/2}}{4301}-\frac{2(8cd-23be)x\left(d+ex^3\right)^{5/2}}{391e^2}+\frac{2cx^4\left(d+ex^3\right.}{23e}$$

$$= \frac{18d\left(391a+\frac{2d(8cd-23be)}{e^2}\right)x\sqrt{d+ex^3}}{21505}+\frac{2\left(391a+\frac{2d(8cd-23be)}{e^2}\right)x\left(d+ex^3\right)^{3/2}}{4301}-\frac{2(8c}{}$$

$$= \frac{18d\left(391a+\frac{2d(8cd-23be)}{e^2}\right)x\sqrt{d+ex^3}}{21505}+\frac{2\left(391a+\frac{2d(8cd-23be)}{e^2}\right)x\left(d+ex^3\right)^{3/2}}{4301}-\frac{2(8c}{}$$

Mathematica [C] time = 0.153007, size = 101, normalized size = 0.28

$$\frac{x\sqrt{d+ex^3}\left(\frac{{}_2F_1\left(-\frac{3}{2},\frac{1}{3};\frac{4}{3};-\frac{ex^3}{d}\right)\left(23de(17ae-2bd)+16cd^3\right)}{\sqrt{\frac{ex^3}{d}+1}}-2\left(d+ex^3\right)^2\left(-23be+8cd-17cex^3\right)\right)}{391e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^3)^(3/2)*(a + b*x^3 + c*x^6),x]
```

```
[Out] (x*Sqrt[d + e*x^3]*(-2*(d + e*x^3)^2*(8*c*d - 23*b*e - 17*c*e*x^3) + ((16*c
*d^3 + 23*d*e*(-2*b*d + 17*a*e))*Hypergeometric2F1[-3/2, 1/3, 4/3, -((e*x^3
)/d)])/Sqrt[1 + (e*x^3)/d]))/(391*e^2)
```

Maple [B] time = 0.029, size = 1010, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^3+d)^(3/2)*(c*x^6+b*x^3+a),x)
```

```
[Out] c*(2/23*e*x^10*(e*x^3+d)^(1/2)+52/391*d*x^7*(e*x^3+d)^(1/2)+54/4301*d^2/e*x
^4*(e*x^3+d)^(1/2)-432/21505*d^3/e^2*x*(e*x^3+d)^(1/2)-288/21505*I*d^4/e^3*
3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^
(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)*((x-1/e*(-d*e^2)^(1/3))/(-3/2/e*(-d*
e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)*(-I*(x+1/2/e*(-d*e^2)^(1/
3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)/(e*x^3+d
)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-
d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2),(I*3^(1/2)/e*(-d*e^2)^(1/3)/(
-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)))+b*(2/17*e*x^
7*(e*x^3+d)^(1/2)+40/187*d*x^4*(e*x^3+d)^(1/2)+54/935*d^2/e*x*(e*x^3+d)^(1/
2)+36/935*I*d^3/e^2*3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I
*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)*((x-1/e*(-d*e^2)
^(1/3))/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)*(-I*(
x+1/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(
1/3))^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*(-d*e^2)^(1/3
)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2),(I*3^(1/2
)/e*(-d*e^2)^(1/3)/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^
(1/2)))+a*(2/11*e*x^4*(e*x^3+d)^(1/2)+28/55*d*x*(e*x^3+d)^(1/2)-18/55*I*d^2
*3^(1/2)/e*(-d*e^2)^(1/3)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^
2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)*((x-1/e*(-d*e^2)^(1/3))/(-3/2/e*(
-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)*(-I*(x+1/2/e*(-d*e^2)^
(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)/(e*x^
3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e
*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2),(I*3^(1/2)/e*(-d*e^2)^(1/3
)/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^6 + bx^3 + a\right)\left(ex^3 + d\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^(3/2)*(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)*(e*x^3 + d)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cex^9 + (cd + be)x^6 + (bd + ae)x^3 + ad\right)\sqrt{ex^3 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^(3/2)*(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] integral((c*e*x^9 + (c*d + b*e)*x^6 + (b*d + a*e)*x^3 + a*d)*sqrt(e*x^3 + d
), x)
```

Sympy [A] time = 5.8478, size = 257, normalized size = 0.72

$$\frac{ad^{\frac{3}{2}}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix} -\frac{1}{2}, \frac{1}{3} \\ \frac{4}{3} \end{matrix}\right| \frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{a\sqrt{d}ex^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix} -\frac{1}{2}, \frac{4}{3} \\ \frac{7}{3} \end{matrix}\right| \frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{bd^{\frac{3}{2}}x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix} -\frac{1}{2}, \frac{4}{3} \\ \frac{7}{3} \end{matrix}\right| \frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{b\sqrt{d}ex^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(-\frac{1}{2}, \ldots\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**3+d)**(3/2)*(c*x**6+b*x**3+a),x)
```

```
[Out] a*d**(3/2)*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), e*x**3*exp_polar(I*pi)/d
)/(3*gamma(4/3)) + a*sqrt(d)*e*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), e
*x**3*exp_polar(I*pi)/d)/(3*gamma(7/3)) + b*d**(3/2)*x**4*gamma(4/3)*hyper(
(-1/2, 4/3), (7/3,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(7/3)) + b*sqrt(d)*e
*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), e*x**3*exp_polar(I*pi)/d)/(3*g
amma(10/3)) + c*d**(3/2)*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), e*x**3
*exp_polar(I*pi)/d)/(3*gamma(10/3)) + c*sqrt(d)*e*x**10*gamma(10/3)*hyper((
-1/2, 10/3), (13/3,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(13/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^6 + bx^3 + a\right)\left(ex^3 + d\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^(3/2)*(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)*(e*x^3 + d)^(3/2), x)
```

3.37 $\int \sqrt{d+ex^3}\left(a+bx^3+cx^6\right)\,dx$

Optimal. Leaf size=316

$$\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}d\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\left(187ae^2-34bde+16cd^2\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x}{\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x}\right),-7-4\sqrt{3}\right)}{935e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}$$

```
[Out] (2*(16*c*d^2 - 34*b*d*e + 187*a*e^2)*x*Sqrt[d + e*x^3])/(935*e^2) - (2*(8*c
*d - 17*b*e)*x*(d + e*x^3)^(3/2))/(187*e^2) + (2*c*x^4*(d + e*x^3)^(3/2))/(
17*e) + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*d*(16*c*d^2 - 34*b*d*e + 187*a*e^2)*(d
^(1/3) + e^(1/3)*x)*Sqrt[(d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2)/((1 +
Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*d^(1/3) +
e^(1/3)*x)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)], -7 - 4*Sqrt[3]])/(935*e^(7
/3)*Sqrt[(d^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x
)^2]*Sqrt[d + e*x^3])
```

Rubi [A] time = 0.246214, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1411, 388, 195, 218}

$$\frac{2x\sqrt{d+ex^3}\left(187ae^2-34bde+16cd^2\right)}{935e^2}+\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}d\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\left(187ae^2-34bde+16cd^2\right)F\left(s\right.}{935e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x^3]*(a + b*x^3 + c*x^6),x]
```


```
[Out] (2*(16*c*d^2 - 34*b*d*e + 187*a*e^2)*x*Sqrt[d + e*x^3])/(935*e^2) - (2*(8*c
*d - 17*b*e)*x*(d + e*x^3)^(3/2))/(187*e^2) + (2*c*x^4*(d + e*x^3)^(3/2))/(
17*e) + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*d*(16*c*d^2 - 34*b*d*e + 187*a*e^2)*(d
^(1/3) + e^(1/3)*x)*Sqrt[(d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2)/((1 +
Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*d^(1/3) +
e^(1/3)*x)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)], -7 - 4*Sqrt[3]])/(935*e^(7
/3)*Sqrt[(d^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x
```

```
)^2]*Sqrt[d + e*x^3])
```

Rule 1411

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[(c*x^(n + 1)*(d + e*x^n)^(q + 1))/(e*(n*(q + 2) + 1))
, x] + Dist[1/(e*(n*(q + 2) + 1)), Int[(d + e*x^n)^q*(a*e*(n*(q + 2) + 1) -
 (c*d*(n + 1) - b*e*(n*(q + 2) + 1))*x^n), x], x] /; FreeQ[{a, b, c, d, e,
n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e
^2, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \sqrt{d+ex^3}\left(a+bx^3+cx^6\right)dx = \frac{2cx^4\left(d+ex^3\right)^{3/2}}{17e} + \frac{2\int \sqrt{d+ex^3}\left(\frac{17ae}{2}-\left(4cd-\frac{17be}{2}\right)x^3\right)dx}{17e}$$

$$= -\frac{2(8cd-17be)x\left(d+ex^3\right)^{3/2}}{187e^2} + \frac{2cx^4\left(d+ex^3\right)^{3/2}}{17e} - \frac{1}{187}\left(-187a-\frac{2d(8cd-17be)}{e^2}\right)\int$$

$$= \frac{2}{935}\left(187a+\frac{2d(8cd-17be)}{e^2}\right)x\sqrt{d+ex^3} - \frac{2(8cd-17be)x\left(d+ex^3\right)^{3/2}}{187e^2} + \frac{2cx^4\left(d+e\right.}{17e}$$

$$= \frac{2}{935}\left(187a+\frac{2d(8cd-17be)}{e^2}\right)x\sqrt{d+ex^3} - \frac{2(8cd-17be)x\left(d+ex^3\right)^{3/2}}{187e^2} + \frac{2cx^4\left(d+e\right.}{17e}$$

Mathematica [C] time = 0.133707, size = 98, normalized size = 0.31

$$\frac{x\sqrt{d+ex^3}\left(\frac{{}_2F_1\left(-\frac{1}{2},\frac{1}{3};\frac{4}{3};-\frac{ex^3}{d}\right)\left(17e(11ae-2bd)+16cd^2\right)}{\sqrt{\frac{ex^3}{d}+1}} - 2\left(d+ex^3\right)\left(-17be+8cd-11cex^3\right)\right)}{187e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x^3]*(a + b*x^3 + c*x^6),x]
```

```
[Out] (x*Sqrt[d + e*x^3]*(-2*(d + e*x^3)*(8*c*d - 17*b*e - 11*c*e*x^3) + ((16*c*d
^2 + 17*e*(-2*b*d + 11*a*e))*Hypergeometric2F1[-1/2, 1/3, 4/3, -((e*x^3)/d)
])/Sqrt[1 + (e*x^3)/d]))/(187*e^2)
```

Maple [B] time = 0.027, size = 956, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^3+d)^(1/2)*(c*x^6+b*x^3+a),x)
```

```
[Out] c*(2/17*x^7*(e*x^3+d)^(1/2)+6/187*d/e*x^4*(e*x^3+d)^(1/2)-48/935*d^2/e^2*x*
(e*x^3+d)^(1/2)-32/935*I*d^3/e^3*3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2/e*(-d*e^2
)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)*((x
-1/e*(-d*e^2)^(1/3))/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))
)^(1/2)*(-I*(x+1/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)
*e/(-d*e^2)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*
(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1
/2),(I*3^(1/2)/e*(-d*e^2)^(1/3)/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*
e^2)^(1/3)))^(1/2)))+b*(2/11*x^4*(e*x^3+d)^(1/2)+6/55*d/e*x*(e*x^3+d)^(1/2)
+4/55*I*d^2/e^2*3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(
1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)*((x-1/e*(-d*e^2)^(1/
3))/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)*(-I*(x+1/
2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3)
)^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/
2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2),(I*3^(1/2)/e*
(-d*e^2)^(1/3)/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2
)))+a*(2/5*x*(e*x^3+d)^(1/2)-2/5*I*d*3^(1/2)/e*(-d*e^2)^(1/3)*(I*(x+1/2/e*(
-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/
2)*((x-1/e*(-d*e^2)^(1/3))/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^
(1/3)))^(1/2)*(-I*(x+1/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3
^(1/2)*e/(-d*e^2)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+
1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/
3))^(1/2),(I*3^(1/2)/e*(-d*e^2)^(1/3)/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/
e*(-d*e^2)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^6 + bx^3 + a\right)\sqrt{ex^3 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^(1/2)*(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)*sqrt(e*x^3 + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^6 + bx^3 + a\right)\sqrt{ex^3 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^(1/2)*(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)*sqrt(e*x^3 + d), x)
```

Sympy [A] time = 3.11466, size = 124, normalized size = 0.39

$$\frac{a\sqrt{d}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{b\sqrt{d}x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{c\sqrt{d}x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**3+d)**(1/2)*(c*x**6+b*x**3+a),x)
```

```
[Out] a*sqrt(d)*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), e*x**3*exp_polar(I*pi)/d)
/(3*gamma(4/3)) + b*sqrt(d)*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), e*x*
*3*exp_polar(I*pi)/d)/(3*gamma(7/3)) + c*sqrt(d)*x**7*gamma(7/3)*hyper((-1/
2, 7/3), (10/3,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(10/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^6 + bx^3 + a\right)\sqrt{ex^3 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^(1/2)*(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)*sqrt(e*x^3 + d), x)
```

3.38 $\int \frac{a+bx^3+cx^6}{\sqrt{d+ex^3}}\,dx$

Optimal. Leaf size=278

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\left(55ae^2-22bde+16cd^2\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x}{\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x}\right),-7-4\sqrt{3}\right)}{55\sqrt[4]{3}e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}-\frac{2x\sqrt{d}}{}$$

```
[Out] (-2*(8*c*d - 11*b*e)*x*Sqrt[d + e*x^3])/(55*e^2) + (2*c*x^4*Sqrt[d + e*x^3]
)/(11*e) + (2*Sqrt[2 + Sqrt[3]]*(16*c*d^2 - 22*b*d*e + 55*a*e^2)*(d^(1/3) +
 e^(1/3)*x)*Sqrt[(d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2)/((1 + Sqrt[3])
*d^(1/3) + e^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*d^(1/3) + e^(1/3)*
x)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)], -7 - 4*Sqrt[3]])/(55*3^(1/4)*e^(7/
3)*Sqrt[(d^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)
^2]*Sqrt[d + e*x^3])
```

Rubi [A] time = 0.182336, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1411, 388, 218}

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\left(55ae^2-22bde+16cd^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{e}x+\left(1-\sqrt{3}\right)\sqrt[3]{d}}{\sqrt[3]{e}x+\left(1+\sqrt{3}\right)\sqrt[3]{d}}\right)|-7-4\sqrt{3}\right)}{55\sqrt[4]{3}e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}-\frac{2x\sqrt{d+ex^3}(8}{55e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)/Sqrt[d + e*x^3],x]
```


```
[Out] (-2*(8*c*d - 11*b*e)*x*Sqrt[d + e*x^3])/(55*e^2) + (2*c*x^4*Sqrt[d + e*x^3]
)/(11*e) + (2*Sqrt[2 + Sqrt[3]]*(16*c*d^2 - 22*b*d*e + 55*a*e^2)*(d^(1/3) +
 e^(1/3)*x)*Sqrt[(d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2)/((1 + Sqrt[3])
*d^(1/3) + e^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*d^(1/3) + e^(1/3)*
x)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)], -7 - 4*Sqrt[3]])/(55*3^(1/4)*e^(7/
3)*Sqrt[(d^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)
^2]*Sqrt[d + e*x^3])
```

Rule 1411

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[(c*x^(n + 1)*(d + e*x^n)^(q + 1))/(e*(n*(q + 2) + 1))
, x] + Dist[1/(e*(n*(q + 2) + 1)), Int[(d + e*x^n)^q*(a*e*(n*(q + 2) + 1) -
 (c*d*(n + 1) - b*e*(n*(q + 2) + 1))*x^n), x], x] /; FreeQ[{a, b, c, d, e,
n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e
^2, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{a+bx^3+cx^6}{\sqrt{d+ex^3}}\,dx = \frac{2cx^4\sqrt{d+ex^3}}{11e} + \frac{2\int \frac{\frac{11ae}{2}-\left(4cd-\frac{11be}{2}\right)x^3}{\sqrt{d+ex^3}}\,dx}{11e}$$

$$= -\frac{2(8cd-11be)x\sqrt{d+ex^3}}{55e^2} + \frac{2cx^4\sqrt{d+ex^3}}{11e} - \frac{1}{55}\left(-55a - \frac{2d(8cd-11be)}{e^2}\right)\int \frac{1}{\sqrt{d+ex^3}}\,dx$$

$$= -\frac{2(8cd-11be)x\sqrt{d+ex^3}}{55e^2} + \frac{2cx^4\sqrt{d+ex^3}}{11e} + \frac{2\sqrt{2+\sqrt{3}}\left(16cd^2-22bde+55ae^2\right)\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\ldots}}{55\sqrt[4]{3}e^{7/3}\sqrt{\frac{\sqrt[3]{d}(\ldots}{\left((1+\sqrt{\ldots}\right.}}}$$

Mathematica [C] time = 0.0905538, size = 98, normalized size = 0.35

$$\frac{x\left(\sqrt{\frac{ex^3}{d}+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{ex^3}{d}\right)\left(11e(5ae-2bd)+16cd^2\right)-2\left(d+ex^3\right)\left(-11be+8cd-5cex^3\right)\right)}{55e^2\sqrt{d+ex^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)/Sqrt[d + e*x^3],x]
```

```
[Out] (x*(-2*(d + e*x^3)*(8*c*d - 11*b*e - 5*c*e*x^3) + (16*c*d^2 + 11*e*(-2*b*d
+ 5*a*e))*Sqrt[1 + (e*x^3)/d]*Hypergeometric2F1[1/3, 1/2, 4/3, -((e*x^3)/d)
]))/(55*e^2*Sqrt[d + e*x^3])
```

Maple [B] time = 0.028, size = 907, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)/(e*x^3+d)^(1/2),x)
```

```
[Out] c*(2/11/e*x^4*(e*x^3+d)^(1/2)-16/55*d/e^2*x*(e*x^3+d)^(1/2)-32/165*I*d^2/e^
3*3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2
)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)*((x-1/e*(-d*e^2)^(1/3))/(-3/2/e*(-
d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)*(-I*(x+1/2/e*(-d*e^2)^(
1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)/(e*x^3
+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*
(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2),(I*3^(1/2)/e*(-d*e^2)^(1/3)
/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)))+b*(2/5/e*x
*(e*x^3+d)^(1/2)+4/15*I*d/e^2*3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2/e*(-d*e^2)^(
1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)*((x-1/
e*(-d*e^2)^(1/3))/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(
1/2)*(-I*(x+1/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/
(-d*e^2)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*(-d
*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)
,(I*3^(1/2)/e*(-d*e^2)^(1/3)/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2
)^(1/3)))^(1/2)))-2/3*I*a*3^(1/2)/e*(-d*e^2)^(1/3)*(I*(x+1/2/e*(-d*e^2)^(1/
3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)*((x-1/e*
(-d*e^2)^(1/3))/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/
2)*(-I*(x+1/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-
```

```
d*e^2)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*(-d*e
^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2),(
I*3^(1/2)/e*(-d*e^2)^(1/3)/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^
(1/3)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{cx^6 + bx^3 + a}{\sqrt{ex^3 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)/sqrt(e*x^3 + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{cx^6 + bx^3 + a}{\sqrt{ex^3 + d}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)/sqrt(e*x^3 + d), x)
```

Sympy [A] time = 2.62862, size = 119, normalized size = 0.43

$$\frac{ax\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\right|\frac{ex^3e^{i\pi}}{d}\right)}{3\sqrt{d}\Gamma\left(\frac{4}{3}\right)} + \frac{bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{4}{3}\\\frac{7}{3}\end{matrix}\right|\frac{ex^3e^{i\pi}}{d}\right)}{3\sqrt{d}\Gamma\left(\frac{7}{3}\right)} + \frac{cx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{7}{3}\\\frac{10}{3}\end{matrix}\right|\frac{ex^3e^{i\pi}}{d}\right)}{3\sqrt{d}\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)/(e*x**3+d)**(1/2),x)
```

```
[Out] a*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), e*x**3*exp_polar(I*pi)/d)/(3*sqrt(
d)*gamma(4/3)) + b*x**4*gamma(4/3)*hyper((1/2, 4/3), (7/3,), e*x**3*exp_pol
ar(I*pi)/d)/(3*sqrt(d)*gamma(7/3)) + c*x**7*gamma(7/3)*hyper((1/2, 7/3), (1
0/3,), e*x**3*exp_polar(I*pi)/d)/(3*sqrt(d)*gamma(10/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{cx^6 + bx^3 + a}{\sqrt{ex^3 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)/sqrt(e*x^3 + d), x)
```

3.39 $\int \frac{a+bx^3+cx^6}{\left(d+ex^3\right)^{3/2}}\,dx$

Optimal. Leaf size=289

$$-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\left(16cd^2-5e(ae+2bd)\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x}{\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x}\right),-7-4\sqrt{3}\right)}{15\sqrt[4]{3}de^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}+\frac{2x(}{3}$$

```
[Out] (2*(c*d^2 - b*d*e + a*e^2)*x)/(3*d*e^2*Sqrt[d + e*x^3]) + (2*c*x*Sqrt[d + e
*x^3])/(5*e^2) - (2*Sqrt[2 + Sqrt[3]]*(16*c*d^2 - 5*e*(2*b*d + a*e))*(d^(1/
3) + e^(1/3)*x)*Sqrt[(d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2)/((1 + Sqrt
[3])*d^(1/3) + e^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*d^(1/3) + e^(1
/3)*x)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)], -7 - 4*Sqrt[3]])/(15*3^(1/4)*d
*e^(7/3)*Sqrt[(d^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + Sqrt[3])*d^(1/3) + e^(1
/3)*x)^2]*Sqrt[d + e*x^3])
```

Rubi [A] time = 0.189006, antiderivative size = 289, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1409, 388, 218}

$$\frac{2x\left(ae^2-bde+cd^2\right)}{3de^2\sqrt{d+ex^3}}-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\left(16cd^2-5e(ae+2bd)\right)F\left(\sin^{-1}\left(\frac{\sqrt[3]{e}x+\left(1-\sqrt{3}\right)\sqrt[3]{d}}{\sqrt[3]{e}x+\left(1+\sqrt{3}\right)\sqrt[3]{d}}\right)|-7-\right.}{15\sqrt[4]{3}de^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)/(d + e*x^3)^(3/2),x]
```


```
[Out] (2*(c*d^2 - b*d*e + a*e^2)*x)/(3*d*e^2*Sqrt[d + e*x^3]) + (2*c*x*Sqrt[d + e
*x^3])/(5*e^2) - (2*Sqrt[2 + Sqrt[3]]*(16*c*d^2 - 5*e*(2*b*d + a*e))*(d^(1/
3) + e^(1/3)*x)*Sqrt[(d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2)/((1 + Sqrt
[3])*d^(1/3) + e^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*d^(1/3) + e^(1
/3)*x)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)], -7 - 4*Sqrt[3]])/(15*3^(1/4)*d
*e^(7/3)*Sqrt[(d^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + Sqrt[3])*d^(1/3) + e^(1
/3)*x)^2]*Sqrt[d + e*x^3])
```

Rule 1409

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> -Simp[((c*d^2 - b*d*e + a*e^2)*x*(d + e*x^n)^(q + 1))/(d*e
^2*n*(q + 1)), x] + Dist[1/(n*(q + 1)*d*e^2), Int[(d + e*x^n)^(q + 1)*Simp[
c*d^2 - b*d*e + a*e^2*(n*(q + 1) + 1) + c*d*e*n*(q + 1)*x^n, x], x], x] /;
FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && LtQ[q, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+bx^3+cx^6}{\left(d+ex^3\right)^{3/2}}\,dx &= \frac{2\left(cd^2-bde+ae^2\right)x}{3de^2\sqrt{d+ex^3}}-\frac{2\int \frac{\frac{1}{2}\left(2cd^2-e(2bd+ae)\right)-\frac{3}{2}cdex^3}{\sqrt{d+ex^3}}\,dx}{3de^2}\\
&= \frac{2\left(cd^2-bde+ae^2\right)x}{3de^2\sqrt{d+ex^3}}+\frac{2cx\sqrt{d+ex^3}}{5e^2}-\frac{\left(16cd^2-5e(2bd+ae)\right)\int \frac{1}{\sqrt{d+ex^3}}\,dx}{15de^2}\\
&= \frac{2\left(cd^2-bde+ae^2\right)x}{3de^2\sqrt{d+ex^3}}+\frac{2cx\sqrt{d+ex^3}}{5e^2}-\frac{2\sqrt{2+\sqrt{3}}\left(16cd^2-5e(2bd+ae)\right)\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}\cdots}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}\cdots\right.}}\cdots}{15\sqrt[4]{3}de^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\cdots}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}\cdots\right.}}}
\end{aligned}
$$

Mathematica [C] time = 0.105869, size = 102, normalized size = 0.35

$$\frac{x\left(\sqrt{\frac{ex^3}{d}+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{ex^3}{d}\right)\left(5e(ae+2bd)-16cd^2\right)+2\left(5e(ae-bd)+cd\left(8d+3ex^3\right)\right)\right)}{15de^2\sqrt{d+ex^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)/(d + e*x^3)^(3/2),x]
```

```
[Out] (x*(2*(5*e*(-(b*d) + a*e) + c*d*(8*d + 3*e*x^3)) + (-16*c*d^2 + 5*e*(2*b*d
+ a*e))*Sqrt[1 + (e*x^3)/d]*Hypergeometric2F1[1/3, 1/2, 4/3, -((e*x^3)/d)])
)/(15*d*e^2*Sqrt[d + e*x^3])
```

Maple [B] time = 0.038, size = 934, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)/(e*x^3+d)^(3/2),x)
```

```
[Out] c*(2/3/e^2*d*x/((x^3+d/e)*e)^(1/2)+2/5/e^2*x*(e*x^3+d)^(1/2)+32/45*I*d/e^3*
3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^
(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)*((x-1/e*(-d*e^2)^(1/3))/(-3/2/e*(-d*
e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)*(-I*(x+1/2/e*(-d*e^2)^(1/
3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)/(e*x^3+d
)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-
d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2),(I*3^(1/2)/e*(-d*e^2)^(1/3)/(
-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)))+b*(-2/3/e*x/
((x^3+d/e)*e)^(1/2)-4/9*I/e^2*3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2/e*(-d*e^2)^(
1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)*((x-1/
e*(-d*e^2)^(1/3))/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(
1/2)*(-I*(x+1/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/
(-d*e^2)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*(-d
*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)
,(I*3^(1/2)/e*(-d*e^2)^(1/3)/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2
)^(1/3)))^(1/2)))+a*(2/3/d*x/((x^3+d/e)*e)^(1/2)-2/9*I/d*3^(1/2)/e*(-d*e^2)
^(1/3)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e
/(-d*e^2)^(1/3))^(1/2)*((x-1/e*(-d*e^2)^(1/3))/(-3/2/e*(-d*e^2)^(1/3)+1/2*I
*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)*(-I*(x+1/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)
```

```
/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*Elliptic
F(1/3*3^(1/2)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^
(1/2)*e/(-d*e^2)^(1/3))^(1/2),(I*3^(1/2)/e*(-d*e^2)^(1/3)/(-3/2/e*(-d*e^2)^
(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{cx^6 + bx^3 + a}{\left(ex^3 + d\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^6 + bx^3 + a\right)\sqrt{ex^3 + d}}{e^2x^6 + 2\,dex^3 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)*sqrt(e*x^3 + d)/(e^2*x^6 + 2*d*e*x^3 + d^2), x
)
```

Sympy [A] time = 18.6978, size = 119, normalized size = 0.41

$$\frac{ax\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3}, \frac{3}{2}\\ \frac{4}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3d^{\frac{3}{2}}\Gamma\left(\frac{4}{3}\right)} + \frac{bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3}, \frac{3}{2}\\ \frac{7}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3d^{\frac{3}{2}}\Gamma\left(\frac{7}{3}\right)} + \frac{cx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{3}{2}, \frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3d^{\frac{3}{2}}\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)/(e*x**3+d)**(3/2),x)
```

```
[Out] a*x*gamma(1/3)*hyper((1/3, 3/2), (4/3,), e*x**3*exp_polar(I*pi)/d)/(3*d**(3
/2)*gamma(4/3)) + b*x**4*gamma(4/3)*hyper((4/3, 3/2), (7/3,), e*x**3*exp_po
lar(I*pi)/d)/(3*d**(3/2)*gamma(7/3)) + c*x**7*gamma(7/3)*hyper((3/2, 7/3),
(10/3,), e*x**3*exp_polar(I*pi)/d)/(3*d**(3/2)*gamma(10/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{cx^6 + bx^3 + a}{\left(ex^3 + d\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(3/2), x)
```

3.40 $\int \frac{a+bx^3+cx^6}{\left(d+ex^3\right)^{5/2}}\,dx$

Optimal. Leaf size=309

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\left(e(7ae+2bd)+16cd^2\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x}{\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x}\right),-7-4\sqrt{3}\right)}{27\sqrt[4]{3}d^2e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}-\frac{2x\left(-7\ldots\right.}{27\ldots}$$

```
[Out] (2*(c*d^2 - b*d*e + a*e^2)*x)/(9*d*e^2*(d + e*x^3)^(3/2)) - (2*(11*c*d^2 -
2*b*d*e - 7*a*e^2)*x)/(27*d^2*e^2*Sqrt[d + e*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(
16*c*d^2 + e*(2*b*d + 7*a*e))*(d^(1/3) + e^(1/3)*x)*Sqrt[(d^(2/3) - d^(1/3)
*e^(1/3)*x + e^(2/3)*x^2)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*EllipticF[
ArcSin[((1 - Sqrt[3])*d^(1/3) + e^(1/3)*x)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)
*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*d^2*e^(7/3)*Sqrt[(d^(1/3)*(d^(1/3) + e^(
1/3)*x))/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*Sqrt[d + e*x^3])
```

Rubi [A] time = 0.210659, antiderivative size = 309, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1409, 385, 218}

$$-\frac{2x\left(-7ae^2-2bde+11cd^2\right)}{27d^2e^2\sqrt{d+ex^3}}+\frac{2x\left(ae^2-bde+cd^2\right)}{9de^2\left(d+ex^3\right)^{3/2}}+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\left(e(7ae+2bd)+16cd^2\right)\ldots}{27\sqrt[4]{3}d^2e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)/(d + e*x^3)^(5/2),x]
```


```
[Out] (2*(c*d^2 - b*d*e + a*e^2)*x)/(9*d*e^2*(d + e*x^3)^(3/2)) - (2*(11*c*d^2 -
2*b*d*e - 7*a*e^2)*x)/(27*d^2*e^2*Sqrt[d + e*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(
16*c*d^2 + e*(2*b*d + 7*a*e))*(d^(1/3) + e^(1/3)*x)*Sqrt[(d^(2/3) - d^(1/3)
*e^(1/3)*x + e^(2/3)*x^2)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*EllipticF[
ArcSin[((1 - Sqrt[3])*d^(1/3) + e^(1/3)*x)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)
*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*d^2*e^(7/3)*Sqrt[(d^(1/3)*(d^(1/3) + e^(
1/3)*x))/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*Sqrt[d + e*x^3])
```

Rule 1409

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> -Simp[((c*d^2 - b*d*e + a*e^2)*x*(d + e*x^n)^(q + 1))/(d*e
^2*n*(q + 1)), x] + Dist[1/(n*(q + 1)*d*e^2), Int[(d + e*x^n)^(q + 1)*Simp[
c*d^2 - b*d*e + a*e^2*(n*(q + 1) + 1) + c*d*e*n*(q + 1)*x^n, x], x], x] /;
FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{a+bx^3+cx^6}{\left(d+ex^3\right)^{5/2}}\,dx = \frac{2\left(cd^2-bde+ae^2\right)x}{9de^2\left(d+ex^3\right)^{3/2}} - \frac{2\int \frac{\frac{1}{2}\left(2cd^2-e(2bd+7ae)\right)-\frac{9}{2}cdex^3}{\left(d+ex^3\right)^{3/2}}\,dx}{9de^2}$$

$$= \frac{2\left(cd^2-bde+ae^2\right)x}{9de^2\left(d+ex^3\right)^{3/2}} - \frac{2\left(11cd^2-2bde-7ae^2\right)x}{27d^2e^2\sqrt{d+ex^3}} - \frac{\left(4\left(-\frac{9}{2}cd^2e+\frac{1}{4}e\left(2cd^2-e(2bd+7ae)\right)\right)\right)\int \ldots}{27d^2e^3}$$

$$= \frac{2\left(cd^2-bde+ae^2\right)x}{9de^2\left(d+ex^3\right)^{3/2}} - \frac{2\left(11cd^2-2bde-7ae^2\right)x}{27d^2e^2\sqrt{d+ex^3}} + \frac{2\sqrt{2+\sqrt{3}}\left(16cd^2+e(2bd+7ae)\right)\left(\sqrt[3]{d}+\sqrt[3]{e}\ldots\right.}{27\sqrt[4]{3}d^2e^{7/3}\ldots}$$

Mathematica [C] time = 0.139048, size = 129, normalized size = 0.42

$$\frac{x\left(d+ex^3\right)\sqrt{\frac{ex^3}{d}+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{ex^3}{d}\right)\left(e(7ae+2bd)+16cd^2\right)-2x\left(e\left(bd\left(d-2ex^3\right)-ae\left(10d+7ex^3\right)\right)+cd^2\left(8d+11ex^3\right)\right)}{27d^2e^2\left(d+ex^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)/(d + e*x^3)^(5/2),x]
```

```
[Out] (-2*x*(c*d^2*(8*d + 11*e*x^3) + e*(b*d*(d - 2*e*x^3) - a*e*(10*d + 7*e*x^3)
)) + (16*c*d^2 + e*(2*b*d + 7*a*e))*x*(d + e*x^3)*Sqrt[1 + (e*x^3)/d]*Hyper
geometric2F1[1/3, 1/2, 4/3, -((e*x^3)/d)])/(27*d^2*e^2*(d + e*x^3)^(3/2))
```

Maple [B] time = 0.042, size = 1005, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)/(e*x^3+d)^(5/2),x)
```

```
[Out] c*(2/9*d*x/e^4*(e*x^3+d)^(1/2)/(x^3+d/e)^2-22/27/e^2*x/((x^3+d/e)*e)^(1/2)-
32/81*I/e^3*3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)
/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)*((x-1/e*(-d*e^2)^(1/3))/
(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)*(-I*(x+1/2/e*
(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1
/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*
3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2),(I*3^(1/2)/e*(-d*
e^2)^(1/3)/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2))+
b*(-2/9*x/e^3*(e*x^3+d)^(1/2)/(x^3+d/e)^2+4/27/e/d*x/((x^3+d/e)*e)^(1/2)-4/
81*I/e^2/d*3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/
e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)*((x-1/e*(-d*e^2)^(1/3))/(
-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)*(-I*(x+1/2/e*(
-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/
2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3
^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2),(I*3^(1/2)/e*(-d*e
^2)^(1/3)/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2))+a
*(2/9/d*x/e^2*(e*x^3+d)^(1/2)/(x^3+d/e)^2+14/27/d^2*x/((x^3+d/e)*e)^(1/2)-1
4/81*I/d^2*3^(1/2)/e*(-d*e^2)^(1/3)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2
)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)*((x-1/e*(-d*e^2)^(1/3))
```

```
/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)*(-I*(x+1/2/e
*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(
1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I
*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2),(I*3^(1/2)/e*(-d
*e^2)^(1/3)/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{cx^6 + bx^3 + a}{\left(ex^3 + d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(cx^6 + bx^3 + a\right)\sqrt{ex^3 + d}}{e^3x^9 + 3\,de^2x^6 + 3\,d^2ex^3 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)*sqrt(e*x^3 + d)/(e^3*x^9 + 3*d*e^2*x^6 + 3*d^2
*e*x^3 + d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)/(e*x**3+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{cx^6 + bx^3 + a}{\left(ex^3 + d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(5/2), x)
```

3.41 $\int \frac{a+bx^3+cx^6}{\left(d+ex^3\right)^{7/2}} dx$

Optimal. Leaf size=349

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\left(91ae^2+14bde+16cd^2\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x}{\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x}\right),-7-4\sqrt{3}\right)}{405\sqrt[4]{3}d^3e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}+\frac{2x\left(\right.}{}$$

```
[Out] (2*(c*d^2 - b*d*e + a*e^2)*x)/(15*d*e^2*(d + e*x^3)^(5/2)) - (2*(17*c*d^2 -
 2*b*d*e - 13*a*e^2)*x)/(135*d^2*e^2*(d + e*x^3)^(3/2)) + (2*(16*c*d^2 + 14
*b*d*e + 91*a*e^2)*x)/(405*d^3*e^2*Sqrt[d + e*x^3]) + (2*Sqrt[2 + Sqrt[3]]*
(16*c*d^2 + 14*b*d*e + 91*a*e^2)*(d^(1/3) + e^(1/3)*x)*Sqrt[(d^(2/3) - d^(1
/3)*e^(1/3)*x + e^(2/3)*x^2)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*Ellipti
cF[ArcSin[((1 - Sqrt[3])*d^(1/3) + e^(1/3)*x)/((1 + Sqrt[3])*d^(1/3) + e^(1
/3)*x)], -7 - 4*Sqrt[3]])/(405*3^(1/4)*d^3*e^(7/3)*Sqrt[(d^(1/3)*(d^(1/3) +
 e^(1/3)*x))/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*Sqrt[d + e*x^3])
```

Rubi [A] time = 0.324033, antiderivative size = 349, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1409, 385, 199, 218}

$$\frac{2x\left(91ae^2+14bde+16cd^2\right)}{405d^3e^2\sqrt{d+ex^3}}-\frac{2x\left(-13ae^2-2bde+17cd^2\right)}{135d^2e^2\left(d+ex^3\right)^{3/2}}+\frac{2x\left(ae^2-bde+cd^2\right)}{15de^2\left(d+ex^3\right)^{5/2}}+\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{}}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{}\right.}}}{40}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)/(d + e*x^3)^(7/2),x]
```


```
[Out] (2*(c*d^2 - b*d*e + a*e^2)*x)/(15*d*e^2*(d + e*x^3)^(5/2)) - (2*(17*c*d^2 -
 2*b*d*e - 13*a*e^2)*x)/(135*d^2*e^2*(d + e*x^3)^(3/2)) + (2*(16*c*d^2 + 14
*b*d*e + 91*a*e^2)*x)/(405*d^3*e^2*Sqrt[d + e*x^3]) + (2*Sqrt[2 + Sqrt[3]]*
(16*c*d^2 + 14*b*d*e + 91*a*e^2)*(d^(1/3) + e^(1/3)*x)*Sqrt[(d^(2/3) - d^(1
/3)*e^(1/3)*x + e^(2/3)*x^2)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*Ellipti
cF[ArcSin[((1 - Sqrt[3])*d^(1/3) + e^(1/3)*x)/((1 + Sqrt[3])*d^(1/3) + e^(1
```

```
/3)*x)], -7 - 4*Sqrt[3]])/(405*3^(1/4)*d^3*e^(7/3)*Sqrt[(d^(1/3)*(d^(1/3) +
 e^(1/3)*x))/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*Sqrt[d + e*x^3])
```

Rule 1409

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> -Simp[((c*d^2 - b*d*e + a*e^2)*x*(d + e*x^n)^(q + 1))/(d*e
^2*n*(q + 1)), x] + Dist[1/(n*(q + 1)*d*e^2), Int[(d + e*x^n)^(q + 1)*Simp[
c*d^2 - b*d*e + a*e^2*(n*(q + 1) + 1) + c*d*e*n*(q + 1)*x^n, x], x], x] /;
FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{a+bx^3+cx^6}{\left(d+ex^3\right)^{7/2}}\,dx = \frac{2\left(cd^2-bde+ae^2\right)x}{15de^2\left(d+ex^3\right)^{5/2}} - \frac{2\int \frac{\frac{1}{2}\left(2cd^2-e(2bd+13ae)\right)-\frac{15}{2}cdex^3}{\left(d+ex^3\right)^{5/2}}dx}{15de^2}$$

$$= \frac{2\left(cd^2-bde+ae^2\right)x}{15de^2\left(d+ex^3\right)^{5/2}} - \frac{2\left(17cd^2-2bde-13ae^2\right)x}{135d^2e^2\left(d+ex^3\right)^{3/2}} + \frac{\left(16cd^2+14bde+91ae^2\right)\int \frac{1}{\left(d+ex^3\right)^{3/2}}dx}{135d^2e^2}$$

$$= \frac{2\left(cd^2-bde+ae^2\right)x}{15de^2\left(d+ex^3\right)^{5/2}} - \frac{2\left(17cd^2-2bde-13ae^2\right)x}{135d^2e^2\left(d+ex^3\right)^{3/2}} + \frac{2\left(16cd^2+14bde+91ae^2\right)x}{405d^3e^2\sqrt{d+ex^3}} + \frac{\left(16cd^2+14\right.}{}$$

$$= \frac{2\left(cd^2-bde+ae^2\right)x}{15de^2\left(d+ex^3\right)^{5/2}} - \frac{2\left(17cd^2-2bde-13ae^2\right)x}{135d^2e^2\left(d+ex^3\right)^{3/2}} + \frac{2\left(16cd^2+14bde+91ae^2\right)x}{405d^3e^2\sqrt{d+ex^3}} + \frac{2\sqrt{2+\sqrt{3}}\left(\right.}{}$$

Mathematica [C] time = 0.191137, size = 166, normalized size = 0.48

$$\frac{2x\left(e\left(ae\left(157d^2+221dex^3+91e^2x^6\right)+bd\left(-7d^2+34dex^3+14e^2x^6\right)\right)+cd^2\left(-8d^2-19dex^3+16e^2x^6\right)\right)+x\sqrt{\frac{ex^3}{d}+1}\left(d\right.}{405d^3e^2\left(d+ex^3\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)/(d + e*x^3)^(7/2),x]
```

```
[Out] (2*x*(c*d^2*(-8*d^2 - 19*d*e*x^3 + 16*e^2*x^6) + e*(b*d*(-7*d^2 + 34*d*e*x^
3 + 14*e^2*x^6) + a*e*(157*d^2 + 221*d*e*x^3 + 91*e^2*x^6))) + (16*c*d^2 +
7*e*(2*b*d + 13*a*e))*x*(d + e*x^3)^2*Sqrt[1 + (e*x^3)/d]*Hypergeometric2F1
[1/3, 1/2, 4/3, -((e*x^3)/d)])/(405*d^3*e^2*(d + e*x^3)^(5/2))
```

Maple [B] time = 0.043, size = 1095, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)/(e*x^3+d)^(7/2),x)
```

```
[Out] c*(2/15*d*x/e^5*(e*x^3+d)^(1/2)/(x^3+d/e)^3-34/135*x/e^4*(e*x^3+d)^(1/2)/(x
^3+d/e)^2+32/405/e^2/d*x/((x^3+d/e)*e)^(1/2)-32/1215*I/e^3/d*3^(1/2)*(-d*e^
2)^(1/3)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)
*e/(-d*e^2)^(1/3))^(1/2)*((x-1/e*(-d*e^2)^(1/3))/(-3/2/e*(-d*e^2)^(1/3)+1/2
*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)*(-I*(x+1/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/
2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*Ellipt
icF(1/3*3^(1/2)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*
3^(1/2)*e/(-d*e^2)^(1/3))^(1/2),(I*3^(1/2)/e*(-d*e^2)^(1/3)/(-3/2/e*(-d*e^2
)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)))+b*(-2/15*x/e^4*(e*x^3+d)^(
1/2)/(x^3+d/e)^3+4/135/d*x/e^3*(e*x^3+d)^(1/2)/(x^3+d/e)^2+28/405/e/d^2*x/(
(x^3+d/e)*e)^(1/2)-28/1215*I/e^2/d^2*3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2/e*(-d
*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)
*((x-1/e*(-d*e^2)^(1/3))/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1
/3)))^(1/2)*(-I*(x+1/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(
1/2)*e/(-d*e^2)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/
2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3)
)^(1/2),(I*3^(1/2)/e*(-d*e^2)^(1/3)/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*
(-d*e^2)^(1/3)))^(1/2)))+a*(2/15/d*x/e^3*(e*x^3+d)^(1/2)/(x^3+d/e)^3+26/135
/d^2*x/e^2*(e*x^3+d)^(1/2)/(x^3+d/e)^2+182/405/d^3*x/((x^3+d/e)*e)^(1/2)-18
2/1215*I/d^3*3^(1/2)/e*(-d*e^2)^(1/3)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1
/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)*((x-1/e*(-d*e^2)^(1/3
))/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)*(-I*(x+1/2
/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))
^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2
*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2),(I*3^(1/2)/e*(
-d*e^2)^(1/3)/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)
))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{cx^6 + bx^3 + a}{\left(ex^3 + d\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(7/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^6+bx^3+a\right)\sqrt{ex^3+d}}{e^4x^{12}+4\,de^3x^9+6\,d^2e^2x^6+4\,d^3ex^3+d^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)*sqrt(e*x^3 + d)/(e^4*x^12 + 4*d*e^3*x^9 + 6*d^
2*e^2*x^6 + 4*d^3*e*x^3 + d^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)/(e*x**3+d)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{cx^6+bx^3+a}{\left(ex^3+d\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(7/2), x)
```

3.42 $\int \frac{a+bx^3+cx^6}{\left(d+ex^3\right)^{9/2}}\,dx$

Optimal. Leaf size=389

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\left(247ae^2+26bde+16cd^2\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\left(1-\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x}{\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x}\right),-7-4\sqrt{3}\right)}{1215\sqrt[4]{3}d^4e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}+\frac{2x\left(\right.}{}$$

```
[Out] (2*(c*d^2 - b*d*e + a*e^2)*x)/(21*d*e^2*(d + e*x^3)^(7/2)) - (2*(23*c*d^2 -
 2*b*d*e - 19*a*e^2)*x)/(315*d^2*e^2*(d + e*x^3)^(5/2)) + (2*(16*c*d^2 + 26
*b*d*e + 247*a*e^2)*x)/(2835*d^3*e^2*(d + e*x^3)^(3/2)) + (2*(16*c*d^2 + 26
*b*d*e + 247*a*e^2)*x)/(1215*d^4*e^2*Sqrt[d + e*x^3]) + (2*Sqrt[2 + Sqrt[3]
]*(16*c*d^2 + 26*b*d*e + 247*a*e^2)*(d^(1/3) + e^(1/3)*x)*Sqrt[(d^(2/3) - d
^(1/3)*e^(1/3)*x + e^(2/3)*x^2)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*Elli
pticF[ArcSin[((1 - Sqrt[3])*d^(1/3) + e^(1/3)*x)/((1 + Sqrt[3])*d^(1/3) + e
^(1/3)*x)], -7 - 4*Sqrt[3]])/(1215*3^(1/4)*d^4*e^(7/3)*Sqrt[(d^(1/3)*(d^(1/
3) + e^(1/3)*x))/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*Sqrt[d + e*x^3])
```

Rubi [A] time = 0.394819, antiderivative size = 389, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1409, 385, 199, 218}

$$\frac{2x\left(247ae^2+26bde+16cd^2\right)}{1215d^4e^2\sqrt{d+ex^3}}+\frac{2x\left(247ae^2+26bde+16cd^2\right)}{2835d^3e^2\left(d+ex^3\right)^{3/2}}-\frac{2x\left(-19ae^2-2bde+23cd^2\right)}{315d^2e^2\left(d+ex^3\right)^{5/2}}+\frac{2x\left(ae^2-bde+cd^2\right)}{21de^2\left(d+ex^3\right)^{7/2}}+\frac{2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)/(d + e*x^3)^(9/2),x]
```


```
[Out] (2*(c*d^2 - b*d*e + a*e^2)*x)/(21*d*e^2*(d + e*x^3)^(7/2)) - (2*(23*c*d^2 -
 2*b*d*e - 19*a*e^2)*x)/(315*d^2*e^2*(d + e*x^3)^(5/2)) + (2*(16*c*d^2 + 26
*b*d*e + 247*a*e^2)*x)/(2835*d^3*e^2*(d + e*x^3)^(3/2)) + (2*(16*c*d^2 + 26
*b*d*e + 247*a*e^2)*x)/(1215*d^4*e^2*Sqrt[d + e*x^3]) + (2*Sqrt[2 + Sqrt[3]
]*(16*c*d^2 + 26*b*d*e + 247*a*e^2)*(d^(1/3) + e^(1/3)*x)*Sqrt[(d^(2/3) - d
```

```
^(1/3)*e^(1/3)*x + e^(2/3)*x^2)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*Elli
pticF[ArcSin[((1 - Sqrt[3])*d^(1/3) + e^(1/3)*x)/((1 + Sqrt[3])*d^(1/3) + e
^(1/3)*x)], -7 - 4*Sqrt[3]])/(1215*3^(1/4)*d^4*e^(7/3)*Sqrt[(d^(1/3)*(d^(1/
3) + e^(1/3)*x))/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*Sqrt[d + e*x^3])
```

Rule 1409

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> -Simp[((c*d^2 - b*d*e + a*e^2)*x*(d + e*x^n)^(q + 1))/(d*e
^2*n*(q + 1)), x] + Dist[1/(n*(q + 1)*d*e^2), Int[(d + e*x^n)^(q + 1)*Simp[
c*d^2 - b*d*e + a*e^2*(n*(q + 1) + 1) + c*d*e*n*(q + 1)*x^n, x], x], x] /;
FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{a+bx^3+cx^6}{\left(d+ex^3\right)^{9/2}}\,dx = \frac{2\left(cd^2-bde+ae^2\right)x}{21de^2\left(d+ex^3\right)^{7/2}} - \frac{2\int \frac{\frac{1}{2}\left(2cd^2-e(2bd+19ae)\right)-\frac{21}{2}cdex^3}{\left(d+ex^3\right)^{7/2}}\,dx}{21de^2}$$

$$= \frac{2\left(cd^2-bde+ae^2\right)x}{21de^2\left(d+ex^3\right)^{7/2}} - \frac{2\left(23cd^2-2bde-19ae^2\right)x}{315d^2e^2\left(d+ex^3\right)^{5/2}} + \frac{\left(16cd^2+26bde+247ae^2\right)\int \frac{1}{\left(d+ex^3\right)^{5/2}}\,dx}{315d^2e^2}$$

$$= \frac{2\left(cd^2-bde+ae^2\right)x}{21de^2\left(d+ex^3\right)^{7/2}} - \frac{2\left(23cd^2-2bde-19ae^2\right)x}{315d^2e^2\left(d+ex^3\right)^{5/2}} + \frac{2\left(16cd^2+26bde+247ae^2\right)x}{2835d^3e^2\left(d+ex^3\right)^{3/2}} + \frac{\left(16cd^2+26\right.}{}$$

$$= \frac{2\left(cd^2-bde+ae^2\right)x}{21de^2\left(d+ex^3\right)^{7/2}} - \frac{2\left(23cd^2-2bde-19ae^2\right)x}{315d^2e^2\left(d+ex^3\right)^{5/2}} + \frac{2\left(16cd^2+26bde+247ae^2\right)x}{2835d^3e^2\left(d+ex^3\right)^{3/2}} + \frac{2\left(16cd^2+2\right.}{1215d}$$

$$= \frac{2\left(cd^2-bde+ae^2\right)x}{21de^2\left(d+ex^3\right)^{7/2}} - \frac{2\left(23cd^2-2bde-19ae^2\right)x}{315d^2e^2\left(d+ex^3\right)^{5/2}} + \frac{2\left(16cd^2+26bde+247ae^2\right)x}{2835d^3e^2\left(d+ex^3\right)^{3/2}} + \frac{2\left(16cd^2+2\right.}{1215d}$$

Mathematica [C] time = 0.243114, size = 200, normalized size = 0.51

$$\frac{2x\left(e\left(ae\left(7182d^2ex^3+3388d^3+5928de^2x^6+1729e^3x^9\right)+bd\left(756d^2ex^3-91d^3+624de^2x^6+182e^3x^9\right)\right)+cd^2\left(-189d^2ex\right.\right.}{8505d^4e^2\left(d+ex^3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)/(d + e*x^3)^(9/2),x]
```

```
[Out] (2*x*(c*d^2*(-56*d^3 - 189*d^2*e*x^3 + 384*d*e^2*x^6 + 112*e^3*x^9) + e*(b*
d*(-91*d^3 + 756*d^2*e*x^3 + 624*d*e^2*x^6 + 182*e^3*x^9) + a*e*(3388*d^3 +
 7182*d^2*e*x^3 + 5928*d*e^2*x^6 + 1729*e^3*x^9))) + 7*(16*c*d^2 + 13*e*(2*
b*d + 19*a*e))*x*(d + e*x^3)^3*Sqrt[1 + (e*x^3)/d]*Hypergeometric2F1[1/3, 1
/2, 4/3, -((e*x^3)/d)])/(8505*d^4*e^2*(d + e*x^3)^(7/2))
```

Maple [B] time = 0.048, size = 1182, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)/(e*x^3+d)^(9/2),x)
```

```
[Out] c*(2/21*d*x/e^6*(e*x^3+d)^(1/2)/(x^3+d/e)^4-46/315*x/e^5*(e*x^3+d)^(1/2)/(x
^3+d/e)^3+32/2835/d*x/e^4*(e*x^3+d)^(1/2)/(x^3+d/e)^2+32/1215/e^2/d^2*x/((x
^3+d/e)*e)^(1/2)-32/3645*I/e^3/d^2*3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2/e*(-d*e
^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)*(
(x-1/e*(-d*e^2)^(1/3))/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3
)))^(1/2)*(-I*(x+1/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/
2)*e/(-d*e^2)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/
e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^
(1/2),(I*3^(1/2)/e*(-d*e^2)^(1/3)/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-
d*e^2)^(1/3)))^(1/2)))+b*(-2/21*x/e^5*(e*x^3+d)^(1/2)/(x^3+d/e)^4+4/315/d*x
/e^4*(e*x^3+d)^(1/2)/(x^3+d/e)^3+52/2835/d^2*x/e^3*(e*x^3+d)^(1/2)/(x^3+d/e
)^2+52/1215/e/d^3*x/((x^3+d/e)*e)^(1/2)-52/3645*I/e^2/d^3*3^(1/2)*(-d*e^2)^
(1/3)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/
(-d*e^2)^(1/3))^(1/2)*((x-1/e*(-d*e^2)^(1/3))/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*
3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)*(-I*(x+1/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/
e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*EllipticF
(1/3*3^(1/2)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(
1/2)*e/(-d*e^2)^(1/3))^(1/2),(I*3^(1/2)/e*(-d*e^2)^(1/3)/(-3/2/e*(-d*e^2)^(
1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)))+a*(2/21/d*x/e^4*(e*x^3+d)^(1/
2)/(x^3+d/e)^4+38/315/d^2*x/e^3*(e*x^3+d)^(1/2)/(x^3+d/e)^3+494/2835/d^3*x/
e^2*(e*x^3+d)^(1/2)/(x^3+d/e)^2+494/1215/d^4*x/((x^3+d/e)*e)^(1/2)-494/3645
*I/d^4*3^(1/2)/e*(-d*e^2)^(1/3)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*
(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)*((x-1/e*(-d*e^2)^(1/3))/(-3
/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)*(-I*(x+1/2/e*(-d
*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)
/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(
1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2),(I*3^(1/2)/e*(-d*e^2
)^(1/3)/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{cx^6 + bx^3 + a}{\left(ex^3 + d\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(9/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^6+bx^3+a\right)\sqrt{ex^3+d}}{e^5x^{15}+5\,de^4x^{12}+10\,d^2e^3x^9+10\,d^3e^2x^6+5\,d^4ex^3+d^5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(9/2),x, algorithm="fricas")
```

```
[Out] integral((c*x^6 + b*x^3 + a)*sqrt(e*x^3 + d)/(e^5*x^15 + 5*d*e^4*x^12 + 10*
d^2*e^3*x^9 + 10*d^3*e^2*x^6 + 5*d^4*e*x^3 + d^5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)/(e*x**3+d)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{cx^6+bx^3+a}{\left(ex^3+d\right)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)/(e*x^3 + d)^(9/2), x)
```

3.43 $\int \frac{x^4\left(d+ex^4\right)}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=433

$$-\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-b\cdots\right.}{2\sqrt[4]{2}c^{5/4}\left(\cdots\right.}$$

```
[Out] (e*x)/c - ((c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan
[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-
b - Sqrt[b^2 - 4*a*c])^(3/4)) - ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqr
t[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])
/(2*2^(1/4)*c^(5/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) - ((c*d - b*e + (b*c*d
- b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqr
t[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4))
- ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/
4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b + Sqr
t[b^2 - 4*a*c])^(3/4))
```

Rubi [A] time = 1.13232, antiderivative size = 433, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1502, 1422, 212, 208, 205}

$$-\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-b\cdots\right.}{2\sqrt[4]{2}c^{5/4}\left(\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```



```
[Out] (e*x)/c - ((c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan
[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-
b - Sqrt[b^2 - 4*a*c])^(3/4)) - ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqr
t[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])
/(2*2^(1/4)*c^(5/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) - ((c*d - b*e + (b*c*d
- b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqr
t[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4))
```

```
- ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/
4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b + Sqr
t[b^2 - 4*a*c])^(3/4))
```

Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(e*f^(n - 1)*(f*x)^(m - n + 1)*(a
+ b*x^n + c*x^(2*n))^(p + 1))/(c*(m + n*(2*p + 1) + 1)), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{x^4\left(d+ex^4\right)}{a+bx^4+cx^8}\,dx = \frac{ex}{c} - \frac{\int \frac{ae-(cd-be)x^4}{a+bx^4+cx^8}\,dx}{c}$$

$$= \frac{ex}{c} + \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2c} + \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2c}$$

$$= \frac{ex}{c} - \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2c\sqrt{-b+\sqrt{b^2-4ac}}} - \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}+\sqrt{2}}}{2c\sqrt{-b+\sqrt{b^2-4ac}}}$$

$$= \frac{ex}{c} - \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} - \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}c^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}$$

Mathematica [C] time = 0.07527, size = 88, normalized size = 0.2

$$\frac{ex}{c} - \frac{\text{RootSum}\left[\#1^4 b + \#1^8 c + a\&, \frac{\#1^4 be\log(x-\#1)+\#1^4(-c)d\log(x-\#1)+ae\log(x-\#1)}{\#1^3 b+2\#1^7 c}\&\right]}{4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

```
[Out] (e*x)/c - RootSum[a + b*#1^4 + c*#1^8 & , (a*e*Log[x - #1] - c*d*Log[x - #1
]*#1^4 + b*e*Log[x - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ]/(4*c)
```

Maple [C] time = 0.004, size = 67, normalized size = 0.2

$$\frac{ex}{c} + \frac{1}{4\,c}\sum_{\_R=\text{RootOf}\left(\_Z^8 c+\_Z^4 b+a\right)} \frac{\left((-be+cd)\_R^4 - ae\right)\ln\left(x-\_R\right)}{2\_R^7 c+\_R^3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

```
[Out] e*x/c+1/4/c*sum(((-b*e+c*d)*_R^4-a*e)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(
_Z^8*c+_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{ex}{c} - \frac{-\int \frac{(cd-be)x^4-ae}{cx^8+bx^4+a}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] e*x/c - integrate(-((c*d - b*e)*x^4 - a*e)/(c*x^8 + b*x^4 + a), x)/c
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x**4+d)/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.44 $\int \frac{x^3(d+ex^4)}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=72

$$\frac{e\log\left(a+bx^4+cx^8\right)}{8c}-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c\sqrt{b^2-4ac}}$$

```
[Out] -((2*c*d - b*e)*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(4*c*Sqrt[b^2 - 4
*a*c]) + (e*Log[a + b*x^4 + c*x^8])/(8*c)
```

Rubi [A] time = 0.0721409, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1468, 634, 618, 206, 628}

$$\frac{e\log\left(a+bx^4+cx^8\right)}{8c}-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

```
[Out] -((2*c*d - b*e)*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(4*c*Sqrt[b^2 - 4
*a*c]) + (e*Log[a + b*x^4 + c*x^8])/(8*c)
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3\left(d+ex^4\right)}{a+bx^4+cx^8}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{d+ex}{a+bx+cx^2}\,dx, x, x^4\right)\\
&= \frac{e\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^4\right)}{8c} + \frac{(2cd-be)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^4\right)}{8c}\\
&= \frac{e\log\left(a+bx^4+cx^8\right)}{8c} - \frac{(2cd-be)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^4\right)}{4c}\\
&= -\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c\sqrt{b^2-4ac}} + \frac{e\log\left(a+bx^4+cx^8\right)}{8c}
\end{aligned}
$$

Mathematica [A] time = 0.0555689, size = 71, normalized size = 0.99

$$
\frac{e\log\left(a+bx^4+cx^8\right) - \frac{2(be-2cd)\tan^{-1}\left(\frac{b+2cx^4}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{8c}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

```
[Out] ((-2*(-2*c*d + b*e)*ArcTan[(b + 2*c*x^4)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4
*a*c] + e*Log[a + b*x^4 + c*x^8])/(8*c)
```

Maple [A] time = 0.003, size = 99, normalized size = 1.4

$$\frac{e\ln\left(cx^8+bx^4+a\right)}{8c}+\frac{d}{2}\arctan\left((2cx^4+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}-\frac{be}{4c}\arctan\left((2cx^4+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/8*e*ln(c*x^8+b*x^4+a)/c+1/2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^4+b)/(4*a*c-b
^2)^(1/2))*d-1/4/(4*a*c-b^2)^(1/2)*arctan((2*c*x^4+b)/(4*a*c-b^2)^(1/2))*e*
b/c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.22427, size = 481, normalized size = 6.68

$$\left[\frac{\left(b^2-4ac\right)e\log\left(cx^8+bx^4+a\right)-\sqrt{b^2-4ac}(2cd-be)\log\left(\frac{2c^2x^8+2bcx^4+b^2-2ac+\left(2cx^4+b\right)\sqrt{b^2-4ac}}{cx^8+bx^4+a}\right)}{8\left(b^2c-4ac^2\right)},\frac{\left(b^2-4ac\right)e\log\left(cx^8+b\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/8*((b^2 - 4*a*c)*e*log(c*x^8 + b*x^4 + a) - sqrt(b^2 - 4*a*c)*(2*c*d - b
*e)*log((2*c^2*x^8 + 2*b*c*x^4 + b^2 - 2*a*c + (2*c*x^4 + b)*sqrt(b^2 - 4*a
*c))/(c*x^8 + b*x^4 + a)))/(b^2*c - 4*a*c^2), 1/8*((b^2 - 4*a*c)*e*log(c*x^
8 + b*x^4 + a) - 2*sqrt(-b^2 + 4*a*c)*(2*c*d - b*e)*arctan(-(2*c*x^4 + b)*s
qrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/(b^2*c - 4*a*c^2)]
```

Sympy [B] time = 7.51907, size = 287, normalized size = 3.99

$$\left(\frac{e}{8c} - \frac{\sqrt{-4ac+b^2}\,(be-2cd)}{8c\left(4ac-b^2\right)}\right)\log\left(x^4 + \frac{-16ac\left(\frac{e}{8c} - \frac{\sqrt{-4ac+b^2}(be-2cd)}{8c\left(4ac-b^2\right)}\right) + 2ae + 4b^2\left(\frac{e}{8c} - \frac{\sqrt{-4ac+b^2}(be-2cd)}{8c\left(4ac-b^2\right)}\right) - bd}{be-2cd}\right) + \left(\frac{e}{8c} + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**4+d)/(c*x**8+b*x**4+a),x)
```

```
[Out] (e/(8*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(8*c*(4*a*c - b**2)))*log(x**4
 + (-16*a*c*(e/(8*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(8*c*(4*a*c - b**2
))) + 2*a*e + 4*b**2*(e/(8*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(8*c*(4*a
*c - b**2))) - b*d)/(b*e - 2*c*d)) + (e/(8*c) + sqrt(-4*a*c + b**2)*(b*e -
2*c*d)/(8*c*(4*a*c - b**2)))*log(x**4 + (-16*a*c*(e/(8*c) + sqrt(-4*a*c + b
**2)*(b*e - 2*c*d)/(8*c*(4*a*c - b**2))) + 2*a*e + 4*b**2*(e/(8*c) + sqrt(-
4*a*c + b**2)*(b*e - 2*c*d)/(8*c*(4*a*c - b**2))) - b*d)/(b*e - 2*c*d))
```

Giac [A] time = 6.16674, size = 95, normalized size = 1.32

$$\frac{e\log\left(cx^8 + bx^4 + a\right)}{8\,c} + \frac{(2\,cd - be)\arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{4\,\sqrt{-b^2+4\,ac}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/8*e*log(c*x^8 + b*x^4 + a)/c + 1/4*(2*c*d - b*e)*arctan((2*c*x^4 + b)/sqr
t(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c)
```

3.45 $\int \frac{x^2(d+ex^4)}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=375

$$\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}\right.}{2\,2^{3/}}$$

```
[Out] ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqr
t[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])^(1/4))
+ ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + S
qrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(3/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)
) - ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b
- Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])^(1
/4)) - ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(
-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(3/4)*(-b + Sqrt[b^2 - 4*a*c])
^(1/4))
```

Rubi [A] time = 0.455776, antiderivative size = 375, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {1510, 298, 205, 208}

$$\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}\right.}{2\,2^{3/}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

```
[Out] ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqr
t[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])^(1/4))
+ ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + S
qrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(3/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)
) - ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b
- Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])^(1
/4)) - ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(
-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(3/4)*(-b + Sqrt[b^2 - 4*a*c])
```

```
^(1/4))
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{x^2\left(d+ex^4\right)}{a+bx^4+cx^8}\,dx = \frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{x^2}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx+\frac{1}{2}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{x^2}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx$$

$$= -\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{2}\sqrt{c}}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{2}\sqrt{c}}-\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\cdots}{\cdots}$$

$$= \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-b-\sqrt{b^2-4ac}}}+\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-b+\sqrt{b^2-4ac}}}-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\cdots\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-b-\cdots}}$$

Mathematica [C] time = 0.0474342, size = 59, normalized size = 0.16

$$\frac{1}{4}\text{RootSum}\left[\#1^4 b+\#1^8 c+a\&,\frac{\#1^4 e\log(x-\#1)+d\log(x-\#1)}{2\#1^5 c+\#1 b}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

```
[Out] RootSum[a + b*#1^4 + c*#1^8 & , (d*Log[x - #1] + e*Log[x - #1]*#1^4)/(b*#1
+ 2*c*#1^5) & ]/4
```

Maple [C] time = 0.003, size = 51, normalized size = 0.1

$$\frac{1}{4}\sum_{\_R=\text{RootOf}\left(\_Z^8 c+\_Z^4 b+a\right)}\frac{\left(\_R^6 e+\_R^2 d\right)\ln\left(x-\_R\right)}{2\_R^7 c+\_R^3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/4*sum((_R^6*e+_R^2*d)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4*b+
a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(ex^4+d\right)x^2}{cx^8+bx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^4 + d)*x^2/(c*x^8 + b*x^4 + a), x)
```

Fricas [B] time = 97.8058, size = 26996, normalized size = 71.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] -sqrt(sqrt(1/2)*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a
*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 - (a*b^4*c^3 - 8*a^2*b^
2*c^4 + 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 -
48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^
2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3
*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*
a^5*c^9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)))*arctan(1/2*((2*(a*b^4
*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6)*d - (a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*
b*c^5)*e)*x*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b
*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^
3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c +
 a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)
) + ((b^2*c^3 - 4*a*c^4)*d^4*e - 6*(a*b^2*c^2 - 4*a^2*c^3)*d^2*e^3 + 4*(a*b
^3*c - 4*a^2*b*c^2)*d*e^4 - (a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*e^5)*x - sqrt
(1/2)*((b^2*c^3 - 4*a*c^4)*d^4*e - 6*(a*b^2*c^2 - 4*a^2*c^3)*d^2*e^3 + 4*(a
*b^3*c - 4*a^2*b*c^2)*d*e^4 - (a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*e^5 + (2*(a
*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6)*d - (a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*
a^3*b*c^5)*e)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2
*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*
a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c
 + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^
9)))*sqrt((2*(c^5*d^8 - 2*b*c^4*d^7*e + 14*a*b*c^3*d^5*e^3 + (b^2*c^3 - 4*a
*c^4)*d^6*e^2 - 5*(3*a*b^2*c^2 + 2*a^2*c^3)*d^4*e^4 + 6*(a*b^3*c + 3*a^2*b*
c^2)*d^3*e^5 - (a*b^4 + 9*a^2*b^2*c + 4*a^3*c^2)*d^2*e^6 + 2*(a^2*b^3 + a^3
*b*c)*d*e^7 - (a^3*b^2 - a^4*c)*e^8)*x^2 - sqrt(1/2)*((b^3*c^4 - 4*a*b*c^5)
*d^6 - 4*(a*b^2*c^4 - 4*a^2*c^5)*d^5*e - 5*(a*b^3*c^3 - 4*a^2*b*c^4)*d^4*e^
2 + 4*(a*b^4*c^2 + 2*a^2*b^2*c^3 - 24*a^3*c^4)*d^3*e^3 - (a*b^5*c + 17*a^2*
b^3*c^2 - 84*a^3*b*c^3)*d^2*e^4 + 4*(2*a^2*b^4*c - 9*a^3*b^2*c^2 + 4*a^4*c^
3)*d*e^5 - (a^2*b^5 - 5*a^3*b^3*c + 4*a^4*b*c^2)*e^6 + ((a*b^6*c^4 - 12*a^2
*b^4*c^5 + 48*a^3*b^2*c^6 - 64*a^4*c^7)*d^2 - (a^2*b^6*c^3 - 12*a^3*b^4*c^4
 + 48*a^4*b^2*c^5 - 64*a^5*c^6)*e^2)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a
*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4
+ 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 +
 (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*
a^4*b^2*c^8 - 64*a^5*c^9)))*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^
2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 - (a*b^4*c^
```

```
3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^
4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(
7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2
*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b
^2*c^8 - 64*a^5*c^9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)))/(c^5*d^8
- 2*b*c^4*d^7*e + 14*a*b*c^3*d^5*e^3 + (b^2*c^3 - 4*a*c^4)*d^6*e^2 - 5*(3*a
*b^2*c^2 + 2*a^2*c^3)*d^4*e^4 + 6*(a*b^3*c + 3*a^2*b*c^2)*d^3*e^5 - (a*b^4
+ 9*a^2*b^2*c + 4*a^3*c^2)*d^2*e^6 + 2*(a^2*b^3 + a^3*b*c)*d*e^7 - (a^3*b^2
 - a^4*c)*e^8)))*sqrt(sqrt(1/2)*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^
2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 - (a*b^
4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*
b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 +
 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 +
(a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a
^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)))/(c^4*
d^6 - b*c^3*d^5*e - 5*a*c^3*d^4*e^2 + 10*a*b*c^2*d^3*e^3 - 5*(a*b^2*c + a^2
*c^2)*d^2*e^4 + (a*b^3 + 3*a^2*b*c)*d*e^5 - (a^2*b^2 - a^3*c)*e^6)) + sqrt(
sqrt(1/2)*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c
 - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 + (a*b^4*c^3 - 8*a^2*b^2*c^4
+ 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2
*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*
a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c
 + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^
9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)))*arctan(1/2*(sqrt(1/2)*((b^2
*c^3 - 4*a*c^4)*d^4*e - 6*(a*b^2*c^2 - 4*a^2*c^3)*d^2*e^3 + 4*(a*b^3*c - 4*
a^2*b*c^2)*d*e^4 - (a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*e^5 - (2*(a*b^4*c^4 -
8*a^2*b^2*c^5 + 16*a^3*c^6)*d - (a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*
e)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*
e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^
2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)
*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))*sqrt(s
qrt(1/2)*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c
- 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 + (a*b^4*c^3 - 8*a^2*b^2*c^4 +
 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*
b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a
^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c
+ a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9
)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)))*sqrt((2*(c^5*d^8 - 2*b*c^4*d
^7*e + 14*a*b*c^3*d^5*e^3 + (b^2*c^3 - 4*a*c^4)*d^6*e^2 - 5*(3*a*b^2*c^2 +
2*a^2*c^3)*d^4*e^4 + 6*(a*b^3*c + 3*a^2*b*c^2)*d^3*e^5 - (a*b^4 + 9*a^2*b^2
*c + 4*a^3*c^2)*d^2*e^6 + 2*(a^2*b^3 + a^3*b*c)*d*e^7 - (a^3*b^2 - a^4*c)*e
^8)*x^2 - sqrt(1/2)*((b^3*c^4 - 4*a*b*c^5)*d^6 - 4*(a*b^2*c^4 - 4*a^2*c^5)*
d^5*e - 5*(a*b^3*c^3 - 4*a^2*b*c^4)*d^4*e^2 + 4*(a*b^4*c^2 + 2*a^2*b^2*c^3
- 24*a^3*c^4)*d^3*e^3 - (a*b^5*c + 17*a^2*b^3*c^2 - 84*a^3*b*c^3)*d^2*e^4 +
 4*(2*a^2*b^4*c - 9*a^3*b^2*c^2 + 4*a^4*c^3)*d*e^5 - (a^2*b^5 - 5*a^3*b^3*c
```

```
 + 4*a^4*b*c^2)*e^6 - ((a*b^6*c^4 - 12*a^2*b^4*c^5 + 48*a^3*b^2*c^6 - 64*a^
4*c^7)*d^2 - (a^2*b^6*c^3 - 12*a^3*b^4*c^4 + 48*a^4*b^2*c^5 - 64*a^5*c^6)*e
^2)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3
*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d
^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2
)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))*sqrt(
-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d
*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 + (a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*s
qrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5
- 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^
6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8
)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^4*c^3
 - 8*a^2*b^2*c^4 + 16*a^3*c^5)))/(c^5*d^8 - 2*b*c^4*d^7*e + 14*a*b*c^3*d^5*
e^3 + (b^2*c^3 - 4*a*c^4)*d^6*e^2 - 5*(3*a*b^2*c^2 + 2*a^2*c^3)*d^4*e^4 + 6
*(a*b^3*c + 3*a^2*b*c^2)*d^3*e^5 - (a*b^4 + 9*a^2*b^2*c + 4*a^3*c^2)*d^2*e^
6 + 2*(a^2*b^3 + a^3*b*c)*d*e^7 - (a^3*b^2 - a^4*c)*e^8)) + ((2*(a*b^4*c^4
- 8*a^2*b^2*c^5 + 16*a^3*c^6)*d - (a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5
)*e)*x*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*
d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3
)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*
c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)) - (
(b^2*c^3 - 4*a*c^4)*d^4*e - 6*(a*b^2*c^2 - 4*a^2*c^3)*d^2*e^3 + 4*(a*b^3*c
- 4*a^2*b*c^2)*d*e^4 - (a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*e^5)*x)*sqrt(sqrt(
1/2)*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*
a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 + (a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*
a^3*c^5)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^
3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c
^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^
4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))/
(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5))))/(c^4*d^6 - b*c^3*d^5*e - 5*a*c^
3*d^4*e^2 + 10*a*b*c^2*d^3*e^3 - 5*(a*b^2*c + a^2*c^2)*d^2*e^4 + (a*b^3 + 3
*a^2*b*c)*d*e^5 - (a^2*b^2 - a^3*c)*e^6)) - 1/4*sqrt(sqrt(1/2)*sqrt(-(b*c^3
*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 +
(a*b^3 - 3*a^2*b*c)*e^4 + (a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*sqrt((c^
6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*
b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(
a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*
b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^4*c^3 - 8*a^
2*b^2*c^4 + 16*a^3*c^5)))*log(1/2*sqrt(1/2)*((b^4*c^5 - 8*a*b^2*c^6 + 16*a^
2*c^7)*d^7 - 9*(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6)*d^5*e^2 + 5*(a*b^5*
c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*d^4*e^3 - (a*b^6*c^2 - 27*a^2*b^4*c^3 +
 168*a^3*b^2*c^4 - 304*a^4*c^5)*d^3*e^4 - 18*(a^2*b^5*c^2 - 8*a^3*b^3*c^3 +
 16*a^4*b*c^4)*d^2*e^5 + (7*a^2*b^6*c - 59*a^3*b^4*c^2 + 136*a^4*b^2*c^3 -
48*a^5*c^4)*d*e^6 - (a^2*b^7 - 9*a^3*b^5*c + 24*a^4*b^3*c^2 - 16*a^5*b*c^3)
*e^7 - ((a*b^7*c^5 - 12*a^2*b^5*c^6 + 48*a^3*b^3*c^7 - 64*a^4*b*c^8)*d^3 -
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6*(a^2*b^6*c^5 - 12*a^3*b^4*c^6 + 48*a^4*b^2*c^7 - 64*a^5*c^8)*d^2*e + 3*(a
^2*b^7*c^4 - 12*a^3*b^5*c^5 + 48*a^4*b^3*c^6 - 64*a^5*b*c^7)*d*e^2 - (a^2*b
^8*c^3 - 14*a^3*b^6*c^4 + 72*a^4*b^4*c^5 - 160*a^5*b^2*c^6 + 128*a^6*c^7)*e
^3)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3
*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d
^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2
)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))*sqrt(
sqrt(1/2)*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c
 - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 + (a*b^4*c^3 - 8*a^2*b^2*c^4
+ 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2
*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*
a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c
 + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^
9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)))*sqrt(-(b*c^3*d^4 - 8*a*c^3*
d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*
b*c)*e^4 + (a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a*c^
5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^
2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3
*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^
3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*
a^3*c^5)) + (c^6*d^10 - 3*b*c^5*d^9*e + 3*(b^2*c^4 - a*c^5)*d^8*e^2 - (b^3*
c^3 - 16*a*b*c^4)*d^7*e^3 - 14*(2*a*b^2*c^3 + a^2*c^4)*d^6*e^4 + 21*(a*b^3*
c^2 + 2*a^2*b*c^3)*d^5*e^5 - 7*(a*b^4*c + 6*a^2*b^2*c^2 + 2*a^3*c^3)*d^4*e^
6 + (a*b^5 + 17*a^2*b^3*c + 24*a^3*b*c^2)*d^3*e^7 - 3*(a^2*b^4 + 4*a^3*b^2*
c + a^4*c^2)*d^2*e^8 + (3*a^3*b^3 + a^4*b*c)*d*e^9 - (a^4*b^2 - a^5*c)*e^10
)*x) + 1/4*sqrt(sqrt(1/2)*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*
e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 + (a*b^4*c^3
- 8*a^2*b^2*c^4 + 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*
d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*
a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b
^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2
*c^8 - 64*a^5*c^9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)))*log(-1/2*sq
rt(1/2)*((b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*d^7 - 9*(a*b^4*c^4 - 8*a^2*b^
2*c^5 + 16*a^3*c^6)*d^5*e^2 + 5*(a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*
d^4*e^3 - (a*b^6*c^2 - 27*a^2*b^4*c^3 + 168*a^3*b^2*c^4 - 304*a^4*c^5)*d^3*
e^4 - 18*(a^2*b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*d^2*e^5 + (7*a^2*b^6*
c - 59*a^3*b^4*c^2 + 136*a^4*b^2*c^3 - 48*a^5*c^4)*d*e^6 - (a^2*b^7 - 9*a^3
*b^5*c + 24*a^4*b^3*c^2 - 16*a^5*b*c^3)*e^7 - ((a*b^7*c^5 - 12*a^2*b^5*c^6
+ 48*a^3*b^3*c^7 - 64*a^4*b*c^8)*d^3 - 6*(a^2*b^6*c^5 - 12*a^3*b^4*c^6 + 48
*a^4*b^2*c^7 - 64*a^5*c^8)*d^2*e + 3*(a^2*b^7*c^4 - 12*a^3*b^5*c^5 + 48*a^4
*b^3*c^6 - 64*a^5*b*c^7)*d*e^2 - (a^2*b^8*c^3 - 14*a^3*b^6*c^4 + 72*a^4*b^4
*c^5 - 160*a^5*b^2*c^6 + 128*a^6*c^7)*e^3)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2
 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^
4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d
*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7
```

+ 48*a^4*b^2*c^8 - 64*a^5*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 + (a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)))*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 + (a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)) + (c^6*d^10 - 3*b*c^5*d^9*e + 3*(b^2*c^4 - a*c^5)*d^8*e^2 - (b^3*c^3 - 16*a*b*c^4)*d^7*e^3 - 14*(2*a*b^2*c^3 + a^2*c^4)*d^6*e^4 + 21*(a*b^3*c^2 + 2*a^2*b*c^3)*d^5*e^5 - 7*(a*b^4*c + 6*a^2*b^2*c^2 + 2*a^3*c^3)*d^4*e^6 + (a*b^5 + 17*a^2*b^3*c + 24*a^3*b*c^2)*d^3*e^7 - 3*(a^2*b^4 + 4*a^3*b^2*c + a^4*c^2)*d^2*e^8 + (3*a^3*b^3 + a^4*b*c)*d*e^9 - (a^4*b^2 - a^5*c)*e^10)*x) - 1/4*sqrt(sqrt(1/2)*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 - (a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)))*log(1/2*sqrt(1/2)*((b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*d^7 - 9*(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6)*d^5*e^2 + 5*(a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*d^4*e^3 - (a*b^6*c^2 - 27*a^2*b^4*c^3 + 168*a^3*b^2*c^4 - 304*a^4*c^5)*d^3*e^4 - 18*(a^2*b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*d^2*e^5 + (7*a^2*b^6*c - 59*a^3*b^4*c^2 + 136*a^4*b^2*c^3 - 48*a^5*c^4)*d*e^6 - (a^2*b^7 - 9*a^3*b^5*c + 24*a^4*b^3*c^2 - 16*a^5*b*c^3)*e^7 + ((a*b^7*c^5 - 12*a^2*b^5*c^6 + 48*a^3*b^3*c^7 - 64*a^4*b*c^8)*d^3 - 6*(a^2*b^6*c^5 - 12*a^3*b^4*c^6 + 48*a^4*b^2*c^7 - 64*a^5*c^8)*d^2*e + 3*(a^2*b^7*c^4 - 12*a^3*b^5*c^5 + 48*a^4*b^3*c^6 - 64*a^5*b*c^7)*d*e^2 - (a^2*b^8*c^3 - 14*a^3*b^6*c^4 + 72*a^4*b^4*c^5 - 160*a^5*b^2*c^6 + 128*a^6*c^7)*e^3)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 - (a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*

```
c^9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)))*sqrt(-(b*c^3*d^4 - 8*a*c^
3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^
2*b*c)*e^4 - (a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a*
c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*
a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a
^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*
a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 1
6*a^3*c^5)) + (c^6*d^10 - 3*b*c^5*d^9*e + 3*(b^2*c^4 - a*c^5)*d^8*e^2 - (b^
3*c^3 - 16*a*b*c^4)*d^7*e^3 - 14*(2*a*b^2*c^3 + a^2*c^4)*d^6*e^4 + 21*(a*b^
3*c^2 + 2*a^2*b*c^3)*d^5*e^5 - 7*(a*b^4*c + 6*a^2*b^2*c^2 + 2*a^3*c^3)*d^4*
e^6 + (a*b^5 + 17*a^2*b^3*c + 24*a^3*b*c^2)*d^3*e^7 - 3*(a^2*b^4 + 4*a^3*b^
2*c + a^4*c^2)*d^2*e^8 + (3*a^3*b^3 + a^4*b*c)*d*e^9 - (a^4*b^2 - a^5*c)*e^
10)*x) + 1/4*sqrt(sqrt(1/2)*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^
2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 - (a*b^4*c^
3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^
4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(
7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2
*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b
^2*c^8 - 64*a^5*c^9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)))*log(-1/2*
sqrt(1/2)*((b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*d^7 - 9*(a*b^4*c^4 - 8*a^2*
b^2*c^5 + 16*a^3*c^6)*d^5*e^2 + 5*(a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5
)*d^4*e^3 - (a*b^6*c^2 - 27*a^2*b^4*c^3 + 168*a^3*b^2*c^4 - 304*a^4*c^5)*d^
3*e^4 - 18*(a^2*b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*d^2*e^5 + (7*a^2*b^
6*c - 59*a^3*b^4*c^2 + 136*a^4*b^2*c^3 - 48*a^5*c^4)*d*e^6 - (a^2*b^7 - 9*a
^3*b^5*c + 24*a^4*b^3*c^2 - 16*a^5*b*c^3)*e^7 + ((a*b^7*c^5 - 12*a^2*b^5*c^
6 + 48*a^3*b^3*c^7 - 64*a^4*b*c^8)*d^3 - 6*(a^2*b^6*c^5 - 12*a^3*b^4*c^6 +
48*a^4*b^2*c^7 - 64*a^5*c^8)*d^2*e + 3*(a^2*b^7*c^4 - 12*a^3*b^5*c^5 + 48*a
^4*b^3*c^6 - 64*a^5*b*c^7)*d*e^2 - (a^2*b^8*c^3 - 14*a^3*b^6*c^4 + 72*a^4*b
^4*c^5 - 160*a^5*b^2*c^6 + 128*a^6*c^7)*e^3)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e
^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*
d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)
*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c
^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))*sqrt(sqrt(1/2)*sqrt(-(b*c^3*d^4 - 8*a*c
^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a
^2*b*c)*e^4 - (a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a
*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19
*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c -
a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12
*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 +
16*a^3*c^5)))*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a*b
^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 - (a*b^4*c^3 - 8*a^2*b^2*
c^4 + 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 - 48
*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^2
- 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3*b
^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^
```

```
5*c^9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)) + (c^6*d^10 - 3*b*c^5*d^
9*e + 3*(b^2*c^4 - a*c^5)*d^8*e^2 - (b^3*c^3 - 16*a*b*c^4)*d^7*e^3 - 14*(2*
a*b^2*c^3 + a^2*c^4)*d^6*e^4 + 21*(a*b^3*c^2 + 2*a^2*b*c^3)*d^5*e^5 - 7*(a*
b^4*c + 6*a^2*b^2*c^2 + 2*a^3*c^3)*d^4*e^6 + (a*b^5 + 17*a^2*b^3*c + 24*a^3
*b*c^2)*d^3*e^7 - 3*(a^2*b^4 + 4*a^3*b^2*c + a^4*c^2)*d^2*e^8 + (3*a^3*b^3
+ a^4*b*c)*d*e^9 - (a^4*b^2 - a^5*c)*e^10)*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**4+d)/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.46 $\int \frac{x(d+ex^4)}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=184

$$\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}$$

```
[Out] ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b
- Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((
e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rubi [A] time = 0.212806, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {1490, 1166, 205}

$$\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```


```
[Out] ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b
- Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((
e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```


Rule 1490

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e
_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subs
t[Int[x^((m + 1)/k - 1)*(d + e*x^(n/k))^q*(a + b*x^(n/k) + c*x^((2*n)/k))^p
, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2,
2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(d+ex^4\right)}{a+bx^4+cx^8}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{d+ex^2}{a+bx^2+cx^4}\,dx, x, x^2\right)\\
&= \frac{1}{4}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, x^2\right)+\frac{1}{4}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{}{\frac{b}{2}}\right.\\
&= \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}}
\end{aligned}$$

Mathematica [A] time = 0.155503, size = 179, normalized size = 0.97

$$\frac{\frac{\left(e\left(\sqrt{b^2-4ac}-b\right)+2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(e\left(\sqrt{b^2-4ac}+b\right)-2cd\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}}}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

```
[Out] (((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b
- Sqrt[b^2 - 4*a*c]]])/Sqrt[b - Sqrt[b^2 - 4*a*c]] + ((-2*c*d + (b + Sqrt[b
^2 - 4*a*c])*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/
Sqrt[b + Sqrt[b^2 - 4*a*c]])/(2*Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Maple [B] time = 0.022, size = 340, normalized size = 1.9

$$\frac{\sqrt{2}e}{4}\arctan\left(cx^{2}\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}\right)\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}+\frac{\sqrt{2}be}{4}\arctan\left(cx^{2}\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}\right)\frac{1}{\sqrt{-4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^4+d)/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/4*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x^2*2^(1/2)/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2))*e+1/4/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctan(c*x^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*e-
1/2*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*
x^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*d-1/4*2^(1/2)/((-b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*arctanh(c*x^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))
*e+1/4/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh
(c*x^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*e-1/2*c/(-4*a*c+b^2)^(1
/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x^2*2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2))*d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(ex^{4}+d\right)x}{cx^{8}+bx^{4}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^4 + d)*x/(c*x^8 + b*x^4 + a), x)
```

Fricas [B] time = 2.27764, size = 3077, normalized size = 16.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(1/2)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + (a*b^2*c - 4*a^2*c^2)*
sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2
*c - 4*a^2*c^2))*log(-(c^2*d^4 - b*c*d^3*e + a*b*d*e^3 - a^2*e^4)*x^2 + 1/2
*sqrt(1/2)*((b^2*c - 4*a*c^2)*d^3 - (a*b^2 - 4*a^2*c)*d*e^2 - ((a*b^3*c - 4
*a^2*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c^2)*e)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2
+ a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2
 + (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*
c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))) - 1/4*sqrt(1/2)*sqrt(-(b*c*d^2 -
 4*a*c*d*e + a*b*e^2 + (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2
+ a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))*log(-(c^2*d^4
 - b*c*d^3*e + a*b*d*e^3 - a^2*e^4)*x^2 - 1/2*sqrt(1/2)*((b^2*c - 4*a*c^2)*
d^3 - (a*b^2 - 4*a^2*c)*d*e^2 - ((a*b^3*c - 4*a^2*b*c^2)*d - 2*(a^2*b^2*c -
 4*a^3*c^2)*e)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^
3*c^3)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + (a*b^2*c - 4*a^2*c^2)*sqrt(
(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c -
4*a^2*c^2))) + 1/4*sqrt(1/2)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 - (a*b^2*
c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^
3*c^3)))/(a*b^2*c - 4*a^2*c^2))*log(-(c^2*d^4 - b*c*d^3*e + a*b*d*e^3 - a^2
*e^4)*x^2 + 1/2*sqrt(1/2)*((b^2*c - 4*a*c^2)*d^3 - (a*b^2 - 4*a^2*c)*d*e^2
+ ((a*b^3*c - 4*a^2*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c^2)*e)*sqrt((c^2*d^4 -
 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))*sqrt(-(b*c*d^2 - 4*a*
c*d*e + a*b*e^2 - (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2
*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))) - 1/4*sqrt(1/2)*s
qrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 - (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 -
 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2)
)*log(-(c^2*d^4 - b*c*d^3*e + a*b*d*e^3 - a^2*e^4)*x^2 - 1/2*sqrt(1/2)*((b^
2*c - 4*a*c^2)*d^3 - (a*b^2 - 4*a^2*c)*d*e^2 + ((a*b^3*c - 4*a^2*b*c^2)*d -
 2*(a^2*b^2*c - 4*a^3*c^2)*e)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2
*b^2*c^2 - 4*a^3*c^3)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 - (a*b^2*c - 4
*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3
)))/(a*b^2*c - 4*a^2*c^2)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**4+d)/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

Giac [C] time = 7.93858, size = 6484, normalized size = 35.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*(3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 -
4*a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2
*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*e*sin(5/4*pi + 1
/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(3/4)*b^2 - 4*
(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cosh(1/2*imag_part(a
rcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*e*sin(5/4*pi + 1/2*real_part(arcsin(1
/2*sqrt(a*c)*b/(a*abs(c)))))^3 - 9*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c
 + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2
*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*a
bs(c)))))^2*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 3*((a*c^3)^(3/
4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cosh(1/2*
imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*e*sin(5/4*pi + 1/2*real_pa
rt(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqr
t(a*c)*b/(a*abs(c))))) + 9*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^
3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*
c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))
)*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/
2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^2 -
 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cosh(1/2*imag_par
t(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*e*sin(5/4*pi + 1/2*real_part(arcsin(
1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(
a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)
*sqrt(b^2 - 4*a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*
abs(c)))))^2*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))
)))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + ((a*c^3)^(3
/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*e*sin(5/
4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + ((a*c^3)^(1/4)*b^2*c^2 - 4*(a*
c^3)^(1/4)*a*c^3 + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b*c^2)*d*cosh(1/2*imag_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5/4*pi + 1/2*real_part(arcsin(
1/2*sqrt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(1/4)*b^2*c^2 - 4*(a*c^3)^(1/4)*a*
```

```
c^3 + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b*c^2)*d*sin(5/4*pi + 1/2*real_part(a
rcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)
*b/(a*abs(c))))))*arctan((x^2 - (a/c)^(1/4)*cos(5/4*pi + 1/2*arcsin(1/2*sqr
t(a*c)*b/(a*abs(c)))))/((a/c)^(1/4)*sin(5/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b
/(a*abs(c))))))/(a*b^2*c^3 - 4*a^2*c^4) + 1/4*(3*((a*c^3)^(3/4)*b^2 - 4*(a*
c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(1/4*pi + 1/2*real_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*imag_part(arcsin(1/2*sq
rt(a*c)*b/(a*abs(c)))))^3*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)
*b/(a*abs(c))))) - ((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)
*sqrt(b^2 - 4*a*c)*b)*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
))^3*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 -
9*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c
)*b)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh
(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*e*sin(1/4*pi + 1/2*re
al_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*
sqrt(a*c)*b/(a*abs(c))))) + 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a
*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/
(a*abs(c)))))^2*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(
c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 9*((a*c^
3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos
(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*cosh(1/2*ima
g_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*e*sin(1/4*pi + 1/2*real_part(ar
csin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*
b/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3
/4)*sqrt(b^2 - 4*a*c)*b)*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c
)))))*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*s
inh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)
*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(1/4*pi
+ 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*e*sin(1/4*pi + 1/2*r
eal_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2
*sqrt(a*c)*b/(a*abs(c)))))^3 + ((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (
a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*s
qrt(a*c)*b/(a*abs(c)))))^3*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs
(c)))))^3 + ((a*c^3)^(1/4)*b^2*c^2 - 4*(a*c^3)^(1/4)*a*c^3 + (a*c^3)^(1/4)*
sqrt(b^2 - 4*a*c)*b*c^2)*d*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs
(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - (
(a*c^3)^(1/4)*b^2*c^2 - 4*(a*c^3)^(1/4)*a*c^3 + (a*c^3)^(1/4)*sqrt(b^2 - 4*
a*c)*b*c^2)*d*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*arctan((x^2 - (
a/c)^(1/4)*cos(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))/((a/c)^(1/
4)*sin(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))/(a*b^2*c^3 - 4*a^
2*c^4) - 1/8*(((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt
(b^2 - 4*a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c
)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*e - 3*((a
*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*
```

```
cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*im
ag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*e*sin(5/4*pi + 1/2*real_part
(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^
(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(5/4*pi + 1/2*real_part(a
rcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*
c)*b/(a*abs(c)))))^2*e*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))
))) + 9*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 -
 4*a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*
cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*e*sin(5/4*pi + 1/
2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsi
n(1/2*sqrt(a*c)*b/(a*abs(c))))) + 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*
c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/
2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*
abs(c)))))*e*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 9*
((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*
b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2
*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*e*sin(5/4*pi + 1/2*real_par
t(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt
(a*c)*b/(a*abs(c)))))^2 - ((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3
)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c
)*b/(a*abs(c)))))^3*e*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
))^3 + 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2
- 4*a*c)*b)*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))
*e*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1
/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + ((a*c^3)^(1/4)*b^2*c^
2 - 4*(a*c^3)^(1/4)*a*c^3 + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b*c^2)*d*cos(5/
4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_par
t(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(1/4)*b^2*c^2 - 4*(a*c^3)
^(1/4)*a*c^3 + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b*c^2)*d*cos(5/4*pi + 1/2*re
al_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*
sqrt(a*c)*b/(a*abs(c))))))*log(x^4 - 2*x^2*(a/c)^(1/4)*cos(5/4*pi + 1/2*arc
sin(1/2*sqrt(a*c)*b/(a*abs(c)))) + sqrt(a/c))/(a*b^2*c^3 - 4*a^2*c^4) - 1/8
*(((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c
)*b)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*cosh
(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*e - 3*((a*c^3)^(3/4)*
b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(1/4*pi +
 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcs
in(1/2*sqrt(a*c)*b/(a*abs(c)))))^3*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2*
sqrt(a*c)*b/(a*abs(c)))))^2 - 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c +
(a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sq
rt(a*c)*b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(
c)))))^2*e*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + 9*((a*
c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*c
os(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*ima
g_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*e*sin(1/4*pi + 1/2*real_part(
```

```
arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a
*c)*b/(a*abs(c))))) + 3*((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^
(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*
b/(a*abs(c)))))^3*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*e
*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2 - 9*((a*c^3)^(3/
4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*cos(1/4*p
i + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(a
rcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*e*sin(1/4*pi + 1/2*real_part(arcsin(1/2
*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*a
bs(c)))))^2 - ((a*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt
(b^2 - 4*a*c)*b)*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c
)))))^3*e*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + 3*((a
*c^3)^(3/4)*b^2 - 4*(a*c^3)^(3/4)*a*c + (a*c^3)^(3/4)*sqrt(b^2 - 4*a*c)*b)*
cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*e*sin(1/4*p
i + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^2*sinh(1/2*imag_part
(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))^3 + ((a*c^3)^(1/4)*b^2*c^2 - 4*(a*c^3
)^(1/4)*a*c^3 + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b*c^2)*d*cos(1/4*pi + 1/2*r
eal_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2
*sqrt(a*c)*b/(a*abs(c))))) - ((a*c^3)^(1/4)*b^2*c^2 - 4*(a*c^3)^(1/4)*a*c^3
 + (a*c^3)^(1/4)*sqrt(b^2 - 4*a*c)*b*c^2)*d*cos(1/4*pi + 1/2*real_part(arcs
in(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/
(a*abs(c))))))*log(x^4 - 2*x^2*(a/c)^(1/4)*cos(1/4*pi + 1/2*arcsin(1/2*sqrt
(a*c)*b/(a*abs(c)))) + sqrt(a/c))/(a*b^2*c^3 - 4*a^2*c^4)
```

3.47 $\int \frac{d+ex^4}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=375

$$-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(\frac{2cd-be}{\sqrt{b^2-4a\cdots}}\right.}{2\sqrt[4]{2}\cdots}$$

```
[Out] -((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sq
rt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4))
 - ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b +
Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4
)) - ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b
 - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(
3/4)) - ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/
(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a*c]
)^(3/4))
```

Rubi [A] time = 0.350982, antiderivative size = 375, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1422, 212, 208, 205}

$$-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(\frac{2cd-be}{\sqrt{b^2-4a\cdots}}\right.}{2\sqrt[4]{2}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^4)/(a + b*x^4 + c*x^8),x]
```

```
[Out] -((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sq
rt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4))
 - ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b +
Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4
)) - ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b
 - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(
3/4)) - ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/
(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a*c]
```

```
)^(3/4))
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{d+ex^4}{a+bx^4+cx^8}\,dx = \frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx+\frac{1}{2}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,d\cdots$$

$$= -\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{-b-\sqrt{b^2-4ac}}}-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{-b-\sqrt{b^2-4ac}}}-\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\cdots}{2\sqrt{\cdots}}$$

$$= -\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}-\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\cdots}{2\sqrt[4]{2}\sqrt[4]{c}\left(-b-\sqrt{\cdots}\right.}$$

Mathematica [C] time = 0.0483642, size = 61, normalized size = 0.16

$$\frac{1}{4}\text{RootSum}\left[\#1^4b+\#1^8c+a\&,\frac{\#1^4e\log(x-\#1)+d\log(x-\#1)}{\#1^3b+2\#1^7c}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^4)/(a + b*x^4 + c*x^8),x]
```

```
[Out] RootSum[a + b*#1^4 + c*#1^8 & , (d*Log[x - #1] + e*Log[x - #1]*#1^4)/(b*#1^
3 + 2*c*#1^7) & ]/4
```

Maple [C] time = 0.003, size = 47, normalized size = 0.1

$$\frac{1}{4}\sum_{\_R=\text{RootOf}\left(\_Z^8c+\_Z^4b+a\right)}\frac{\left(\_R^4e+d\right)\ln\left(x-\_R\right)}{2\_R^7c+\_R^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^4+d)/(c*x^8+b*x^4+a),x)
```

```
[Out] 1/4*sum((_R^4*e+d)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{ex^4+d}{cx^8+bx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate((e*x^4 + d)/(c*x^8 + b*x^4 + a), x)
```

Fricas [B] time = 20.9388, size = 26437, normalized size = 70.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] sqrt(sqrt(1/2)*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + a^3*b*e^4 + (b^3*
c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e - (a^3*b^4*c - 8*a^4*b^2
*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5
*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2
 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c
- 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*
a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))*arctan(1/4*(2*sqrt(1/
2)*(((a^3*b^8*c^2 - 14*a^4*b^6*c^3 + 72*a^5*b^4*c^4 - 160*a^6*b^2*c^5 + 128
*a^7*c^6)*d^3 - 3*(a^4*b^7*c^2 - 12*a^5*b^5*c^3 + 48*a^6*b^3*c^4 - 64*a^7*b
*c^5)*d^2*e + 6*(a^5*b^6*c^2 - 12*a^6*b^4*c^3 + 48*a^7*b^2*c^4 - 64*a^8*c^5
)*d*e^2 - (a^5*b^7*c - 12*a^6*b^5*c^2 + 48*a^7*b^3*c^3 - 64*a^8*b*c^4)*e^3)
*x*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6
*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^
7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^
4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)) + ((b^
7*c^2 - 9*a*b^5*c^3 + 24*a^2*b^3*c^4 - 16*a^3*b*c^5)*d^7 - (7*a*b^6*c^2 - 5
9*a^2*b^4*c^3 + 136*a^3*b^2*c^4 - 48*a^4*c^5)*d^6*e + 18*(a^2*b^5*c^2 - 8*a
^3*b^3*c^3 + 16*a^4*b*c^4)*d^5*e^2 + (a^2*b^6*c - 27*a^3*b^4*c^2 + 168*a^4*
b^2*c^3 - 304*a^5*c^4)*d^4*e^3 - 5*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^
3)*d^3*e^4 + 9*(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*d^2*e^5 - (a^5*b^4
- 8*a^6*b^2*c + 16*a^7*c^2)*e^7)*x)*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^
3 + a^3*b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e - (
a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4
*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^
4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^
6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 +
 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)) -
 ((b^7*c^2 - 9*a*b^5*c^3 + 24*a^2*b^3*c^4 - 16*a^3*b*c^5)*d^7 - (7*a*b^6*c^
2 - 59*a^2*b^4*c^3 + 136*a^3*b^2*c^4 - 48*a^4*c^5)*d^6*e + 18*(a^2*b^5*c^2
- 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*d^5*e^2 + (a^2*b^6*c - 27*a^3*b^4*c^2 + 168
*a^4*b^2*c^3 - 304*a^5*c^4)*d^4*e^3 - 5*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5
*b*c^3)*d^3*e^4 + 9*(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*d^2*e^5 - (a^5
*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*e^7 + ((a^3*b^8*c^2 - 14*a^4*b^6*c^3 + 72*
a^5*b^4*c^4 - 160*a^6*b^2*c^5 + 128*a^7*c^6)*d^3 - 3*(a^4*b^7*c^2 - 12*a^5*
b^5*c^3 + 48*a^6*b^3*c^4 - 64*a^7*b*c^5)*d^2*e + 6*(a^5*b^6*c^2 - 12*a^6*b^
4*c^3 + 48*a^7*b^2*c^4 - 64*a^8*c^5)*d*e^2 - (a^5*b^7*c - 12*a^6*b^5*c^2 +
48*a^7*b^3*c^3 - 64*a^8*b*c^4)*e^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c
*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d
^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^
2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*
```

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a^8*b^2*c^4 - 64*a^9*c^5)))*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + a^3*
b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e - (a^3*b^4*
c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3
*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 +
 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 +
2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*
b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3))*sqrt((2*(
14*a^3*b*c*d^3*e^5 - 2*a^4*b*d*e^7 + a^5*e^8 - (b^2*c^3 - a*c^4)*d^8 + 2*(b
^3*c^2 + a*b*c^3)*d^7*e - (b^4*c + 9*a*b^2*c^2 + 4*a^2*c^3)*d^6*e^2 + 6*(a*
b^3*c + 3*a^2*b*c^2)*d^5*e^3 - 5*(3*a^2*b^2*c + 2*a^3*c^2)*d^4*e^4 + (a^3*b
^2 - 4*a^4*c)*d^2*e^6)*x^2 - sqrt(1/2)*((b^6*c - 7*a*b^4*c^2 + 14*a^2*b^2*c
^3 - 8*a^3*c^4)*d^6 - 2*(3*a*b^5*c - 17*a^2*b^3*c^2 + 20*a^3*b*c^3)*d^5*e +
 2*(8*a^2*b^4*c - 39*a^3*b^2*c^2 + 28*a^4*c^3)*d^4*e^2 - 20*(a^3*b^3*c - 4*
a^4*b*c^2)*d^3*e^3 - (a^3*b^4 - 18*a^4*b^2*c + 56*a^5*c^2)*d^2*e^4 + 2*(a^4
*b^3 - 4*a^5*b*c)*d*e^5 - 2*(a^5*b^2 - 4*a^6*c)*e^6 + ((a^3*b^7*c - 12*a^4*
b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*d^2 - 2*(a^4*b^6*c - 12*a^5*b^4*c^
2 + 48*a^6*b^2*c^3 - 64*a^7*c^4)*d*e)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b
*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)
*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*
e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 4
8*a^8*b^2*c^4 - 64*a^9*c^5)))*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + a^
3*b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e - (a^3*b^
4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d
^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8
 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2
+ 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^
8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))/(14*a^
3*b*c*d^3*e^5 - 2*a^4*b*d*e^7 + a^5*e^8 - (b^2*c^3 - a*c^4)*d^8 + 2*(b^3*c^
2 + a*b*c^3)*d^7*e - (b^4*c + 9*a*b^2*c^2 + 4*a^2*c^3)*d^6*e^2 + 6*(a*b^3*c
 + 3*a^2*b*c^2)*d^5*e^3 - 5*(3*a^2*b^2*c + 2*a^3*c^2)*d^4*e^4 + (a^3*b^2 -
4*a^4*c)*d^2*e^6)))*sqrt(sqrt(1/2)*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3
 + a^3*b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e - (a
^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*
b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4
)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6
*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 +
48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))/(
3*a^5*b*d*e^9 - a^6*e^10 + (b^2*c^4 - a*c^5)*d^10 - (3*b^3*c^3 + a*b*c^4)*d
^9*e + 3*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*d^8*e^2 - (b^5*c + 17*a*b^3*c^2
+ 24*a^2*b*c^3)*d^7*e^3 + 7*(a*b^4*c + 6*a^2*b^2*c^2 + 2*a^3*c^3)*d^6*e^4 -
 21*(a^2*b^3*c + 2*a^3*b*c^2)*d^5*e^5 + 14*(2*a^3*b^2*c + a^4*c^2)*d^4*e^6
+ (a^3*b^3 - 16*a^4*b*c)*d^3*e^7 - 3*(a^4*b^2 - a^5*c)*d^2*e^8)) - sqrt(sqr
t(1/2)*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + a^3*b*e^4 + (b^3*c - 3*a*
b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e + (a^3*b^4*c - 8*a^4*b^2*c^2 + 1
6*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e
```

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^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b
*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4
*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)
))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))*arctan(1/4*(2*sqrt(1/2)*(((a^
3*b^8*c^2 - 14*a^4*b^6*c^3 + 72*a^5*b^4*c^4 - 160*a^6*b^2*c^5 + 128*a^7*c^6
)*d^3 - 3*(a^4*b^7*c^2 - 12*a^5*b^5*c^3 + 48*a^6*b^3*c^4 - 64*a^7*b*c^5)*d^
2*e + 6*(a^5*b^6*c^2 - 12*a^6*b^4*c^3 + 48*a^7*b^2*c^4 - 64*a^8*c^5)*d*e^2
- (a^5*b^7*c - 12*a^6*b^5*c^2 + 48*a^7*b^3*c^3 - 64*a^8*b*c^4)*e^3)*x*sqrt(
-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (
b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*
(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(
a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)) - ((b^7*c^2 -
9*a*b^5*c^3 + 24*a^2*b^3*c^4 - 16*a^3*b*c^5)*d^7 - (7*a*b^6*c^2 - 59*a^2*b^
4*c^3 + 136*a^3*b^2*c^4 - 48*a^4*c^5)*d^6*e + 18*(a^2*b^5*c^2 - 8*a^3*b^3*c
^3 + 16*a^4*b*c^4)*d^5*e^2 + (a^2*b^6*c - 27*a^3*b^4*c^2 + 168*a^4*b^2*c^3
- 304*a^5*c^4)*d^4*e^3 - 5*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^3*e
^4 + 9*(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*d^2*e^5 - (a^5*b^4 - 8*a^6*
b^2*c + 16*a^7*c^2)*e^7)*x)*sqrt(sqrt(1/2)*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3
*c*d*e^3 + a^3*b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^
3*e + (a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3
- 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 +
 a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*
c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^
4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*
c^3)))*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + a^3*b*e^4 + (b^3*c - 3*a*
b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e + (a^3*b^4*c - 8*a^4*b^2*c^2 + 1
6*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e
^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b
*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4
*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)
))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)) + ((b^7*c^2 - 9*a*b^5*c^3 + 24
*a^2*b^3*c^4 - 16*a^3*b*c^5)*d^7 - (7*a*b^6*c^2 - 59*a^2*b^4*c^3 + 136*a^3*
b^2*c^4 - 48*a^4*c^5)*d^6*e + 18*(a^2*b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4*b*c^
4)*d^5*e^2 + (a^2*b^6*c - 27*a^3*b^4*c^2 + 168*a^4*b^2*c^3 - 304*a^5*c^4)*d
^4*e^3 - 5*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d^3*e^4 + 9*(a^4*b^4*
c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*d^2*e^5 - (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c
^2)*e^7 - ((a^3*b^8*c^2 - 14*a^4*b^6*c^3 + 72*a^5*b^4*c^4 - 160*a^6*b^2*c^5
 + 128*a^7*c^6)*d^3 - 3*(a^4*b^7*c^2 - 12*a^5*b^5*c^3 + 48*a^6*b^3*c^4 - 64
*a^7*b*c^5)*d^2*e + 6*(a^5*b^6*c^2 - 12*a^6*b^4*c^3 + 48*a^7*b^2*c^4 - 64*a
^8*c^5)*d*e^2 - (a^5*b^7*c - 12*a^6*b^5*c^2 + 48*a^7*b^3*c^3 - 64*a^8*b*c^4
)*e^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 -
 a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3
)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2
)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))*s
qrt(sqrt(1/2)*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + a^3*b*e^4 + (b^3*c
```

- 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e + (a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + a^3*b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e + (a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3))*sqrt((2*(14*a^3*b*c*d^3*e^5 - 2*a^4*b*d*e^7 + a^5*e^8 - (b^2*c^3 - a*c^4)*d^8 + 2*(b^3*c^2 + a*b*c^3)*d^7*e - (b^4*c + 9*a*b^2*c^2 + 4*a^2*c^3)*d^6*e^2 + 6*(a*b^3*c + 3*a^2*b*c^2)*d^5*e^3 - 5*(3*a^2*b^2*c + 2*a^3*c^2)*d^4*e^4 + (a^3*b^2 - 4*a^4*c)*d^2*e^6)*x^2 - sqrt(1/2)*((b^6*c - 7*a*b^4*c^2 + 14*a^2*b^2*c^3 - 8*a^3*c^4)*d^6 - 2*(3*a*b^5*c - 17*a^2*b^3*c^2 + 20*a^3*b*c^3)*d^5*e + 2*(8*a^2*b^4*c - 39*a^3*b^2*c^2 + 28*a^4*c^3)*d^4*e^2 - 20*(a^3*b^3*c - 4*a^4*b*c^2)*d^3*e^3 - (a^3*b^4 - 18*a^4*b^2*c + 56*a^5*c^2)*d^2*e^4 + 2*(a^4*b^3 - 4*a^5*b*c)*d*e^5 - 2*(a^5*b^2 - 4*a^6*c)*e^6 - ((a^3*b^7*c - 12*a^4*b^5*c^2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*d^2 - 2*(a^4*b^6*c - 12*a^5*b^4*c^2 + 48*a^6*b^2*c^3 - 64*a^7*c^4)*d*e)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + a^3*b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e + (a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))/(14*a^3*b*c*d^3*e^5 - 2*a^4*b*d*e^7 + a^5*e^8 - (b^2*c^3 - a*c^4)*d^8 + 2*(b^3*c^2 + a*b*c^3)*d^7*e - (b^4*c + 9*a*b^2*c^2 + 4*a^2*c^3)*d^6*e^2 + 6*(a*b^3*c + 3*a^2*b*c^2)*d^5*e^3 - 5*(3*a^2*b^2*c + 2*a^3*c^2)*d^4*e^4 + (a^3*b^2 - 4*a^4*c)*d^2*e^6)))/(3*a^5*b*d*e^9 - a^6*e^10 + (b^2*c^4 - a*c^5)*d^10 - (3*b^3*c^3 + a*b*c^4)*d^9*e + 3*(b^4*c^2 + 4*a*b^2*c^3 + a^2*c^4)*d^8*e^2 - (b^5*c + 17*a*b^3*c^2 + 24*a^2*b*c^3)*d^7*e^3 + 7*(a*b^4*c + 6*a^2*b^2*c^2 + 2*a^3*c^3)*d^6*e^4 - 21*(a^2*b^3*c + 2*a^3*b*c^2)*d^5*e^5 + 14*(2*a^3*b^2*c + a^4*c^2)*d^4*e^6 + (a^3*b^3 - 16*a^4*b*c)*d^3*e^7 - 3*(a^4*b^2 - a^5*c)*d^2*e^8)) + 1/4*sqrt(sqrt(1/2)*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + a^3*b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e + (a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^

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2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c
^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2
*c^2 + 16*a^5*c^3)))*log((10*a^2*b*c*d^3*e^3 - 5*a^3*c*d^2*e^4 - a^3*b*d*e^
5 + a^4*e^6 - (b^2*c^2 - a*c^3)*d^6 + (b^3*c + 3*a*b*c^2)*d^5*e - 5*(a*b^2*
c + a^2*c^2)*d^4*e^2)*x + 1/2*((b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d^5 - 4*(a
*b^3*c - 4*a^2*b*c^2)*d^4*e + 6*(a^2*b^2*c - 4*a^3*c^2)*d^3*e^2 - (a^3*b^2
- 4*a^4*c)*d*e^4 - ((a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d - 2*(a^4*b
^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*e)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b
*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)
*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*
e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 4
8*a^8*b^2*c^4 - 64*a^9*c^5)))*sqrt(sqrt(1/2)*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a
^3*c*d*e^3 + a^3*b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*
d^3*e + (a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^
3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3
 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^
3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*
b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^
5*c^3)))) - 1/4*sqrt(sqrt(1/2)*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + a
^3*b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e + (a^3*b
^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*
d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^
8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2
 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a
^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))*log((
10*a^2*b*c*d^3*e^3 - 5*a^3*c*d^2*e^4 - a^3*b*d*e^5 + a^4*e^6 - (b^2*c^2 - a
*c^3)*d^6 + (b^3*c + 3*a*b*c^2)*d^5*e - 5*(a*b^2*c + a^2*c^2)*d^4*e^2)*x -
1/2*((b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d^5 - 4*(a*b^3*c - 4*a^2*b*c^2)*d^4*
e + 6*(a^2*b^2*c - 4*a^3*c^2)*d^3*e^2 - (a^3*b^2 - 4*a^4*c)*d*e^4 - ((a^3*b
^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d - 2*(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*
a^6*c^3)*e)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*
e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*
b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^
4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5
)))*sqrt(sqrt(1/2)*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + a^3*b*e^4 + (
b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e + (a^3*b^4*c - 8*a^4
*b^2*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12
*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3
*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^
2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 -
 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))) + 1/4*sqrt(sqrt(
1/2)*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + a^3*b*e^4 + (b^3*c - 3*a*b*
c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e - (a^3*b^4*c - 8*a^4*b^2*c^2 + 16*
a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6
 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c
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^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c
^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))
/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))*log((10*a^2*b*c*d^3*e^3 - 5*a^3
*c*d^2*e^4 - a^3*b*d*e^5 + a^4*e^6 - (b^2*c^2 - a*c^3)*d^6 + (b^3*c + 3*a*b
*c^2)*d^5*e - 5*(a*b^2*c + a^2*c^2)*d^4*e^2)*x + 1/2*((b^4*c - 5*a*b^2*c^2
+ 4*a^2*c^3)*d^5 - 4*(a*b^3*c - 4*a^2*b*c^2)*d^4*e + 6*(a^2*b^2*c - 4*a^3*c
^2)*d^3*e^2 - (a^3*b^2 - 4*a^4*c)*d*e^4 + ((a^3*b^5*c - 8*a^4*b^3*c^2 + 16*
a^5*b*c^3)*d - 2*(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*e)*sqrt(-(48*a^3*
b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 -
 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^
2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c
^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))*sqrt(sqrt(1/2)*sqrt(-(
6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + a^3*b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4
*(a*b^2*c - 2*a^2*c^2)*d^3*e - (a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*sqr
t(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 -
 (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e -
4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)
/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c
- 8*a^4*b^2*c^2 + 16*a^5*c^3)))) - 1/4*sqrt(sqrt(1/2)*sqrt(-(6*a^2*b*c*d^2*
e^2 - 8*a^3*c*d*e^3 + a^3*b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*
a^2*c^2)*d^3*e - (a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c
^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*
a*b^2*c^3 + a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c
^2 - 3*a^3*c^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2
- 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^
2 + 16*a^5*c^3)))*log((10*a^2*b*c*d^3*e^3 - 5*a^3*c*d^2*e^4 - a^3*b*d*e^5 +
 a^4*e^6 - (b^2*c^2 - a*c^3)*d^6 + (b^3*c + 3*a*b*c^2)*d^5*e - 5*(a*b^2*c +
 a^2*c^2)*d^4*e^2)*x - 1/2*((b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d^5 - 4*(a*b^
3*c - 4*a^2*b*c^2)*d^4*e + 6*(a^2*b^2*c - 4*a^3*c^2)*d^3*e^2 - (a^3*b^2 - 4
*a^4*c)*d*e^4 + ((a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d - 2*(a^4*b^4*
c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*e)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*
d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^
8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2
 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a
^8*b^2*c^4 - 64*a^9*c^5)))*sqrt(sqrt(1/2)*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*
c*d*e^3 + a^3*b*e^4 + (b^3*c - 3*a*b*c^2)*d^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3
*e - (a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 -
 8*a^4*b*c*d^3*e^5 + 12*a^5*c*d^2*e^6 - a^6*e^8 - (b^4*c^2 - 2*a*b^2*c^3 +
a^2*c^4)*d^8 + 8*(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c
^3)*d^6*e^2 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4
*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c
^3))))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**4+d)/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.48 $\int \frac{d+ex^4}{x\left(a+bx^4+cx^8\right)}\,dx$

Optimal. Leaf size=78

$$\frac{(bd-2ae)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a\sqrt{b^2-4ac}}-\frac{d\log\left(a+bx^4+cx^8\right)}{8a}+\frac{d\log(x)}{a}$$

```
[Out] ((b*d - 2*a*e)*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(4*a*Sqrt[b^2 - 4*
a*c]) + (d*Log[x])/a - (d*Log[a + b*x^4 + c*x^8])/(8*a)
```

Rubi [A] time = 0.125543, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1474, 800, 634, 618, 206, 628}

$$\frac{(bd-2ae)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a\sqrt{b^2-4ac}}-\frac{d\log\left(a+bx^4+cx^8\right)}{8a}+\frac{d\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^4)/(x*(a + b*x^4 + c*x^8)),x]
```

```
[Out] ((b*d - 2*a*e)*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(4*a*Sqrt[b^2 - 4*
a*c]) + (d*Log[x])/a - (d*Log[a + b*x^4 + c*x^8])/(8*a)
```

Rule 1474

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{d+ex^4}{x\left(a+bx^4+cx^8\right)}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{d+ex}{x\left(a+bx+cx^2\right)}\,dx, x, x^4\right)\\
&= \frac{1}{4}\operatorname{Subst}\left(\int \left(\frac{d}{ax}+\frac{-bd+ae-cdx}{a\left(a+bx+cx^2\right)}\right)dx, x, x^4\right)\\
&= \frac{d\log(x)}{a}+\frac{\operatorname{Subst}\left(\int \frac{-bd+ae-cdx}{a+bx+cx^2}\,dx, x, x^4\right)}{4a}\\
&= \frac{d\log(x)}{a}-\frac{d\operatorname{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^4\right)}{8a}+\frac{(-bd+2ae)\operatorname{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^4\right)}{8a}\\
&= \frac{d\log(x)}{a}-\frac{d\log\left(a+bx^4+cx^8\right)}{8a}-\frac{(-bd+2ae)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^4\right)}{4a}\\
&= \frac{(bd-2ae)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a\sqrt{b^2-4ac}}+\frac{d\log(x)}{a}-\frac{d\log\left(a+bx^4+cx^8\right)}{8a}
\end{aligned}
$$

Mathematica [C] time = 0.0339713, size = 80, normalized size = 1.03

$$\frac{d\log(x)}{a}-\frac{\text{RootSum}\left[\#1^4b+\#1^8c+a\&,\frac{\#1^4cd\log(x-\#1)-ae\log(x-\#1)+bd\log(x-\#1)}{2\#1^4c+b}\&\right]}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^4)/(x*(a + b*x^4 + c*x^8)),x]
```

```
[Out] (d*Log[x])/a - RootSum[a + b*#1^4 + c*#1^8 & , (b*d*Log[x - #1] - a*e*Log[x
 - #1] + c*d*Log[x - #1]*#1^4)/(b + 2*c*#1^4) & ]/(4*a)
```

Maple [A] time = 0.007, size = 106, normalized size = 1.4

$$\frac{d\ln(x)}{a}-\frac{d\ln\left(cx^8+bx^4+a\right)}{8a}+\frac{e}{2}\arctan\left((2cx^4+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4ac-b^2}}-\frac{bd}{4a}\arctan\left((2cx^4+b)\frac{1}{\sqrt{4ac-b^2}}\right)\frac{1}{\sqrt{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^4+d)/x/(c*x^8+b*x^4+a),x)
```

```
[Out] d*ln(x)/a-1/8*d*ln(c*x^8+b*x^4+a)/a+1/2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^4+b
)/(4*a*c-b^2)^(1/2))*e-1/4/a/(4*a*c-b^2)^(1/2)*arctan((2*c*x^4+b)/(4*a*c-b^
2)^(1/2))*b*d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/x/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 4.89877, size = 556, normalized size = 7.13

$$\left[-\frac{\left(b^2-4\,ac\right)d\log\left(cx^8+bx^4+a\right)-8\left(b^2-4\,ac\right)d\log\left(x\right)+\sqrt{b^2-4\,ac}(bd-2\,ae)\log\left(\frac{2\,c^2x^8+2\,bcx^4+b^2-2\,ac-\left(2\,cx^4+b\right)\sqrt{b^2-4\,ac}}{cx^8+bx^4+a}\right.}{8\left(ab^2-4\,a^2c\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/x/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] [-1/8*((b^2 - 4*a*c)*d*log(c*x^8 + b*x^4 + a) - 8*(b^2 - 4*a*c)*d*log(x) +
sqrt(b^2 - 4*a*c)*(b*d - 2*a*e)*log((2*c^2*x^8 + 2*b*c*x^4 + b^2 - 2*a*c -
(2*c*x^4 + b)*sqrt(b^2 - 4*a*c))/(c*x^8 + b*x^4 + a)))/(a*b^2 - 4*a^2*c), -
1/8*((b^2 - 4*a*c)*d*log(c*x^8 + b*x^4 + a) - 8*(b^2 - 4*a*c)*d*log(x) - 2*
sqrt(-b^2 + 4*a*c)*(b*d - 2*a*e)*arctan(-(2*c*x^4 + b)*sqrt(-b^2 + 4*a*c)/(
b^2 - 4*a*c)))/(a*b^2 - 4*a^2*c)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**4+d)/x/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

Giac [A] time = 6.59181, size = 105, normalized size = 1.35

$$-\frac{d\log\left(cx^8+bx^4+a\right)}{8\,a}+\frac{d\log\left(x^4\right)}{4\,a}-\frac{(bd-2\,ae)\arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{4\,\sqrt{-b^2+4\,ac}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/x/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/8*d*log(c*x^8 + b*x^4 + a)/a + 1/4*d*log(x^4)/a - 1/4*(b*d - 2*a*e)*arct
an((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a)
```

3.49 $\int \frac{d+ex^4}{x^2\left(a+bx^4+cx^8\right)}\,dx$

Optimal. Leaf size=392

$$-\frac{\sqrt[4]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}a\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-b}}$$

```
[Out] -(d/(a*x)) - (c^(1/4)*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)
*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*a*(-b - Sqrt[b^2 -
4*a*c])^(1/4)) - (c^(1/4)*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(
1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*a*(-b + Sqrt[b^
2 - 4*a*c])^(1/4)) + (c^(1/4)*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTanh
[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*a*(-b - Sq
rt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Ar
cTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*a*(-b
 + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rubi [A] time = 0.682775, antiderivative size = 392, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1504, 1510, 298, 205, 208}

$$-\frac{\sqrt[4]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}a\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-b}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^4)/(x^2*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -(d/(a*x)) - (c^(1/4)*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)
*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*a*(-b - Sqrt[b^2 -
4*a*c])^(1/4)) - (c^(1/4)*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(
1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*a*(-b + Sqrt[b^
2 - 4*a*c])^(1/4)) + (c^(1/4)*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTanh
[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*a*(-b - Sq
rt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Ar
cTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*a*(-b
```

```
 + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rule 1510

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{d+ex^4}{x^2\left(a+bx^4+cx^8\right)}\,dx = -\frac{d}{ax} - \frac{\int \frac{x^2\left(bd-ae+cdx^4\right)}{a+bx^4+cx^8}\,dx}{a}$$
$$= -\frac{d}{ax} - \frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{x^2}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2a} - \frac{\left(c\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{x^2}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2a}$$
$$= -\frac{d}{ax} + \frac{\left(\sqrt{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2\sqrt{2}a} - \frac{\left(\sqrt{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}}{2\sqrt{2}a}$$
$$= -\frac{d}{ax} - \frac{\sqrt[4]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}a\sqrt[4]{-b-\sqrt{b^2-4ac}}} - \frac{\sqrt[4]{c}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}a\sqrt[4]{-b+\sqrt{b^2-4ac}}} + \frac{\sqrt[4]{c}\,(}{}$$

Mathematica [C] time = 0.0653445, size = 85, normalized size = 0.22

$$-\frac{\text{RootSum}\left[\#1^4b+\#1^8c+a\&,\frac{\#1^4cd\log(x-\#1)-ae\log(x-\#1)+bd\log(x-\#1)}{2\#1^5c+\#1b}\&\right]}{4a} - \frac{d}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^4)/(x^2*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -(d/(a*x)) - RootSum[a + b*#1^4 + c*#1^8 & , (b*d*Log[x - #1] - a*e*Log[x -
 #1] + c*d*Log[x - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ]/(4*a)
```

Maple [C] time = 0.006, size = 72, normalized size = 0.2

$$-\frac{1}{4a}\sum_{\_R=\text{RootOf}\left(\_Z^8c+\_Z^4b+a\right)}\frac{\left(cd\_R^6+(-ae+bd)\_R^2\right)\ln(x-\_R)}{2\_R^7c+\_R^3b} - \frac{d}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^4+d)/x^2/(c*x^8+b*x^4+a),x)
```

```
[Out] -1/4/a*sum((c*d*_R^6+(-a*e+b*d)*_R^2)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(
_Z^8*c+_Z^4*b+a))-d/a/x
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/x^2/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/x^2/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**4+d)/x**2/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/x^2/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.50 $\int \frac{d+ex^4}{x^3\left(a+bx^4+cx^8\right)}\,dx$

Optimal. Leaf size=199

$$-\frac{\sqrt{c}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{d}{2ax^2}$$

```
[Out] -d/(2*a*x^2) - (Sqrt[c]*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[b - Sqrt[b^
2 - 4*a*c]]) - (Sqrt[c]*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[b + Sqrt[b^
2 - 4*a*c]])
```

Rubi [A] time = 0.311309, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {1490, 1281, 1166, 205}

$$-\frac{\sqrt{c}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{d}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^4)/(x^3*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -d/(2*a*x^2) - (Sqrt[c]*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[b - Sqrt[b^
2 - 4*a*c]]) - (Sqrt[c]*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[b + Sqrt[b^
2 - 4*a*c]])
```

Rule 1490

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e
_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subs
t[Int[x^((m + 1)/k - 1)*(d + e*x^(n/k))^q*(a + b*x^(n/k) + c*x^((2*n)/k))^p
, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2,
```

```
2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{d+ex^4}{x^3\left(a+bx^4+cx^8\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{d+ex^2}{x^2\left(a+bx^2+cx^4\right)}\,dx,x,x^2\right)$$

$$= -\frac{d}{2ax^2}-\frac{\text{Subst}\left(\int \frac{bd-ae+cdx^2}{a+bx^2+cx^4}\,dx,x,x^2\right)}{2a}$$

$$= -\frac{d}{2ax^2}-\frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx,x,x^2\right)}{4a}-\frac{\left(c\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{}{\frac{b}{2}\cdots}\right.}{4a}$$

$$= -\frac{d}{2ax^2}-\frac{\sqrt{c}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [C] time = 0.0490785, size = 89, normalized size = 0.45

$$-\frac{\text{RootSum}\left[\#1^4 b+\#1^8 c+a\&,\frac{\#1^4 cd\log(x-\#1)-ae\log(x-\#1)+bd\log(x-\#1)}{\#1^2 b+2\#1^6 c}\&\right]}{4a}-\frac{d}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^4)/(x^3*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -d/(2*a*x^2) - RootSum[a + b*#1^4 + c*#1^8 & , (b*d*Log[x - #1] - a*e*Log[x
 - #1] + c*d*Log[x - #1]*#1^4)/(b*#1^2 + 2*c*#1^6) & ]/(4*a)
```

Maple [B] time = 0.02, size = 365, normalized size = 1.8

$$-\frac{c\sqrt{2}d}{4a}\arctan\left(cx^2\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{c\sqrt{2}e}{2}\arctan\left(cx^2\sqrt{2}\frac{1}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^4+d)/x^3/(c*x^8+b*x^4+a),x)
```

```
[Out] -1/4/a*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x^2*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2))*d-1/2*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*arctan(c*x^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
))*e+1/4/a*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
ctan(c*x^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*d+1/4/a*c*2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/
2))*c)^(1/2))*d-1/2*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctanh(c*x^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*e+1/4/a*c/
(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x^2*
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))*b*d-1/2*d/a/x^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/x^3/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] -integrate((c*d*x^4 + b*d - a*e)*x/(c*x^8 + b*x^4 + a), x)/a - 1/2*d/(a*x^2
)
```

Fricas [B] time = 5.41222, size = 5485, normalized size = 27.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/x^3/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(1/2)*a*x^2*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*
a^2*c)*d*e + (a^3*b^2 - 4*a^4*c)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*
a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*
d^2*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log((3*a*b^2*c*d^2*e^2
- 3*a^2*b*c*d*e^3 + a^3*c*e^4 + (b^2*c^2 - a*c^3)*d^4 - (b^3*c + a*b*c^2)*d
^3*e)*x^2 + 1/2*sqrt(1/2)*((b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*d^3 - (3*a*b^4 -
 13*a^2*b^2*c + 4*a^3*c^2)*d^2*e + 3*(a^2*b^3 - 4*a^3*b*c)*d*e^2 - (a^3*b^2
 - 4*a^4*c)*e^3 - ((a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*d - (a^4*b^3 - 4*a^5
*b*c)*e)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 +
 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*b^2 - 4*a^
7*c)))*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e + (
a^3*b^2 - 4*a^4*c)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*
c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*
b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x^2*sqrt(-(a^2*b*e^2 +
 (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e + (a^3*b^2 - 4*a^4*c)*sqrt(-
(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2
*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2
 - 4*a^4*c))*log((3*a*b^2*c*d^2*e^2 - 3*a^2*b*c*d*e^3 + a^3*c*e^4 + (b^2*c^
2 - a*c^3)*d^4 - (b^3*c + a*b*c^2)*d^3*e)*x^2 - 1/2*sqrt(1/2)*((b^5 - 5*a*b
^3*c + 4*a^2*b*c^2)*d^3 - (3*a*b^4 - 13*a^2*b^2*c + 4*a^3*c^2)*d^2*e + 3*(a
^2*b^3 - 4*a^3*b*c)*d*e^2 - (a^3*b^2 - 4*a^4*c)*e^3 - ((a^3*b^4 - 6*a^4*b^2
*c + 8*a^5*c^2)*d - (a^4*b^3 - 4*a^5*b*c)*e)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4
 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b
^2 - a^3*c)*d^2*e^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*
c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e + (a^3*b^2 - 4*a^4*c)*sqrt(-(4*a^3*b*d*e^3
 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e -
2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)))
+ sqrt(1/2)*a*x^2*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2
*c)*d*e - (a^3*b^2 - 4*a^4*c)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b
```

```
^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2
*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log((3*a*b^2*c*d^2*e^2 - 3
*a^2*b*c*d*e^3 + a^3*c*e^4 + (b^2*c^2 - a*c^3)*d^4 - (b^3*c + a*b*c^2)*d^3*
e)*x^2 + 1/2*sqrt(1/2)*((b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*d^3 - (3*a*b^4 - 13
*a^2*b^2*c + 4*a^3*c^2)*d^2*e + 3*(a^2*b^3 - 4*a^3*b*c)*d*e^2 - (a^3*b^2 -
4*a^4*c)*e^3 + ((a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*d - (a^4*b^3 - 4*a^5*b*
c)*e)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*
(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*b^2 - 4*a^7*c
)))*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e - (a^3
*b^2 - 4*a^4*c)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2
)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*b^2
 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x^2*sqrt(-(a^2*b*e^2 + (b
^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e - (a^3*b^2 - 4*a^4*c)*sqrt(-(4*
a^3*b*d*e^3 - a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*
c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 -
4*a^4*c))*log((3*a*b^2*c*d^2*e^2 - 3*a^2*b*c*d*e^3 + a^3*c*e^4 + (b^2*c^2 -
 a*c^3)*d^4 - (b^3*c + a*b*c^2)*d^3*e)*x^2 - 1/2*sqrt(1/2)*((b^5 - 5*a*b^3*
c + 4*a^2*b*c^2)*d^3 - (3*a*b^4 - 13*a^2*b^2*c + 4*a^3*c^2)*d^2*e + 3*(a^2*
b^3 - 4*a^3*b*c)*d*e^2 - (a^3*b^2 - 4*a^4*c)*e^3 + ((a^3*b^4 - 6*a^4*b^2*c
+ 8*a^5*c^2)*d - (a^4*b^3 - 4*a^5*b*c)*e)*sqrt(-(4*a^3*b*d*e^3 - a^4*e^4 -
(b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2
- a^3*c)*d^2*e^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*
d^2 - 2*(a*b^2 - 2*a^2*c)*d*e - (a^3*b^2 - 4*a^4*c)*sqrt(-(4*a^3*b*d*e^3 -
a^4*e^4 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(
3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - 2
*d)/(a*x^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**4+d)/x**3/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

Giac [C] time = 7.0813, size = 2365, normalized size = 11.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/x^3/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] -((a*c^3)^(1/4)*a*c*d*x^4*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(
c)))))*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - (a
*c^3)^(1/4)*a*c*d*x^4*sin(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*
abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + (a*c^3
)^(1/4)*a*b*d*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sin(5
/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - (a*c^3)^(1/4)*
a^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*e*sin(5/4*pi +
1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - (a*c^3)^(1/4)*a*b*d*si
n(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag
_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + (a*c^3)^(1/4)*a^2*e*sin(5/4*pi
 + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(ar
csin(1/2*sqrt(a*c)*b/(a*abs(c))))))*arctan((x^2 - (a/c)^(1/4)*cos(5/4*pi +
1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))/((a/c)^(1/4)*sin(5/4*pi + 1/2*arcs
in(1/2*sqrt(a*c)*b/(a*abs(c))))))/(sqrt(b^2 - 4*a*c)*a*b*abs(a) - (b^2 - 4*
a*c)*a^2) - ((a*c^3)^(1/4)*a*c*d*x^4*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c
)*b/(a*abs(c)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(
c))))) - (a*c^3)^(1/4)*a*c*d*x^4*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt
(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
)) + (a*c^3)^(1/4)*a*b*d*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c
)))))*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - (a*
c^3)^(1/4)*a^2*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*e*si
n(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - (a*c^3)^(1/
4)*a*b*d*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*si
nh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + (a*c^3)^(1/4)*a^2*e
*sin(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*i
mag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*arctan((x^2 - (a/c)^(1/4)*co
s(1/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))/((a/c)^(1/4)*sin(1/4*pi
 + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))/(sqrt(b^2 - 4*a*c)*a*b*abs(a)
- (b^2 - 4*a*c)*a^2) + 1/2*((a*c^3)^(1/4)*a*c*d*x^4*cos(5/4*pi + 1/2*real_p
art(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt
(a*c)*b/(a*abs(c))))) - (a*c^3)^(1/4)*a*c*d*x^4*cos(5/4*pi + 1/2*real_part(
arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c
)*b/(a*abs(c))))) + (a*c^3)^(1/4)*a*b*d*cos(5/4*pi + 1/2*real_part(arcsin(1
/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*a
bs(c))))) - (a*c^3)^(1/4)*a^2*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*
c)*b/(a*abs(c)))))*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*
e - (a*c^3)^(1/4)*a*b*d*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(
a*abs(c)))))*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + (a*c
^3)^(1/4)*a^2*cos(5/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))
))*e*sinh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))))*log(x^4 - 2*x
```

```
^2*(a/c)^(1/4)*cos(5/4*pi + 1/2*arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))) + sqrt(
a/c))/(sqrt(b^2 - 4*a*c)*a*b*abs(a) - (b^2 - 4*a*c)*a^2) + 1/2*((a*c^3)^(1/
4)*a*c*d*x^4*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))
)*cosh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - (a*c^3)^(1/4)*a
*c*d*x^4*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*si
nh(1/2*imag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) + (a*c^3)^(1/4)*a*b*d
*cos(1/4*pi + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*i
mag_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c))))) - (a*c^3)^(1/4)*a^2*cos(1/4*p
i + 1/2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*cosh(1/2*imag_part(a
rcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*e - (a*c^3)^(1/4)*a*b*d*cos(1/4*pi + 1/
2*real_part(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*sinh(1/2*imag_part(arcsin(
1/2*sqrt(a*c)*b/(a*abs(c))))) + (a*c^3)^(1/4)*a^2*cos(1/4*pi + 1/2*real_par
t(arcsin(1/2*sqrt(a*c)*b/(a*abs(c)))))*e*sinh(1/2*imag_part(arcsin(1/2*sqrt
(a*c)*b/(a*abs(c))))))*log(x^4 - 2*x^2*(a/c)^(1/4)*cos(1/4*pi + 1/2*arcsin(
1/2*sqrt(a*c)*b/(a*abs(c)))) + sqrt(a/c))/(sqrt(b^2 - 4*a*c)*a*b*abs(a) - (
b^2 - 4*a*c)*a^2) - 1/2*d/(a*x^2)
```

3.51 $\int \frac{d+ex^4}{x^4\left(a+bx^4+cx^8\right)}\,dx$

Optimal. Leaf size=394

$$\frac{c^{3/4}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}a\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}a\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}a\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}$$

```
[Out] -d/(3*a*x^3) + (c^(3/4)*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/
4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*a*(-b - Sqrt[b^2
- 4*a*c])^(3/4)) + (c^(3/4)*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2
^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*a*(-b + Sqrt[
b^2 - 4*a*c])^(3/4)) + (c^(3/4)*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTa
nh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*a*(-b -
Sqrt[b^2 - 4*a*c])^(3/4)) + (c^(3/4)*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*
ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*a*(
-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rubi [A] time = 0.625247, antiderivative size = 394, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1504, 1422, 212, 208, 205}

$$\frac{c^{3/4}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}a\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}a\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}a\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^4)/(x^4*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -d/(3*a*x^3) + (c^(3/4)*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/
4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*a*(-b - Sqrt[b^2
- 4*a*c])^(3/4)) + (c^(3/4)*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2
^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*a*(-b + Sqrt[
b^2 - 4*a*c])^(3/4)) + (c^(3/4)*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTa
nh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*a*(-b -
Sqrt[b^2 - 4*a*c])^(3/4)) + (c^(3/4)*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*
ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*a*(
```

```
-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rule 1422

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{d+ex^4}{x^4\left(a+bx^4+cx^8\right)}\,dx = -\frac{d}{3ax^3} - \frac{\int \frac{3(bd-ae)+3cdx^4}{a+bx^4+cx^8}\,dx}{3a}$$

$$= -\frac{d}{3ax^3} - \frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2a} - \frac{\left(c\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2a}$$

$$= -\frac{d}{3ax^3} + \frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}x^2}\,dx}{2a\sqrt{-b-\sqrt{b^2-4ac}}} + \frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}+\sqrt{2}\sqrt{c}x^2}}{2a\sqrt{-b-\sqrt{b^2-4ac}}}$$

$$= -\frac{d}{3ax^3} + \frac{c^{3/4}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}a\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} + \frac{c^{3/4}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}a\left(-b+\sqrt{b^2-4ac}\right)^{3/4}} + c$$

Mathematica [C] time = 0.0735016, size = 86, normalized size = 0.22

$$-\frac{3\text{RootSum}\left[\#1^4b+\#1^8c+a\&,\frac{\#1^4cd\log(x-\#1)-ae\log(x-\#1)+bd\log(x-\#1)}{\#1^3b+2\#1^7c}\&\right]+\frac{4d}{x^3}}{12a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^4)/(x^4*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -((4*d)/x^3 + 3*RootSum[a + b*#1^4 + c*#1^8 & , (b*d*Log[x - #1] - a*e*Log[
x - #1] + c*d*Log[x - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ])/(12*a)
```

Maple [C] time = 0.007, size = 68, normalized size = 0.2

$$-\frac{d}{3\,ax^3}+\frac{1}{4\,a}\sum_{\_R=\text{RootOf}\left(\_Z^8c+\_Z^4b+a\right)}\frac{\left(-cd\_R^4+ae-bd\right)\ln\left(x-\_R\right)}{2\_R^7c+\_R^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^4+d)/x^4/(c*x^8+b*x^4+a),x)
```

```
[Out] -1/3*d/a/x^3+1/4/a*sum((-_R^4*c*d+a*e-b*d)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=Ro
otOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\text{sage}_2}{a} - \frac{d}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/x^4/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] -integrate((c*d*x^4 + b*d - a*e)/(c*x^8 + b*x^4 + a), x)/a - 1/3*d/(a*x^3)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/x^4/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**4+d)/x**4/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/x^4/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.52 $\int \frac{x^4(1-x^4)}{1-x^4+x^8}\,dx$

Optimal. Leaf size=278

$$-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}}-x-$$

```
[Out] -x - ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) - ArcT
an[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) + ArcTan[(Sqrt[
2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) + ArcTan[(Sqrt[2 + Sqrt[
3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) - Log[1 - Sqrt[2 - Sqrt[3]]*x + x
^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) - Log[1 -
Sqrt[2 + Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2]
/(4*Sqrt[6])
```

Rubi [A] time = 0.299574, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {1502, 1346, 1169, 634, 618, 204, 628}

$$-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}}-x-$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(1 - x^4))/(1 - x^4 + x^8),x]
```


```
[Out] -x - ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) - ArcT
an[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) + ArcTan[(Sqrt[
2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) + ArcTan[(Sqrt[2 + Sqrt[
3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) - Log[1 - Sqrt[2 - Sqrt[3]]*x + x
^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) - Log[1 -
Sqrt[2 + Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2]
/(4*Sqrt[6])
```


Rule 1502

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(e*f^(n - 1)*(f*x)^(m - n + 1)*(a
```

```
+ b*x^n + c*x^(2*n))^(p + 1))/(c*(m + n*(2*p + 1) + 1)), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rule 1346

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x^(n
/2))/(q - r*x^(n/2) + x^n), x], x] + Dist[1/(2*c*q*r), Int[(r + x^(n/2))/(q
 + r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^4(1-x^4)}{1-x^4+x^8}\,dx = -x+\int \frac{1}{1-x^4+x^8}\,dx$$

$$= -x+\frac{\int \frac{\sqrt{3}-x^2}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}+\frac{\int \frac{\sqrt{3}+x^2}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}$$

$$= -x+\frac{\int \frac{\sqrt{3(2-\sqrt{3})}-(-1+\sqrt{3})x}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{3(2-\sqrt{3})}}+\frac{\int \frac{\sqrt{3(2-\sqrt{3})}+(-1+\sqrt{3})x}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{3(2-\sqrt{3})}}+\frac{\int \frac{\sqrt{3(2+\sqrt{3})}-(1+\sqrt{3})x}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{3(2+\sqrt{3})}}+\frac{\int \frac{\sqrt{3(2+\sqrt{3})}+(1+\sqrt{}}{1+\sqrt{2+\sqrt{3}}x+x}}{4\sqrt{3(2+\sqrt{}}}$$

$$= -x-\frac{\int \frac{-\sqrt{2-\sqrt{3}}+2x}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{6}}+\frac{\int \frac{\sqrt{2-\sqrt{3}}+2x}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{6}}-\frac{\int \frac{-\sqrt{2+\sqrt{3}}+2x}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{6}}+\frac{\int \frac{\sqrt{2+\sqrt{3}}+2x}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{6}}+\frac{\int \frac{1}{1-\sqrt{2-\sqrt{3}}}}{4\sqrt{6(2-}}$$

$$= -x-\frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}}+\frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}}-\frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}}+\frac{\log\left(1+\right.}{}$$

$$= -x-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\log\left(1-\right.}{}$$

Mathematica [C] time = 0.0162919, size = 46, normalized size = 0.17

$$\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\log(x-\#1)}{2\#1^7-\#1^3}\&\right]-x$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(1 - x^4))/(1 - x^4 + x^8),x]
```

```
[Out] -x + RootSum[1 - #1^4 + #1^8 & , Log[x - #1]/(-#1^3 + 2*#1^7) & ]/4
```

Maple [C] time = 0.006, size = 34, normalized size = 0.1

$$-x+\frac{\sum_{\_R=\mathrm{RootOf}\left(9\_Z^{4}+1\right)}\_R \ln\left(3\_R^{2}+3\_R x+x^{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-x^4+1)/(x^8-x^4+1),x)
```

```
[Out] -x+1/4*sum(_R*ln(3*_R^2+3*_R*x+x^2),_R=RootOf(9*_Z^4+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-x+\int \frac{1}{x^{8}-x^{4}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-x^4+1)/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -x + integrate(1/(x^8 - x^4 + 1), x)
```

Fricas [A] time = 1.83203, size = 613, normalized size = 2.21

$$-\frac{1}{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{\sqrt{3}\sqrt{2}\left(x^{3}-x\right)+x^{2}-\sqrt{x^{4}+\sqrt{3}\sqrt{2}\left(x^{3}+x\right)+3\,x^{2}+1}\left(\sqrt{3}\sqrt{2}x-2\right)}{3\,x^{2}-2}\right)-\frac{1}{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{\sqrt{3}\sqrt{2}\left(x^{3}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-x^4+1)/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3)*sqrt(2)*arctan(-(sqrt(3)*sqrt(2)*(x^3 - x) + x^2 - sqrt(x^4 +
sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)*(sqrt(3)*sqrt(2)*x - 2))/(3*x^2 - 2)
) - 1/6*sqrt(3)*sqrt(2)*arctan(-(sqrt(3)*sqrt(2)*(x^3 - x) - x^2 - sqrt(x^4
 - sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)*(sqrt(3)*sqrt(2)*x + 2))/(3*x^2 -
 2)) + 1/24*sqrt(3)*sqrt(2)*log(x^4 + sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1
) - 1/24*sqrt(3)*sqrt(2)*log(x^4 - sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1) -
```

```
x
```

Sympy [A] time = 0.328117, size = 170, normalized size = 0.61

$$-x-\frac{\sqrt{6}\left(-2\operatorname{atan}\left(\frac{\sqrt{6}x}{3}-\frac{1}{3}\right)-2\operatorname{atan}\left(\sqrt{6}x^3-4x^2+2\sqrt{6}x-3\right)\right)}{24}-\frac{\sqrt{6}\left(-2\operatorname{atan}\left(\frac{\sqrt{6}x}{3}+\frac{1}{3}\right)-2\operatorname{atan}\left(\sqrt{6}x^3+4x^2+2\right.\right.}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-x**4+1)/(x**8-x**4+1),x)
```

```
[Out] -x - sqrt(6)*(-2*atan(sqrt(6)*x/3 - 1/3) - 2*atan(sqrt(6)*x**3 - 4*x**2 + 2
*sqrt(6)*x - 3))/24 - sqrt(6)*(-2*atan(sqrt(6)*x/3 + 1/3) - 2*atan(sqrt(6)*
x**3 + 4*x**2 + 2*sqrt(6)*x + 3))/24 - sqrt(6)*log(x**4 - sqrt(6)*x**3 + 3*
x**2 - sqrt(6)*x + 1)/24 + sqrt(6)*log(x**4 + sqrt(6)*x**3 + 3*x**2 + sqrt(
6)*x + 1)/24
```

Giac [A] time = 1.15097, size = 281, normalized size = 1.01

$$\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-x^4+1)/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(6)*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt(2))) + 1/12*s
qrt(6)*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6) + sqrt(2))) + 1/12*sqrt(6)
*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) + 1/12*sqrt(6)*arcta
n((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(2))) + 1/24*sqrt(6)*log(x^2 + 1
/2*x*(sqrt(6) + sqrt(2)) + 1) - 1/24*sqrt(6)*log(x^2 - 1/2*x*(sqrt(6) + sqr
t(2)) + 1) + 1/24*sqrt(6)*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1/24*s
qrt(6)*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1) - x
```

3.53 $\int \frac{x^3(1-x^4)}{1-x^4+x^8} dx$

Optimal. Leaf size=39

$$-\frac{1}{8}\log\left(x^8-x^4+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}$$

```
[Out] -ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 - x^4 + x^8]/8
```

Rubi [A] time = 0.0423471, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1468, 634, 618, 204, 628}

$$-\frac{1}{8}\log\left(x^8-x^4+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(1 - x^4))/(1 - x^4 + x^8),x]
```

```
[Out] -ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 - x^4 + x^8]/8
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\left(1-x^4\right)}{1-x^4+x^8}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1-x}{1-x+x^2}\,dx, x, x^4\right)\\ &= \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^4\right)-\frac{1}{8}\,\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, x^4\right)\\ &= -\frac{1}{8}\log\left(1-x^4+x^8\right)-\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^4\right)\\ &= -\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{8}\log\left(1-x^4+x^8\right)\end{aligned}$$

Mathematica [A] time = 0.0126138, size = 39, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{2x^4-1}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{8}\log\left(x^8-x^4+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(1 - x^4))/(1 - x^4 + x^8),x]
```

```
[Out] ArcTan[(-1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 - x^4 + x^8]/8
```

Maple [A] time = 0.004, size = 33, normalized size = 0.9

$$-\frac{\ln\left(x^{8}-x^{4}+1\right)}{8}+\frac{\sqrt{3}}{12}\arctan\left(\frac{\left(2x^{4}-1\right)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-x^4+1)/(x^8-x^4+1),x)
```

```
[Out] -1/8*ln(x^8-x^4+1)+1/12*3^(1/2)*arctan(1/3*(2*x^4-1)*3^(1/2))
```

Maxima [A] time = 1.48673, size = 43, normalized size = 1.1

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{4}-1\right)\right)-\frac{1}{8}\log\left(x^{8}-x^{4}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-x^4+1)/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1)
```

Fricas [A] time = 1.75197, size = 96, normalized size = 2.46

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{4}-1\right)\right)-\frac{1}{8}\log\left(x^{8}-x^{4}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-x^4+1)/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1)
```

Sympy [A] time = 0.182488, size = 37, normalized size = 0.95

$$-\frac{\log\left(x^{8}-x^{4}+1\right)}{8}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^{4}}{3}-\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-x**4+1)/(x**8-x**4+1),x)
```

```
[Out] -log(x**8 - x**4 + 1)/8 + sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/3)/12
```

Giac [A] time = 1.11937, size = 43, normalized size = 1.1

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)-\frac{1}{8}\log\left(x^8-x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-x^4+1)/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1)
```

3.54 $\int \frac{x^2(1-x^4)}{1-x^4+x^8}\,dx$

Optimal. Leaf size=355

$$\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\sqrt{2+\sqrt{}}\right.$$

```
[Out] ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2 - Sqrt[3])
]) - ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[3*(2 + Sqr
t[3])]) - ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2
- Sqrt[3])]) + ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[
3*(2 + Sqrt[3])]) + (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 - Sqrt[3]]*x + x^
2])/8 - (Sqrt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/8 - (Sqr
t[(2 + Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8 + (Sqrt[(2 + Sqrt[
3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```

Rubi [A] time = 0.289485, antiderivative size = 355, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {1506, 1279, 1169, 634, 618, 204, 628}

$$\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\sqrt{2+\sqrt{}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 - x^4))/(1 - x^4 + x^8),x]
```


```
[Out] ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2 - Sqrt[3])
]) - ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[3*(2 + Sqr
t[3])]) - ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2
- Sqrt[3])]) + ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[
3*(2 + Sqrt[3])]) + (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 - Sqrt[3]]*x + x^
2])/8 - (Sqrt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/8 - (Sqr
t[(2 + Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8 + (Sqrt[(2 + Sqrt[
3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```

Rule 1506

```
Int[(((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
(c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[a*c, 2]}, With[{r = Rt[2*c*q -
 b*c, 2]}, Dist[c/(2*q*r), Int[((f*x)^m*Simp[d*r - (c*d - e*q)*x^(n/2), x])
/(q - r*x^(n/2) + c*x^n), x], x] + Dist[c/(2*q*r), Int[((f*x)^m*Simp[d*r +
(c*d - e*q)*x^(n/2), x])/(q + r*x^(n/2) + c*x^n), x], x]] /;  !LtQ[2*c*q -
b*c, 0]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[n2, 2*n] && LtQ[b^2 - 4*a*c
, 0] && IntegersQ[m, n/2] && LtQ[0, m, n] && PosQ[a*c]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^2\left(1-x^4\right)}{1-x^4+x^8}\,dx = \frac{\int \frac{x^2\left(\sqrt{3}-2x^2\right)}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}} + \frac{\int \frac{x^2\left(\sqrt{3}+2x^2\right)}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}$$

$$= -\frac{\int \frac{-2+\sqrt{3}x^2}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}} - \frac{\int \frac{2+\sqrt{3}x^2}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}$$

$$= -\frac{\int \frac{2\sqrt{2-\sqrt{3}}-\left(2-\sqrt{3}\right)x}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\int \frac{2\sqrt{2-\sqrt{3}}+\left(2-\sqrt{3}\right)x}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\int \frac{-2\sqrt{2+\sqrt{3}}-\left(-2-\sqrt{3}\right)x}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}} - \frac{\int \frac{-2\sqrt{2+\sqrt{3}}+\left(-2-\sqrt{3}\right)x}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}}$$

$$= \frac{1}{8}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\int \frac{1}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx + \frac{1}{8}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\int \frac{1}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx - \frac{1}{8}\sqrt{\frac{1}{3}\left(2\right.}$$

$$= \frac{1}{8}\sqrt{\frac{2}{3}-\frac{1}{\sqrt{3}}}\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right) - \frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right) - \frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}$$

$$= \frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right) - \frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right) - \frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\,\mathrm{t}$$

Mathematica [C] time = 0.0156995, size = 55, normalized size = 0.15

$$-\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1^4\log(x-\#1)-\log(x-\#1)}{2\#1^5-\#1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(1 - x^4))/(1 - x^4 + x^8),x]
```

```
[Out] -RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(-#1 + 2*#1^
5) & ]/4
```

Maple [C] time = 0.007, size = 46, normalized size = 0.1

$$-\frac{1}{4}\sum_{\_R=\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)}\frac{\left(\_R^6-\_R^2\right)\ln\left(x-\_R\right)}{2\_R^7-\_R^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-x^4+1)/(x^8-x^4+1),x)
```

```
[Out] -1/4*sum((_R^6-_R^2)/(2*_R^7-_R^3)*ln(x-_R),_R=RootOf(_Z^8-_Z^4+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\left(x^4-1\right)x^2}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+1)/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^4 - 1)*x^2/(x^8 - x^4 + 1), x)
```

Fricas [B] time = 2.07034, size = 2313, normalized size = 6.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+1)/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/48*sqrt(6)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2)*log(12*x^2 + 2
*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12) - 1/48
*sqrt(6)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2)*log(12*x^2 - 2*sqr
```

```
t(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12) + 1/96*sqr
t(6)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8)*log(12*x^2 + sqrt(6
)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + 12) - 1/96*sqr
t(6)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8)*log(12*x^2 - sqrt(6
)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + 12) - 1/12*sqr
t(6)*sqrt(2)*sqrt(sqrt(3) + 2)*arctan(1/6*sqrt(6)*sqrt(12*x^2 + 2*sqrt(6)*(
2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12)*(sqrt(3)*sqrt(2)
 - 2*sqrt(2))*sqrt(sqrt(3) + 2) + 1/3*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt
(2)*x)*sqrt(sqrt(3) + 2) - sqrt(3) + 2) - 1/12*sqrt(6)*sqrt(2)*sqrt(sqrt(3)
 + 2)*arctan(1/6*sqrt(6)*sqrt(12*x^2 - 2*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*s
qrt(2)*x)*sqrt(sqrt(3) + 2) + 12)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3
) + 2) + 1/3*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2)
+ sqrt(3) - 2) - 1/24*sqrt(6)*sqrt(2)*sqrt(-4*sqrt(3) + 8)*arctan(1/12*sqrt
(6)*sqrt(12*x^2 + sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(
3) + 8) + 12)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8) - 1/6*sqrt
(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) - sqrt(3) - 2)
 - 1/24*sqrt(6)*sqrt(2)*sqrt(-4*sqrt(3) + 8)*arctan(1/12*sqrt(6)*sqrt(12*x^
2 - sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + 12)*
(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8) - 1/6*sqrt(6)*(2*sqrt(3)
*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + sqrt(3) + 2)
```

Sympy [A] time = 1.35041, size = 27, normalized size = 0.08

$$-\operatorname{RootSum}\left(5308416t^8 - 2304t^4 + 1, \left(t \mapsto t\log\left(442368t^7 - 384t^3 + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-x**4+1)/(x**8-x**4+1),x)
```

```
[Out] -RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(442368*_t**7 - 3
84*_t**3 + x)))
```

Giac [A] time = 1.14034, size = 342, normalized size = 0.96

$$-\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+}{\sqrt{6}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+1)/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] -1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqr
t(2))) - 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(
6) + sqrt(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2)
)/(sqrt(6) - sqrt(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) -
 sqrt(2))/(sqrt(6) - sqrt(2))) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x
*(sqrt(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqr
t(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6)
- sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqr
t(2)) + 1)
```

3.55 $\int \frac{x(1-x^4)}{1-x^4+x^8}\,dx$

Optimal. Leaf size=50

$$\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{4\sqrt{3}}-\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{4\sqrt{3}}$$

```
[Out] -Log[1 - Sqrt[3]*x^2 + x^4]/(4*Sqrt[3]) + Log[1 + Sqrt[3]*x^2 + x^4]/(4*Sqr
t[3])
```

Rubi [A] time = 0.0399463, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1490, 1164, 628}

$$\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{4\sqrt{3}}-\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 - x^4))/(1 - x^4 + x^8),x]
```

```
[Out] -Log[1 - Sqrt[3]*x^2 + x^4]/(4*Sqrt[3]) + Log[1 + Sqrt[3]*x^2 + x^4]/(4*Sqr
t[3])
```

Rule 1490

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e
_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subs
t[Int[x^((m + 1)/k - 1)*(d + e*x^(n/k))^q*(a + b*x^(n/k) + c*x^((2*n)/k))^p
, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2,
2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(1-x^4\right)}{1-x^4+x^8}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1-x^2}{1-x^2+x^4}\,dx, x, x^2\right)\\ &= -\frac{\operatorname{Subst}\left(\int \frac{\sqrt{3}+2x}{-1-\sqrt{3}x-x^2}\,dx, x, x^2\right)}{4\sqrt{3}}-\frac{\operatorname{Subst}\left(\int \frac{\sqrt{3}-2x}{-1+\sqrt{3}x-x^2}\,dx, x, x^2\right)}{4\sqrt{3}}\\ &= -\frac{\log\left(1-\sqrt{3}x^2+x^4\right)}{4\sqrt{3}}+\frac{\log\left(1+\sqrt{3}x^2+x^4\right)}{4\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.0161647, size = 44, normalized size = 0.88

$$\frac{\log\left(x^4+\sqrt{3}x^2+1\right)-\log\left(-x^4+\sqrt{3}x^2-1\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 - x^4))/(1 - x^4 + x^8),x]
```

```
[Out] (-Log[-1 + Sqrt[3]*x^2 - x^4] + Log[1 + Sqrt[3]*x^2 + x^4])/(4*Sqrt[3])
```

Maple [A] time = 0.012, size = 39, normalized size = 0.8

$$-\frac{\ln\left(1+x^4-x^2\sqrt{3}\right)\sqrt{3}}{12}+\frac{\ln\left(1+x^4+x^2\sqrt{3}\right)\sqrt{3}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^4+1)/(x^8-x^4+1),x)
```

```
[Out] -1/12*ln(1+x^4-x^2*3^(1/2))*3^(1/2)+1/12*ln(1+x^4+x^2*3^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(x^4-1)x}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+1)/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^4 - 1)*x/(x^8 - x^4 + 1), x)
```

Fricas [A] time = 1.72637, size = 104, normalized size = 2.08

$$\frac{1}{12}\,\sqrt{3}\log\left(\frac{x^8+5\,x^4+2\,\sqrt{3}\left(x^6+x^2\right)+1}{x^8-x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+1)/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*log((x^8 + 5*x^4 + 2*sqrt(3)*(x^6 + x^2) + 1)/(x^8 - x^4 + 1))
```

Sympy [A] time = 0.162072, size = 42, normalized size = 0.84

$$-\frac{\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right)}{12}+\frac{\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**4+1)/(x**8-x**4+1),x)
```

```
[Out] -sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/12 + sqrt(3)*log(x**4 + sqrt(3)*x**2
+ 1)/12
```

Giac [A] time = 1.11162, size = 42, normalized size = 0.84

$$-\frac{1}{12}\sqrt{3}\log\left(\frac{x^2-\sqrt{3}+\frac{1}{x^2}}{x^2+\sqrt{3}+\frac{1}{x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+1)/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*log((x^2 - sqrt(3) + 1/x^2)/(x^2 + sqrt(3) + 1/x^2))
```

3.56 $\int \frac{1-x^4}{1-x^4+x^8}\,dx$

Optimal. Leaf size=355

$$\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\sqrt{2+\sqrt{}}\right.$$

```
[Out] -ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2 - Sqrt[3]
)]) + ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[3*(2 + Sq
rt[3])]) + ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2
 - Sqrt[3])]) - ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt
[3*(2 + Sqrt[3])]) + (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 - Sqrt[3]]*x + x
^2])/8 - (Sqrt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/8 - (Sq
rt[(2 + Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8 + (Sqrt[(2 + Sqrt
[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```

Rubi [A] time = 0.216203, antiderivative size = 355, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {1421, 1169, 634, 618, 204, 628}

$$\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\sqrt{2+\sqrt{}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)/(1 - x^4 + x^8),x]
```


```
[Out] -ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2 - Sqrt[3]
)]) + ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[3*(2 + Sq
rt[3])]) + ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2
 - Sqrt[3])]) - ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt
[3*(2 + Sqrt[3])]) + (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 - Sqrt[3]]*x + x
^2])/8 - (Sqrt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/8 - (Sq
rt[(2 + Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8 + (Sqrt[(2 + Sqrt
[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```


Rule 1421

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x^
(n/2))/Simp[d/e + q*x^(n/2) - x^n, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*
x^(n/2))/Simp[d/e - q*x^(n/2) - x^n, x], x], x]] /; FreeQ[{a, b, c, d, e},
x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[
n/2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1-x^4}{1-x^4+x^8}\,dx = -\frac{\int \frac{\sqrt{3}+2x^2}{-1-\sqrt{3}x^2-x^4}\,dx}{2\sqrt{3}} - \frac{\int \frac{\sqrt{3}-2x^2}{-1+\sqrt{3}x^2-x^4}\,dx}{2\sqrt{3}}$$

$$= \frac{\int \frac{\sqrt{3(2-\sqrt{3})}-(-2+\sqrt{3})x}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{3(2-\sqrt{3})}+(-2+\sqrt{3})x}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{3(2+\sqrt{3})}-(2+\sqrt{3})x}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{3(2+\sqrt{3})}+(2+\sqrt{3})x}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}}$$

$$= -\left(\frac{1}{8}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\int \frac{1}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx\right) - \frac{1}{8}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\int \frac{1}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx + \frac{1}{8}\sqrt{\frac{1}{3}}$$

$$= \frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right) - \frac{1}{8}\sqrt{\frac{2}{3}-\frac{1}{\sqrt{3}}}\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right) - \frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}$$

$$= -\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right) + \frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right) + \frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}$$

Mathematica [C] time = 0.0129269, size = 57, normalized size = 0.16

$$-\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1^4\log(x-\#1)-\log(x-\#1)}{2\#1^7-\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)/(1 - x^4 + x^8),x]
```

```
[Out] -RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(-#1^3 + 2*#
1^7) & ]/4
```

Maple [C] time = 0., size = 44, normalized size = 0.1

$$\frac{1}{4}\sum_{\_R=\text{RootOf}\left(\_Z^8-\_Z^4+1\right)}\frac{\left(-\_R^4+1\right)\ln\left(x-\_R\right)}{2\_R^7-\_R^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)/(x^8-x^4+1),x)
```

```
[Out] 1/4*sum((-_R^4+1)/(2*_R^7-_R^3)*ln(x-_R),_R=RootOf(_Z^8-_Z^4+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^4-1}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^4 - 1)/(x^8 - x^4 + 1), x)
```

Fricas [B] time = 1.86818, size = 2313, normalized size = 6.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/48*sqrt(6)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2)*log(12*x^2 + 2
*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12) - 1/48
*sqrt(6)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2)*log(12*x^2 - 2*sqr
t(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12) + 1/96*sqr
t(6)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8)*log(12*x^2 + sqrt(6
)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + 12) - 1/96*sqr
t(6)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8)*log(12*x^2 - sqrt(6
)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + 12) + 1/12*sqr
t(6)*sqrt(2)*sqrt(sqrt(3) + 2)*arctan(1/6*sqrt(6)*sqrt(12*x^2 + 2*sqrt(6)*(
2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 12)*(sqrt(3)*sqrt(2)
 - 2*sqrt(2))*sqrt(sqrt(3) + 2) + 1/3*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt
(2)*x)*sqrt(sqrt(3) + 2) - sqrt(3) + 2) + 1/12*sqrt(6)*sqrt(2)*sqrt(sqrt(3)
 + 2)*arctan(1/6*sqrt(6)*sqrt(12*x^2 - 2*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*s
qrt(2)*x)*sqrt(sqrt(3) + 2) + 12)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3
) + 2) + 1/3*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2)
+ sqrt(3) - 2) + 1/24*sqrt(6)*sqrt(2)*sqrt(-4*sqrt(3) + 8)*arctan(1/12*sqrt
```

```
(6)*sqrt(12*x^2 + sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(
3) + 8) + 12)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8) - 1/6*sqrt
(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) - sqrt(3) - 2)
 + 1/24*sqrt(6)*sqrt(2)*sqrt(-4*sqrt(3) + 8)*arctan(1/12*sqrt(6)*sqrt(12*x^
2 - sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + 12)*
(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8) - 1/6*sqrt(6)*(2*sqrt(3)
*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + sqrt(3) + 2)
```

Sympy [A] time = 1.54208, size = 26, normalized size = 0.07

$$-\operatorname{RootSum}\left(5308416t^8-2304t^4+1,\left(t\mapsto t\log\left(9216t^5-8t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)/(x**8-x**4+1),x)
```

```
[Out] -RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(9216*_t**5 - 8*_
t + x)))
```

Giac [A] time = 1.12705, size = 342, normalized size = 0.96

$$\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}}{\sqrt{6}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt
(2))) + 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6
) + sqrt(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2))
/(sqrt(6) - sqrt(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) -
sqrt(2))/(sqrt(6) - sqrt(2))) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x*
(sqrt(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt
(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6) -
 sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqrt
(2)) + 1)
```

3.57 $\int \frac{1-x^4}{x\left(1-x^4+x^8\right)} \, dx$

Optimal. Leaf size=41

$$-\frac{1}{8}\log\left(x^8-x^4+1\right)+\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\log(x)$$

```
[Out] ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[x] - Log[1 - x^4 + x^8]/8
```

Rubi [A] time = 0.0526827, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {1474, 800, 634, 618, 204, 628}

$$-\frac{1}{8}\log\left(x^8-x^4+1\right)+\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)/(x*(1 - x^4 + x^8)),x]
```

```
[Out] ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[x] - Log[1 - x^4 + x^8]/8
```

Rule 1474

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1-x^4}{x\left(1-x^4+x^8\right)}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{1-x}{x\left(1-x+x^2\right)}\,dx, x, x^4\right)\\
&= \frac{1}{4}\,\text{Subst}\left(\int \left(\frac{1}{x}-\frac{x}{1-x+x^2}\right)dx, x, x^4\right)\\
&= \log(x)-\frac{1}{4}\,\text{Subst}\left(\int \frac{x}{1-x+x^2}\,dx, x, x^4\right)\\
&= \log(x)-\frac{1}{8}\,\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^4\right)-\frac{1}{8}\,\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, x^4\right)\\
&= \log(x)-\frac{1}{8}\log\left(1-x^4+x^8\right)+\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^4\right)\\
&= \frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\log(x)-\frac{1}{8}\log\left(1-x^4+x^8\right)
\end{aligned}$$

Mathematica [C] time = 0.0128189, size = 44, normalized size = 1.07

$$\log(x)-\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1^4\log(x-\#1)}{2\#1^4-1}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)/(x*(1 - x^4 + x^8)),x]
```

```
[Out] Log[x] - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^4)/(-1 + 2*#1^4) & ]/4
```

Maple [A] time = 0.006, size = 35, normalized size = 0.9

$$\ln(x) - \frac{\ln\left(x^8 - x^4 + 1\right)}{8} - \frac{\sqrt{3}}{12}\arctan\left(\frac{\left(2x^4 - 1\right)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)/x/(x^8-x^4+1),x)
```

```
[Out] ln(x)-1/8*ln(x^8-x^4+1)-1/12*3^(1/2)*arctan(1/3*(2*x^4-1)*3^(1/2))
```

Maxima [A] time = 1.54539, size = 51, normalized size = 1.24

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4 - 1\right)\right) - \frac{1}{8}\log\left(x^8 - x^4 + 1\right) + \frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/x/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1) + 1/
4*log(x^4)
```

Fricas [A] time = 1.46601, size = 109, normalized size = 2.66

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4 - 1\right)\right) - \frac{1}{8}\log\left(x^8 - x^4 + 1\right) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/x/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1) + lo
g(x)
```

Sympy [A] time = 0.201201, size = 41, normalized size = 1.

$$\log(x) - \frac{\log\left(x^8 - x^4 + 1\right)}{8} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^4}{3} - \frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)/x/(x**8-x**4+1),x)
```

```
[Out] log(x) - log(x**8 - x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/3
)/12
```

Giac [A] time = 1.10762, size = 51, normalized size = 1.24

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4 - 1\right)\right) - \frac{1}{8}\log\left(x^8 - x^4 + 1\right) + \frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/x/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1) + 1/
4*log(x^4)
```

3.58 $\int \frac{1-x^4}{x^2\left(1-x^4+x^8\right)}\,dx$

Optimal. Leaf size=280

$$-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}}-\frac{1}{x}+$$

```
[Out] -x^(-1) + ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) +
 ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) - ArcTan[(
Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) - ArcTan[(Sqrt[2 +
Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) - Log[1 - Sqrt[2 - Sqrt[3]]*
x + x^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) - Log
[1 - Sqrt[2 + Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 + Sqrt[3]]*x +
 x^2]/(4*Sqrt[6])
```

Rubi [A] time = 0.207993, antiderivative size = 280, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {1504, 1372, 1169, 634, 618, 204, 628}

$$-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}}-\frac{1}{x}+$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)/(x^2*(1 - x^4 + x^8)),x]
```


```
[Out] -x^(-1) + ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) +
 ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) - ArcTan[(
Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) - ArcTan[(Sqrt[2 +
Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) - Log[1 - Sqrt[2 - Sqrt[3]]*
x + x^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) - Log
[1 - Sqrt[2 + Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 + Sqrt[3]]*x +
 x^2]/(4*Sqrt[6])
```


Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
```

```
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rule 1372

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, -Dist[1/(2*c*r), Int[(x^
(m - 3*(n/2))*(q - r*x^(n/2)))/(q - r*x^(n/2) + x^n), x], x] + Dist[1/(2*c*
r), Int[(x^(m - 3*(n/2))*(q + r*x^(n/2)))/(q + r*x^(n/2) + x^n), x], x]]] /
; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n/2, 0
] && IGtQ[m, 0] && GeQ[m, (3*n)/2] && LtQ[m, 2*n] && NegQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1-x^4}{x^2\left(1-x^4+x^8\right)}\,dx &= -\frac{1}{x}-\int \frac{x^6}{1-x^4+x^8}\,dx \\
&= -\frac{1}{x}+\frac{\int \frac{1-\sqrt{3}x^2}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}-\frac{\int \frac{1+\sqrt{3}x^2}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}} \\
&= -\frac{1}{x}-\frac{\int \frac{\sqrt{2-\sqrt{3}}-\left(1-\sqrt{3}\right)x}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\int \frac{\sqrt{2-\sqrt{3}}+\left(1-\sqrt{3}\right)x}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\int \frac{\sqrt{2+\sqrt{3}}-\left(1+\sqrt{3}\right)x}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{\int \frac{\sqrt{2+\sqrt{3}}+\left(1+\sqrt{3}\right)x}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}} \\
&= -\frac{1}{x}-\frac{\int \frac{-\sqrt{2-\sqrt{3}}+2x}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{6}}+\frac{\int \frac{\sqrt{2-\sqrt{3}}+2x}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{6}}-\frac{\int \frac{-\sqrt{2+\sqrt{3}}+2x}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{6}}+\frac{\int \frac{\sqrt{2+\sqrt{3}}+2x}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{6}}-\frac{\int \frac{}{1-}}{4\sqrt{}} \\
&= -\frac{1}{x}-\frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}}+\frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}}-\frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}}+\frac{\log}{} \\
&= -\frac{1}{x}+\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\log}{}
\end{aligned}
$$

Mathematica [C] time = 0.015771, size = 47, normalized size = 0.17

$$-\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1^3\log(x-\#1)}{2\#1^4-1}\&\right]-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)/(x^2*(1 - x^4 + x^8)),x]
```

```
[Out] -x^(-1) - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^3)/(-1 + 2*#1^4) & ]/
4
```

Maple [C] time = 0.007, size = 38, normalized size = 0.1

$$-\frac{\sum_{\_R=\mathrm{RootOf}(9\_Z^4+1)} \_R \ln\left(9\_R^3x - 3\_R^2 + x^2\right)}{4} - x^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)/x^2/(x^8-x^4+1),x)
```

```
[Out] -1/4*sum(_R*ln(9*_R^3*x-3*_R^2+x^2),_R=RootOf(9*_Z^4+1))-1/x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{x} - \int \frac{x^6}{x^8 - x^4 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/x^2/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -1/x - integrate(x^6/(x^8 - x^4 + 1), x)
```

Fricas [A] time = 1.62903, size = 617, normalized size = 2.2

$$\frac{4\sqrt{3}\sqrt{2}x\arctan\left(-\frac{\sqrt{3}\sqrt{2}(x^3-x)+x^2-\sqrt{x^4+\sqrt{3}\sqrt{2}(x^3+x)+3x^2+1}\left(\sqrt{3}\sqrt{2}x-2\right)}{3x^2-2}\right)+4\sqrt{3}\sqrt{2}x\arctan\left(-\frac{\sqrt{3}\sqrt{2}(x^3-x)-x^2-\sqrt{x^4-\sqrt{3}\sqrt{2}(x^3+x)+3x^2-}{3x^2-2}\right.}{24x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/x^2/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/24*(4*sqrt(3)*sqrt(2)*x*arctan(-(sqrt(3)*sqrt(2)*(x^3 - x) + x^2 - sqrt(x
^4 + sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)*(sqrt(3)*sqrt(2)*x - 2))/(3*x^2
 - 2)) + 4*sqrt(3)*sqrt(2)*x*arctan(-(sqrt(3)*sqrt(2)*(x^3 - x) - x^2 - sqr
t(x^4 - sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)*(sqrt(3)*sqrt(2)*x + 2))/(3*
x^2 - 2)) + sqrt(3)*sqrt(2)*x*log(x^4 + sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 +
```

```
 1) - sqrt(3)*sqrt(2)*x*log(x^4 - sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1) -
24)/x
```

Sympy [A] time = 0.305509, size = 168, normalized size = 0.6

$$-\frac{\sqrt{6}\left(2\,\text{atan}\left(\frac{\sqrt{6}x}{3}-\frac{1}{3}\right)+2\,\text{atan}\left(\sqrt{6}x^3-4x^2+2\sqrt{6}x-3\right)\right)}{24}-\frac{\sqrt{6}\left(2\,\text{atan}\left(\frac{\sqrt{6}x}{3}+\frac{1}{3}\right)+2\,\text{atan}\left(\sqrt{6}x^3+4x^2+2\sqrt{6}x+\right.\right.}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)/x**2/(x**8-x**4+1),x)
```

```
[Out] -sqrt(6)*(2*atan(sqrt(6)*x/3 - 1/3) + 2*atan(sqrt(6)*x**3 - 4*x**2 + 2*sqrt
(6)*x - 3))/24 - sqrt(6)*(2*atan(sqrt(6)*x/3 + 1/3) + 2*atan(sqrt(6)*x**3 +
 4*x**2 + 2*sqrt(6)*x + 3))/24 - sqrt(6)*log(x**4 - sqrt(6)*x**3 + 3*x**2 -
 sqrt(6)*x + 1)/24 + sqrt(6)*log(x**4 + sqrt(6)*x**3 + 3*x**2 + sqrt(6)*x +
 1)/24 - 1/x
```

Giac [A] time = 1.13547, size = 284, normalized size = 1.01

$$-\frac{1}{12}\sqrt{6}\arctan\left(\frac{4\,x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{12}\sqrt{6}\arctan\left(\frac{4\,x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{12}\sqrt{6}\arctan\left(\frac{4\,x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)-\frac{1}{12}\sqrt{6}\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/x^2/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(6)*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt(2))) - 1/12*
sqrt(6)*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6) + sqrt(2))) - 1/12*sqrt(6
)*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) - 1/12*sqrt(6)*arct
an((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(2))) + 1/24*sqrt(6)*log(x^2 +
1/2*x*(sqrt(6) + sqrt(2)) + 1) - 1/24*sqrt(6)*log(x^2 - 1/2*x*(sqrt(6) + sq
rt(2)) + 1) + 1/24*sqrt(6)*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1/24*
sqrt(6)*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1/x
```

3.59 $\int \frac{1-x^4}{x^3(1-x^4+x^8)}\,dx$

Optimal. Leaf size=89

$$-\frac{1}{2x^2}-\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{8\sqrt{3}}+\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{8\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x^2\right)-\frac{1}{4}\tan^{-1}\left(2x^2+\sqrt{3}\right)$$

```
[Out] -1/(2*x^2) + ArcTan[Sqrt[3] - 2*x^2]/4 - ArcTan[Sqrt[3] + 2*x^2]/4 - Log[1
- Sqrt[3]*x^2 + x^4]/(8*Sqrt[3]) + Log[1 + Sqrt[3]*x^2 + x^4]/(8*Sqrt[3])
```

Rubi [A] time = 0.0903328, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {1490, 1281, 1127, 1161, 618, 204, 1164, 628}

$$-\frac{1}{2x^2}-\frac{\log\left(x^4-\sqrt{3}x^2+1\right)}{8\sqrt{3}}+\frac{\log\left(x^4+\sqrt{3}x^2+1\right)}{8\sqrt{3}}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x^2\right)-\frac{1}{4}\tan^{-1}\left(2x^2+\sqrt{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)/(x^3*(1 - x^4 + x^8)),x]
```

```
[Out] -1/(2*x^2) + ArcTan[Sqrt[3] - 2*x^2]/4 - ArcTan[Sqrt[3] + 2*x^2]/4 - Log[1
- Sqrt[3]*x^2 + x^4]/(8*Sqrt[3]) + Log[1 + Sqrt[3]*x^2 + x^4]/(8*Sqrt[3])
```

Rule 1490

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e
_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subs
t[Int[x^((m + 1)/k - 1)*(d + e*x^(n/k))^q*(a + b*x^(n/k) + c*x^((2*n)/k))^p
, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2,
2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
```

```
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1127

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1-x^4}{x^3\left(1-x^4+x^8\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1-x^2}{x^2\left(1-x^2+x^4\right)}\,dx,x,x^2\right)$$
$$= -\frac{1}{2x^2}-\frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{1-x^2+x^4}\,dx,x,x^2\right)$$
$$= -\frac{1}{2x^2}+\frac{1}{4}\,\text{Subst}\left(\int \frac{1-x^2}{1-x^2+x^4}\,dx,x,x^2\right)-\frac{1}{4}\,\text{Subst}\left(\int \frac{1+x^2}{1-x^2+x^4}\,dx,x,x^2\right)$$
$$= -\frac{1}{2x^2}-\frac{1}{8}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{3}x+x^2}\,dx,x,x^2\right)-\frac{1}{8}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{3}x+x^2}\,dx,x,x^2\right)-\frac{\text{Subst}\left(\int\right.}{}$$
$$= -\frac{1}{2x^2}-\frac{\log\left(1-\sqrt{3}x^2+x^4\right)}{8\sqrt{3}}+\frac{\log\left(1+\sqrt{3}x^2+x^4\right)}{8\sqrt{3}}+\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,-\sqrt{3}+2x^2\right)+$$
$$= -\frac{1}{2x^2}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x^2\right)-\frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x^2\right)-\frac{\log\left(1-\sqrt{3}x^2+x^4\right)}{8\sqrt{3}}+\frac{\log\left(1+\sqrt{3}x^2+x^4\right.}{8\sqrt{3}}$$

Mathematica [C] time = 0.0158076, size = 49, normalized size = 0.55

$$-\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1^2\log(x-\#1)}{2\#1^4-1}\&\right]-\frac{1}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)/(x^3*(1 - x^4 + x^8)),x]
```

```
[Out] -1/(2*x^2) - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^2)/(-1 + 2*#1^4) &
 ]/4
```

Maple [A] time = 0.01, size = 70, normalized size = 0.8

$$-\frac{1}{2\,x^2}-\frac{\arctan\left(2\,x^2-\sqrt{3}\right)}{4}-\frac{\arctan\left(2\,x^2+\sqrt{3}\right)}{4}-\frac{\ln\left(1+x^4-x^2\sqrt{3}\right)\sqrt{3}}{24}+\frac{\ln\left(1+x^4+x^2\sqrt{3}\right)\sqrt{3}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)/x^3/(x^8-x^4+1),x)
```

```
[Out] -1/2/x^2-1/4*arctan(2*x^2-3^(1/2))-1/4*arctan(2*x^2+3^(1/2))-1/24*ln(1+x^4-
x^2*3^(1/2))*3^(1/2)+1/24*ln(1+x^4+x^2*3^(1/2))*3^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2\,x^2}-\int \frac{x^5}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/x^3/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -1/2/x^2 - integrate(x^5/(x^8 - x^4 + 1), x)
```

Fricas [B] time = 1.56864, size = 566, normalized size = 6.36

$$\frac{4\sqrt{6}\sqrt{3}\sqrt{2}x^2\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x^2+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2\,x^4+\sqrt{6}\sqrt{2}x^2+2}-\sqrt{3}\right)+4\sqrt{6}\sqrt{3}\sqrt{2}x^2\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}x^2+\right.}{48\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/x^3/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/48*(4*sqrt(6)*sqrt(3)*sqrt(2)*x^2*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x^2
 + 1/3*sqrt(6)*sqrt(3)*sqrt(2*x^4 + sqrt(6)*sqrt(2)*x^2 + 2) - sqrt(3)) + 4
*sqrt(6)*sqrt(3)*sqrt(2)*x^2*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x^2 + 1/3*
sqrt(6)*sqrt(3)*sqrt(2*x^4 - sqrt(6)*sqrt(2)*x^2 + 2) + sqrt(3)) + sqrt(6)*
sqrt(2)*x^2*log(2*x^4 + sqrt(6)*sqrt(2)*x^2 + 2) - sqrt(6)*sqrt(2)*x^2*log(
2*x^4 - sqrt(6)*sqrt(2)*x^2 + 2) - 24)/x^2
```

Sympy [A] time = 0.262795, size = 76, normalized size = 0.85

$$-\frac{\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right)}{24}+\frac{\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)}{24}-\frac{\operatorname{atan}\left(2x^2-\sqrt{3}\right)}{4}-\frac{\operatorname{atan}\left(2x^2+\sqrt{3}\right)}{4}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)/x**3/(x**8-x**4+1),x)
```

```
[Out] -sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/24 + sqrt(3)*log(x**4 + sqrt(3)*x**2
+ 1)/24 - atan(2*x**2 - sqrt(3))/4 - atan(2*x**2 + sqrt(3))/4 - 1/(2*x**2)
```

Giac [B] time = 1.26255, size = 348, normalized size = 3.91

$$-\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+}{\sqrt{6}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/x^3/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] -1/48*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqr
t(2))) - 1/48*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(
6) + sqrt(2))) - 1/48*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2)
)/(sqrt(6) - sqrt(2))) - 1/48*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) -
 sqrt(2))/(sqrt(6) - sqrt(2))) - 1/96*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x
*(sqrt(6) + sqrt(2)) + 1) + 1/96*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqr
t(6) + sqrt(2)) + 1) - 1/96*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6)
- sqrt(2)) + 1) + 1/96*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqr
t(2)) + 1) - 1/2/x^2
```

3.60 $\int \frac{1-x^4}{x^4\left(1-x^4+x^8\right)}\,dx$

Optimal. Leaf size=370

$$-\frac{1}{3x^3}+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2\right.$$

```
[Out] -1/(3*x^3) - (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2
 + Sqrt[3]]])/4 + (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/S
qrt[2 - Sqrt[3]]])/4 + (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] + 2
*x)/Sqrt[2 + Sqrt[3]]])/4 - (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[3]
] + 2*x)/Sqrt[2 - Sqrt[3]]])/4 + (Sqrt[(2 + Sqrt[3])/3]*Log[1 - Sqrt[2 - Sq
rt[3]]*x + x^2])/8 - (Sqrt[(2 + Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x
^2])/8 - (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8 + (Sq
rt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```

Rubi [A] time = 0.269125, antiderivative size = 370, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {1504, 12, 1373, 1127, 1161, 618, 204, 1164, 628}

$$-\frac{1}{3x^3}+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2\right.$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)/(x^4*(1 - x^4 + x^8)),x]
```


```
[Out] -1/(3*x^3) - (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2
 + Sqrt[3]]])/4 + (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/S
qrt[2 - Sqrt[3]]])/4 + (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] + 2
*x)/Sqrt[2 + Sqrt[3]]])/4 - (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[3]
] + 2*x)/Sqrt[2 - Sqrt[3]]])/4 + (Sqrt[(2 + Sqrt[3])/3]*Log[1 - Sqrt[2 - Sq
rt[3]]*x + x^2])/8 - (Sqrt[(2 + Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x
^2])/8 - (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8 + (Sq
rt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```


Rule 1504

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^n + c*x^
(2*n))^(p + 1))/(a*f*(m + 1)), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1373

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m
 - n/2)/(q - r*x^(n/2) + x^n), x], x] - Dist[1/(2*c*r), Int[x^(m - n/2)/(q
+ r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, n/2] && LtQ[m, (3*n
)/2] && NegQ[b^2 - 4*a*c]
```

Rule 1127

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1164

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1-x^4}{x^4\left(1-x^4+x^8\right)}\,dx = -\frac{1}{3x^3}-\frac{1}{3}\int \frac{3x^4}{1-x^4+x^8}\,dx$$

$$= -\frac{1}{3x^3}-\int \frac{x^4}{1-x^4+x^8}\,dx$$

$$= -\frac{1}{3x^3}-\frac{\int \frac{x^2}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}+\frac{\int \frac{x^2}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}$$

$$= -\frac{1}{3x^3}+\frac{\int \frac{1-x^2}{1-\sqrt{3}x^2+x^4}\,dx}{4\sqrt{3}}-\frac{\int \frac{1+x^2}{1-\sqrt{3}x^2+x^4}\,dx}{4\sqrt{3}}-\frac{\int \frac{1-x^2}{1+\sqrt{3}x^2+x^4}\,dx}{4\sqrt{3}}+\frac{\int \frac{1+x^2}{1+\sqrt{3}x^2+x^4}\,dx}{4\sqrt{3}}$$

$$= -\frac{1}{3x^3}+\frac{\int \frac{1}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx}{8\sqrt{3}}+\frac{\int \frac{1}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx}{8\sqrt{3}}-\frac{\int \frac{1}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx}{8\sqrt{3}}-\frac{\int \frac{1}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx}{8\sqrt{3}}+\frac{\int \frac{-1}{}}{8}$$

$$= -\frac{1}{3x^3}+\frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{8\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{\mathrm{lo}}{}$$

$$= -\frac{1}{3x^3}-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\mathrm{log}}{}$$

Mathematica [C] time = 0.0146211, size = 47, normalized size = 0.13

$$-\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\#1\log(x-\#1)}{2\#1^4-1}\&\right]-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)/(x^4*(1 - x^4 + x^8)),x]
```

```
[Out] -1/(3*x^3) - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1)/(-1 + 2*#1^4) & ]
/4
```

Maple [C] time = 0.01, size = 46, normalized size = 0.1

$$-\frac{1}{4}\sum_{\_R=\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)}\frac{\_R^4\ln\left(x-\_R\right)}{2\_R^7-\_R^3}-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)/x^4/(x^8-x^4+1),x)
```

```
[Out] -1/4*sum(_R^4/(2*_R^7-_R^3)*ln(x-_R),_R=RootOf(_Z^8-_Z^4+1))-1/3/x^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{3\,x^3}-\int\frac{x^4}{x^8-x^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/x^4/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -1/3/x^3 - integrate(x^4/(x^8 - x^4 + 1), x)
```

Fricas [B] time = 1.79793, size = 1962, normalized size = 5.3

$$8\,\sqrt{6}\sqrt{2}x^3\sqrt{\sqrt{3}+2}\arctan\left(-\tfrac{1}{3}\,\sqrt{6}\sqrt{3}\sqrt{2}x\sqrt{\sqrt{3}+2}+\tfrac{1}{6}\,\sqrt{6}\sqrt{2}\sqrt{2\,\sqrt{6}\sqrt{3}\sqrt{2}x\sqrt{\sqrt{3}+2}+12\,x^2+12}\sqrt{\sqrt{3}+2}-\sqrt{3}-2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/x^4/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/96*(8*sqrt(6)*sqrt(2)*x^3*sqrt(sqrt(3) + 2)*arctan(-1/3*sqrt(6)*sqrt(3)*s
qrt(2)*x*sqrt(sqrt(3) + 2) + 1/6*sqrt(6)*sqrt(2)*sqrt(2*sqrt(6)*sqrt(3)*sqr
t(2)*x*sqrt(sqrt(3) + 2) + 12*x^2 + 12)*sqrt(sqrt(3) + 2) - sqrt(3) - 2) +
8*sqrt(6)*sqrt(2)*x^3*sqrt(sqrt(3) + 2)*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)
*x*sqrt(sqrt(3) + 2) + 1/6*sqrt(6)*sqrt(2)*sqrt(-2*sqrt(6)*sqrt(3)*sqrt(2)*
x*sqrt(sqrt(3) + 2) + 12*x^2 + 12)*sqrt(sqrt(3) + 2) + sqrt(3) + 2) - 4*sqr
t(6)*sqrt(2)*x^3*sqrt(-4*sqrt(3) + 8)*arctan(-1/6*sqrt(6)*sqrt(3)*sqrt(2)*x
*sqrt(-4*sqrt(3) + 8) + 1/12*sqrt(6)*sqrt(2)*sqrt(sqrt(6)*sqrt(3)*sqrt(2)*x
*sqrt(-4*sqrt(3) + 8) + 12*x^2 + 12)*sqrt(-4*sqrt(3) + 8) + sqrt(3) - 2) -
4*sqrt(6)*sqrt(2)*x^3*sqrt(-4*sqrt(3) + 8)*arctan(-1/6*sqrt(6)*sqrt(3)*sqrt
(2)*x*sqrt(-4*sqrt(3) + 8) + 1/12*sqrt(6)*sqrt(2)*sqrt(-sqrt(6)*sqrt(3)*sqr
t(2)*x*sqrt(-4*sqrt(3) + 8) + 12*x^2 + 12)*sqrt(-4*sqrt(3) + 8) - sqrt(3) +
```

```
 2) - 2*sqrt(6)*(sqrt(3)*sqrt(2)*x^3 - 2*sqrt(2)*x^3)*sqrt(sqrt(3) + 2)*log
(2*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(sqrt(3) + 2) + 12*x^2 + 12) + 2*sqrt(6)*(
sqrt(3)*sqrt(2)*x^3 - 2*sqrt(2)*x^3)*sqrt(sqrt(3) + 2)*log(-2*sqrt(6)*sqrt(
3)*sqrt(2)*x*sqrt(sqrt(3) + 2) + 12*x^2 + 12) - sqrt(6)*(sqrt(3)*sqrt(2)*x^
3 + 2*sqrt(2)*x^3)*sqrt(-4*sqrt(3) + 8)*log(sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(
-4*sqrt(3) + 8) + 12*x^2 + 12) + sqrt(6)*(sqrt(3)*sqrt(2)*x^3 + 2*sqrt(2)*x
^3)*sqrt(-4*sqrt(3) + 8)*log(-sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-4*sqrt(3) + 8
) + 12*x^2 + 12) - 32)/x^3
```

Sympy [A] time = 1.59861, size = 32, normalized size = 0.09

$$-\operatorname{RootSum}\left(5308416t^8-2304t^4+1,\left(t\mapsto t\log\left(-18432t^5+4t+x\right)\right)\right)-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)/x**4/(x**8-x**4+1),x)
```

```
[Out] -RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(-18432*_t**5 + 4
*_t + x))) - 1/(3*x**3)
```

Giac [A] time = 1.16982, size = 348, normalized size = 0.94

$$-\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+}{\sqrt{6}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/x^4/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] -1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqr
t(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(
6) + sqrt(2))) - 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2)
)/(sqrt(6) - sqrt(2))) - 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) -
 sqrt(2))/(sqrt(6) - sqrt(2))) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x
*(sqrt(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqr
t(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6)
- sqrt(2)) + 1) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqr
t(2)) + 1) - 1/3/x^3
```

3.61 $\int \frac{x^3}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx$

Optimal. Leaf size=280

$$\frac{\left(a^2c^2d-3ab^2cd+2abc^2e-b^3ce+b^4d\right)\log\left(ax^2+bx+c\right)}{2a^4\left(ad^2-e(bd-ce)\right)}+\frac{\left(5a^2bc^2d-2a^2c^3e+4ab^2c^2e-5ab^3cd-b^4ce+b^5d\right)\tanh}{a^4\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}$$

```
[Out] ((a^2*d^2 + b^2*e^2 + a*e*(b*d - c*e))*x)/(a^3*e^3) - ((a*d + b*e)*x^2)/(2*
a^2*e^2) + x^3/(3*a*e) + ((b^5*d - 5*a*b^3*c*d + 5*a^2*b*c^2*d - b^4*c*e +
4*a*b^2*c^2*e - 2*a^2*c^3*e)*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(a^4*S
qrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))) - (d^5*Log[d + e*x])/(e^4*(a*d^2
- e*(b*d - c*e))) + ((b^4*d - 3*a*b^2*c*d + a^2*c^2*d - b^3*c*e + 2*a*b*c^2
*e)*Log[c + b*x + a*x^2])/(2*a^4*(a*d^2 - e*(b*d - c*e)))
```

Rubi [A] time = 0.59737, antiderivative size = 280, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1569, 1628, 634, 618, 206, 628}

$$\frac{\left(a^2c^2d-3ab^2cd+2abc^2e-b^3ce+b^4d\right)\log\left(ax^2+bx+c\right)}{2a^4\left(ad^2-e(bd-ce)\right)}+\frac{\left(5a^2bc^2d-2a^2c^3e+4ab^2c^2e-5ab^3cd-b^4ce+b^5d\right)\tanh}{a^4\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + c/x^2 + b/x)*(d + e*x)),x]
```

```
[Out] ((a^2*d^2 + b^2*e^2 + a*e*(b*d - c*e))*x)/(a^3*e^3) - ((a*d + b*e)*x^2)/(2*
a^2*e^2) + x^3/(3*a*e) + ((b^5*d - 5*a*b^3*c*d + 5*a^2*b*c^2*d - b^4*c*e +
4*a*b^2*c^2*e - 2*a^2*c^3*e)*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(a^4*S
qrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))) - (d^5*Log[d + e*x])/(e^4*(a*d^2
- e*(b*d - c*e))) + ((b^4*d - 3*a*b^2*c*d + a^2*c^2*d - b^3*c*e + 2*a*b*c^2
*e)*Log[c + b*x + a*x^2])/(2*a^4*(a*d^2 - e*(b*d - c*e)))
```

Rule 1569

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^3}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx = \int \frac{x^5}{(d+ex)\left(c+bx+ax^2\right)}\,dx$$

$$= \int \left(\frac{a^2d^2+b^2e^2+ae(bd-ce)}{a^3e^3} - \frac{(ad+be)x}{a^2e^2} + \frac{x^2}{ae} + \frac{d^5}{e^3\left(-ad^2+e(bd-ce)\right)(d+ex)} + \frac{c\left(b^3\right.}{}\right.$$

$$= \frac{\left(a^2d^2+b^2e^2+ae(bd-ce)\right)x}{a^3e^3} - \frac{(ad+be)x^2}{2a^2e^2} + \frac{x^3}{3ae} - \frac{d^5\log(d+ex)}{e^4\left(ad^2-e(bd-ce)\right)} + \int \frac{c\left(b^3d-2abcd-\right.}{}$$

$$= \frac{\left(a^2d^2+b^2e^2+ae(bd-ce)\right)x}{a^3e^3} - \frac{(ad+be)x^2}{2a^2e^2} + \frac{x^3}{3ae} - \frac{d^5\log(d+ex)}{e^4\left(ad^2-e(bd-ce)\right)} + \frac{\left(b^4d-3ab^2c\right.}{}$$

$$= \frac{\left(a^2d^2+b^2e^2+ae(bd-ce)\right)x}{a^3e^3} - \frac{(ad+be)x^2}{2a^2e^2} + \frac{x^3}{3ae} - \frac{d^5\log(d+ex)}{e^4\left(ad^2-e(bd-ce)\right)} + \frac{\left(b^4d-3ab^2c\right.}{}$$

$$= \frac{\left(a^2d^2+b^2e^2+ae(bd-ce)\right)x}{a^3e^3} - \frac{(ad+be)x^2}{2a^2e^2} + \frac{x^3}{3ae} + \frac{\left(b^5d-5ab^3cd+5a^2bc^2d-b^4ce+4a\right.}{a^4\sqrt{b^2-4ac}\left(ad\right.}$$

Mathematica [A] time = 0.241123, size = 283, normalized size = 1.01

$$\frac{\left(a^2c^2d-3ab^2cd+2abc^2e-b^3ce+b^4d\right)\log\left(ax^2+bx+c\right)}{2a^4\left(ad^2-bde+ce^2\right)} + \frac{\left(5a^2bc^2d-2a^2c^3e+4ab^2c^2e-5ab^3cd-b^4ce+b^5d\right)\tan^{-1}}{a^4\sqrt{4ac-b^2}\left(-ad^2+bde-ce^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + c/x^2 + b/x)*(d + e*x)),x]
```

```
[Out] ((a^2*d^2 + a*b*d*e + b^2*e^2 - a*c*e^2)*x)/(a^3*e^3) - ((a*d + b*e)*x^2)/(
2*a^2*e^2) + x^3/(3*a*e) + ((b^5*d - 5*a*b^3*c*d + 5*a^2*b*c^2*d - b^4*c*e
+ 4*a*b^2*c^2*e - 2*a^2*c^3*e)*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/(a^4
*Sqrt[-b^2 + 4*a*c]*(-(a*d^2) + b*d*e - c*e^2)) - (d^5*Log[d + e*x])/(e^4*(
a*d^2 - b*d*e + c*e^2)) + ((b^4*d - 3*a*b^2*c*d + a^2*c^2*d - b^3*c*e + 2*a
*b*c^2*e)*Log[c + b*x + a*x^2])/(2*a^4*(a*d^2 - b*d*e + c*e^2))
```

Maple [B] time = 0.014, size = 662, normalized size = 2.4

$$\frac{x^3}{3\,ae} - \frac{x^2d}{2\,ae^2} - \frac{x^2b}{2\,ea^2} + \frac{d^2x}{e^3a} + \frac{bdx}{a^2e^2} - \frac{cx}{ea^2} + \frac{b^2x}{ea^3} - \frac{d^5\ln(ex+d)}{e^4\left(ad^2-bde+e^2c\right)} + \frac{\ln\left(ax^2+bx+c\right)c^2d}{\left(2\,ad^2-2\,bde+2\,e^2c\right)a^2} - \frac{3\,\ln\left(ax^2+bx+\right.}{\left(2\,ad^2-2\,bde+2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+c/x^2+b/x)/(e*x+d),x)
```

```
[Out] 1/3*x^3/a/e-1/2/e^2/a*x^2*d-1/2/e/a^2*x^2*b+1/e^3/a*d^2*x+1/e^2/a^2*b*d*x-1
/e/a^2*c*x+1/e/a^3*b^2*x-1/e^4*d^5/(a*d^2-b*d*e+c*e^2)*ln(e*x+d)+1/2/(a*d^2
-b*d*e+c*e^2)/a^2*ln(a*x^2+b*x+c)*c^2*d-3/2/(a*d^2-b*d*e+c*e^2)/a^3*ln(a*x^
2+b*x+c)*b^2*c*d+1/(a*d^2-b*d*e+c*e^2)/a^3*ln(a*x^2+b*x+c)*b*c^2*e+1/2/(a*d
^2-b*d*e+c*e^2)/a^4*ln(a*x^2+b*x+c)*b^4*d-1/2/(a*d^2-b*d*e+c*e^2)/a^4*ln(a*
x^2+b*x+c)*b^3*c*e-5/(a*d^2-b*d*e+c*e^2)/a^2/(4*a*c-b^2)^(1/2)*arctan((2*a*
x+b)/(4*a*c-b^2)^(1/2))*b*c^2*d+2/(a*d^2-b*d*e+c*e^2)/a^2/(4*a*c-b^2)^(1/2)
*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*c^3*e+5/(a*d^2-b*d*e+c*e^2)/a^3/(4*a*c
-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^3*c*d-4/(a*d^2-b*d*e+c*e^
2)/a^3/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^2*c^2*e-1/(a
*d^2-b*d*e+c*e^2)/a^4/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))
*b^5*d+1/(a*d^2-b*d*e+c*e^2)/a^4/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-
b^2)^(1/2))*b^4*c*e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+c/x^2+b/x)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 137.062, size = 2079, normalized size = 7.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+c/x^2+b/x)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [-1/6*(6*(a^4*b^2 - 4*a^5*c)*d^5*log(e*x + d) - 2*((a^4*b^2 - 4*a^5*c)*d^2*
e^3 - (a^3*b^3 - 4*a^4*b*c)*d*e^4 + (a^3*b^2*c - 4*a^4*c^2)*e^5)*x^3 + 3*((
a^4*b^2 - 4*a^5*c)*d^3*e^2 - (a^2*b^4 - 5*a^3*b^2*c + 4*a^4*c^2)*d*e^4 + (a
```

```
^2*b^3*c - 4*a^3*b*c^2)*e^5)*x^2 + 3*((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d*e^4
 - (b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*e^5)*sqrt(b^2 - 4*a*c)*log((2*a^2*x^2
+ 2*a*b*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x + c))
 - 6*((a^4*b^2 - 4*a^5*c)*d^4*e - (a*b^5 - 6*a^2*b^3*c + 8*a^3*b*c^2)*d*e^4
 + (a*b^4*c - 5*a^2*b^2*c^2 + 4*a^3*c^3)*e^5)*x - 3*((b^6 - 7*a*b^4*c + 13*
a^2*b^2*c^2 - 4*a^3*c^3)*d*e^4 - (b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*e^5)*l
og(a*x^2 + b*x + c))/((a^5*b^2 - 4*a^6*c)*d^2*e^4 - (a^4*b^3 - 4*a^5*b*c)*d
*e^5 + (a^4*b^2*c - 4*a^5*c^2)*e^6), -1/6*(6*(a^4*b^2 - 4*a^5*c)*d^5*log(e*
x + d) - 2*((a^4*b^2 - 4*a^5*c)*d^2*e^3 - (a^3*b^3 - 4*a^4*b*c)*d*e^4 + (a^
3*b^2*c - 4*a^4*c^2)*e^5)*x^3 + 3*((a^4*b^2 - 4*a^5*c)*d^3*e^2 - (a^2*b^4 -
 5*a^3*b^2*c + 4*a^4*c^2)*d*e^4 + (a^2*b^3*c - 4*a^3*b*c^2)*e^5)*x^2 - 6*((
b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d*e^4 - (b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*e^
5)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c))
 - 6*((a^4*b^2 - 4*a^5*c)*d^4*e - (a*b^5 - 6*a^2*b^3*c + 8*a^3*b*c^2)*d*e^4
 + (a*b^4*c - 5*a^2*b^2*c^2 + 4*a^3*c^3)*e^5)*x - 3*((b^6 - 7*a*b^4*c + 13*
a^2*b^2*c^2 - 4*a^3*c^3)*d*e^4 - (b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*e^5)*l
og(a*x^2 + b*x + c))/((a^5*b^2 - 4*a^6*c)*d^2*e^4 - (a^4*b^3 - 4*a^5*b*c)*d
*e^5 + (a^4*b^2*c - 4*a^5*c^2)*e^6)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+c/x**2+b/x)/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.10794, size = 398, normalized size = 1.42

$$-\frac{d^5 \log(|xe+d|)}{ad^2e^4 - bde^5 + ce^6} + \frac{\left(b^4d - 3\,ab^2cd + a^2c^2d - b^3ce + 2\,abc^2e\right)\log\left(ax^2+bx+c\right)}{2\left(a^5d^2 - a^4bde + a^4ce^2\right)} - \frac{\left(b^5d - 5\,ab^3cd + 5\,a^2bc^2d - b^4ce +\right.}{\left(a^5d^2 - a^4bde +\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+c/x^2+b/x)/(e*x+d),x, algorithm="giac")
```

```
[Out] -d^5*log(abs(x*e + d))/(a*d^2*e^4 - b*d*e^5 + c*e^6) + 1/2*(b^4*d - 3*a*b^2
*c*d + a^2*c^2*d - b^3*c*e + 2*a*b*c^2*e)*log(a*x^2 + b*x + c)/(a^5*d^2 - a
^4*b*d*e + a^4*c*e^2) - (b^5*d - 5*a*b^3*c*d + 5*a^2*b*c^2*d - b^4*c*e + 4*
a*b^2*c^2*e - 2*a^2*c^3*e)*arctan((2*a*x + b)/sqrt(-b^2 + 4*a*c))/((a^5*d^2
 - a^4*b*d*e + a^4*c*e^2)*sqrt(-b^2 + 4*a*c)) + 1/6*(2*a^2*x^3*e^2 - 3*a^2*
d*x^2*e + 6*a^2*d^2*x - 3*a*b*x^2*e^2 + 6*a*b*d*x*e + 6*b^2*x*e^2 - 6*a*c*x
*e^2)*e^(-3)/a^3
```

3.62 $\int \frac{x^2}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx$

Optimal. Leaf size=218

$$-\frac{\left(-2abcd+ac^2e-b^2ce+b^3d\right)\log\left(ax^2+bx+c\right)}{2a^3\left(ad^2-e(bd-ce)\right)}-\frac{\left(2a^2c^2d-4ab^2cd+3abc^2e-b^3ce+b^4d\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}-\frac{x(ad}{a^2}$$

```
[Out] -(((a*d + b*e)*x)/(a^2*e^2)) + x^2/(2*a*e) - ((b^4*d - 4*a*b^2*c*d + 2*a^2*
c^2*d - b^3*c*e + 3*a*b*c^2*e)*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(a^3
*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))) + (d^4*Log[d + e*x])/(e^3*(a*d^
2 - e*(b*d - c*e))) - ((b^3*d - 2*a*b*c*d - b^2*c*e + a*c^2*e)*Log[c + b*x
+ a*x^2])/(2*a^3*(a*d^2 - e*(b*d - c*e)))
```

Rubi [A] time = 0.395218, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1569, 1628, 634, 618, 206, 628}

$$-\frac{\left(-2abcd+ac^2e-b^2ce+b^3d\right)\log\left(ax^2+bx+c\right)}{2a^3\left(ad^2-e(bd-ce)\right)}-\frac{\left(2a^2c^2d-4ab^2cd+3abc^2e-b^3ce+b^4d\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}-\frac{x(ad}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + c/x^2 + b/x)*(d + e*x)),x]
```

```
[Out] -(((a*d + b*e)*x)/(a^2*e^2)) + x^2/(2*a*e) - ((b^4*d - 4*a*b^2*c*d + 2*a^2*
c^2*d - b^3*c*e + 3*a*b*c^2*e)*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(a^3
*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))) + (d^4*Log[d + e*x])/(e^3*(a*d^
2 - e*(b*d - c*e))) - ((b^3*d - 2*a*b*c*d - b^2*c*e + a*c^2*e)*Log[c + b*x
+ a*x^2])/(2*a^3*(a*d^2 - e*(b*d - c*e)))
```

Rule 1569

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx = \int \frac{x^4}{(d+ex)\left(c+bx+ax^2\right)}\,dx$$
$$= \int \left(\frac{-ad-be}{a^2e^2}+\frac{x}{ae}+\frac{d^4}{e^2\left(ad^2-e(bd-ce)\right)(d+ex)}+\frac{-c\left(b^2d-acd-bce\right)-\left(b^3d-2abcd\right.}{a^2\left(ad^2-e(bd-ce)\right)\left(c+bx\right.}\right.$$
$$= -\frac{(ad+be)x}{a^2e^2}+\frac{x^2}{2ae}+\frac{d^4\log(d+ex)}{e^3\left(ad^2-e(bd-ce)\right)}+\frac{\int \frac{-c\left(b^2d-acd-bce\right)-\left(b^3d-2abcd-b^2ce+ac^2e\right)x}{c+bx+ax^2}\,dx}{a^2\left(ad^2-e(bd-ce)\right)}$$
$$= -\frac{(ad+be)x}{a^2e^2}+\frac{x^2}{2ae}+\frac{d^4\log(d+ex)}{e^3\left(ad^2-e(bd-ce)\right)}-\frac{\left(b^3d-2abcd-b^2ce+ac^2e\right)\int \frac{b+2ax}{c+bx+ax^2}\,dx}{2a^3\left(ad^2-e(bd-ce)\right)}+$$
$$= -\frac{(ad+be)x}{a^2e^2}+\frac{x^2}{2ae}+\frac{d^4\log(d+ex)}{e^3\left(ad^2-e(bd-ce)\right)}-\frac{\left(b^3d-2abcd-b^2ce+ac^2e\right)\log\left(c+bx+ax\right.}{2a^3\left(ad^2-e(bd-ce)\right)}$$
$$= -\frac{(ad+be)x}{a^2e^2}+\frac{x^2}{2ae}-\frac{\left(b^4d-4ab^2cd+2a^2c^2d-b^3ce+3abc^2e\right)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}+\frac{d^4}{e^3\left(ad\right.}$$

Mathematica [A] time = 0.18102, size = 218, normalized size = 1.

$$\frac{\left(2abcd-ac^2e+b^2ce+b^3(-d)\right)\log(x(ax+b)+c)}{2a^3\left(ad^2+e(ce-bd)\right)}+\frac{\left(2a^2c^2d-4ab^2cd+3abc^2e-b^3ce+b^4d\right)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{a^3\sqrt{4ac-b^2}\left(ad^2+e(ce-bd)\right)}-\frac{x(ad+}{a^2e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + c/x^2 + b/x)*(d + e*x)),x]
```

```
[Out] -(((a*d + b*e)*x)/(a^2*e^2)) + x^2/(2*a*e) + ((b^4*d - 4*a*b^2*c*d + 2*a^2*
c^2*d - b^3*c*e + 3*a*b*c^2*e)*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/(a^3
*Sqrt[-b^2 + 4*a*c]*(a*d^2 + e*(-(b*d) + c*e))) + (d^4*Log[d + e*x])/(e^3*(
a*d^2 + e*(-(b*d) + c*e))) + ((-(b^3*d) + 2*a*b*c*d + b^2*c*e - a*c^2*e)*Lo
g[c + x*(b + a*x)])/(2*a^3*(a*d^2 + e*(-(b*d) + c*e)))
```

Maple [B] time = 0.006, size = 512, normalized size = 2.4

$$\frac{x^2}{2\,ae}-\frac{dx}{ae^2}-\frac{bx}{a^2e}+\frac{d^4\ln(ex+d)}{e^3\left(ad^2-bde+e^2c\right)}+\frac{\ln\left(ax^2+bx+c\right)bcd}{\left(ad^2-bde+e^2c\right)a^2}-\frac{\ln\left(ax^2+bx+c\right)c^2e}{\left(2\,ad^2-2\,bde+2\,e^2c\right)a^2}-\frac{\ln\left(ax^2+bx+c\right)b^3d}{\left(2\,ad^2-2\,bde+2\,e^2c\right)a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+c/x^2+b/x)/(e*x+d),x)
```

```
[Out] 1/2*x^2/a/e-1/a/e^2*x*d-1/a^2/e*b*x+1/e^3*d^4/(a*d^2-b*d*e+c*e^2)*ln(e*x+d)
+1/(a*d^2-b*d*e+c*e^2)/a^2*ln(a*x^2+b*x+c)*b*c*d-1/2/(a*d^2-b*d*e+c*e^2)/a^
2*ln(a*x^2+b*x+c)*c^2*e-1/2/(a*d^2-b*d*e+c*e^2)/a^3*ln(a*x^2+b*x+c)*b^3*d+1
/2/(a*d^2-b*d*e+c*e^2)/a^3*ln(a*x^2+b*x+c)*b^2*c*e+2/(a*d^2-b*d*e+c*e^2)/a/
(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*c^2*d-4/(a*d^2-b*d*e+
c*e^2)/a^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^2*c*d+3/
(a*d^2-b*d*e+c*e^2)/a^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2
))*b*c^2*e+1/(a*d^2-b*d*e+c*e^2)/a^3/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*
a*c-b^2)^(1/2))*b^4*d-1/(a*d^2-b*d*e+c*e^2)/a^3/(4*a*c-b^2)^(1/2)*arctan((2
*a*x+b)/(4*a*c-b^2)^(1/2))*b^3*c*e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+c/x^2+b/x)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 84.0832, size = 1623, normalized size = 7.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+c/x^2+b/x)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a^3*b^2 - 4*a^4*c)*d^4*log(e*x + d) + ((a^3*b^2 - 4*a^4*c)*d^2*e^2
 - (a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*x^2 + ((b^4 -
 4*a*b^2*c + 2*a^2*c^2)*d*e^3 - (b^3*c - 3*a*b*c^2)*e^4)*sqrt(b^2 - 4*a*c)*
log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a*
x^2 + b*x + c)) - 2*((a^3*b^2 - 4*a^4*c)*d^3*e - (a*b^4 - 5*a^2*b^2*c + 4*a
^3*c^2)*d*e^3 + (a*b^3*c - 4*a^2*b*c^2)*e^4)*x - ((b^5 - 6*a*b^3*c + 8*a^2*
```

```
b*c^2)*d*e^3 - (b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*e^4)*log(a*x^2 + b*x + c))
/((a^4*b^2 - 4*a^5*c)*d^2*e^3 - (a^3*b^3 - 4*a^4*b*c)*d*e^4 + (a^3*b^2*c -
4*a^4*c^2)*e^5), 1/2*(2*(a^3*b^2 - 4*a^4*c)*d^4*log(e*x + d) + ((a^3*b^2 -
4*a^4*c)*d^2*e^2 - (a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^
4)*x^2 - 2*((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d*e^3 - (b^3*c - 3*a*b*c^2)*e^4)*
sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)) -
2*((a^3*b^2 - 4*a^4*c)*d^3*e - (a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*d*e^3 + (a
*b^3*c - 4*a^2*b*c^2)*e^4)*x - ((b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*d*e^3 - (b^
4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*e^4)*log(a*x^2 + b*x + c))/((a^4*b^2 - 4*a^5
*c)*d^2*e^3 - (a^3*b^3 - 4*a^4*b*c)*d*e^4 + (a^3*b^2*c - 4*a^4*c^2)*e^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+c/x**2+b/x)/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11492, size = 302, normalized size = 1.39

$$\frac{d^4\log(|xe+d|)}{ad^2e^3-bde^4+ce^5}-\frac{\left(b^3d-2\,abcd-b^2ce+ac^2e\right)\log\left(ax^2+bx+c\right)}{2\left(a^4d^2-a^3bde+a^3ce^2\right)}+\frac{\left(b^4d-4\,ab^2cd+2\,a^2c^2d-b^3ce+3\,abc^2e\right)\mathrm{arcta}}{\left(a^4d^2-a^3bde+a^3ce^2\right)\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+c/x^2+b/x)/(e*x+d),x, algorithm="giac")
```

```
[Out] d^4*log(abs(x*e + d))/(a*d^2*e^3 - b*d*e^4 + c*e^5) - 1/2*(b^3*d - 2*a*b*c*
d - b^2*c*e + a*c^2*e)*log(a*x^2 + b*x + c)/(a^4*d^2 - a^3*b*d*e + a^3*c*e^
2) + (b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*d - b^3*c*e + 3*a*b*c^2*e)*arctan((2*
a*x + b)/sqrt(-b^2 + 4*a*c))/((a^4*d^2 - a^3*b*d*e + a^3*c*e^2)*sqrt(-b^2 +
 4*a*c)) + 1/2*(a*x^2*e - 2*a*d*x - 2*b*x*e)*e^(-2)/a^2
```

3.63 $\int \frac{x}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx$

Optimal. Leaf size=176

$$\frac{\left(-3abcd+2ac^2e-b^2ce+b^3d\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}+\frac{\left(-acd+b^2d-bce\right)\log\left(ax^2+bx+c\right)}{2a^2\left(ad^2-e(bd-ce)\right)}-\frac{d^3\log(d+ex)}{e^2\left(ad^2-e(bd-ce)\right)}+\frac{x}{ae}$$

```
[Out] x/(a*e) + ((b^3*d - 3*a*b*c*d - b^2*c*e + 2*a*c^2*e)*ArcTanh[(b + 2*a*x)/Sq
rt[b^2 - 4*a*c]])/(a^2*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))) - (d^3*Lo
g[d + e*x])/(e^2*(a*d^2 - e*(b*d - c*e))) + ((b^2*d - a*c*d - b*c*e)*Log[c
+ b*x + a*x^2])/(2*a^2*(a*d^2 - e*(b*d - c*e)))
```

Rubi [A] time = 0.285269, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {1569, 1628, 634, 618, 206, 628}

$$\frac{\left(-3abcd+2ac^2e-b^2ce+b^3d\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}+\frac{\left(-acd+b^2d-bce\right)\log\left(ax^2+bx+c\right)}{2a^2\left(ad^2-e(bd-ce)\right)}-\frac{d^3\log(d+ex)}{e^2\left(ad^2-e(bd-ce)\right)}+\frac{x}{ae}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + c/x^2 + b/x)*(d + e*x)),x]
```

```
[Out] x/(a*e) + ((b^3*d - 3*a*b*c*d - b^2*c*e + 2*a*c^2*e)*ArcTanh[(b + 2*a*x)/Sq
rt[b^2 - 4*a*c]])/(a^2*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))) - (d^3*Lo
g[d + e*x])/(e^2*(a*d^2 - e*(b*d - c*e))) + ((b^2*d - a*c*d - b*c*e)*Log[c
+ b*x + a*x^2])/(2*a^2*(a*d^2 - e*(b*d - c*e)))
```

Rule 1569

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
```

```
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx = \int \frac{x^3}{(d+ex)\left(c+bx+ax^2\right)}\,dx$$
$$= \int \left(\frac{1}{ae}+\frac{d^3}{e\left(-ad^2+e(bd-ce)\right)(d+ex)}+\frac{c(bd-ce)+\left(b^2d-acd-bce\right)x}{a\left(ad^2-e(bd-ce)\right)\left(c+bx+ax^2\right)}\right)dx$$
$$= \frac{x}{ae}-\frac{d^3\log(d+ex)}{e^2\left(ad^2-e(bd-ce)\right)}+\frac{\int \frac{c(bd-ce)+\left(b^2d-acd-bce\right)x}{c+bx+ax^2}\,dx}{a\left(ad^2-bde+ce^2\right)}$$
$$= \frac{x}{ae}-\frac{d^3\log(d+ex)}{e^2\left(ad^2-e(bd-ce)\right)}+\frac{\left(b^2d-acd-bce\right)\int \frac{b+2ax}{c+bx+ax^2}\,dx}{2a^2\left(ad^2-e(bd-ce)\right)}-\frac{\left(b^3d-3abcd-b^2ce+2ac^2e\right.}{2a^2\left(ad^2-e(bd-\right.}$$
$$= \frac{x}{ae}-\frac{d^3\log(d+ex)}{e^2\left(ad^2-e(bd-ce)\right)}+\frac{\left(b^2d-acd-bce\right)\log\left(c+bx+ax^2\right)}{2a^2\left(ad^2-e(bd-ce)\right)}+\frac{\left(b^3d-3abcd-b^2ce+\right.}{a}$$
$$= \frac{x}{ae}+\frac{\left(b^3d-3abcd-b^2ce+2ac^2e\right)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}-\frac{d^3\log(d+ex)}{e^2\left(ad^2-e(bd-ce)\right)}+\frac{\left(b^2d-acd-\right.}{2a^2\left(\right.}$$

Mathematica [A] time = 0.187473, size = 178, normalized size = 1.01

$$\frac{\left(-3abcd+2ac^2e-b^2ce+b^3d\right)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{a^2\sqrt{4ac-b^2}\left(-ad^2+bde-ce^2\right)}+\frac{\left(-acd+b^2d-bce\right)\log\left(ax^2+bx+c\right)}{2a^2\left(ad^2-bde+ce^2\right)}-\frac{d^3\log(d+ex)}{e^2\left(ad^2-bde+ce^2\right)}+\frac{x}{ae}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + c/x^2 + b/x)*(d + e*x)),x]
```

```
[Out] x/(a*e) + ((b^3*d - 3*a*b*c*d - b^2*c*e + 2*a*c^2*e)*ArcTan[(b + 2*a*x)/Sqr
t[-b^2 + 4*a*c]])/(a^2*Sqrt[-b^2 + 4*a*c]*(-(a*d^2) + b*d*e - c*e^2)) - (d^
3*Log[d + e*x])/(e^2*(a*d^2 - b*d*e + c*e^2)) + ((b^2*d - a*c*d - b*c*e)*Lo
g[c + b*x + a*x^2])/(2*a^2*(a*d^2 - b*d*e + c*e^2))
```

Maple [B] time = 0.007, size = 388, normalized size = 2.2

$$\frac{x}{ae}-\frac{d^3\ln(ex+d)}{e^2\left(ad^2-bde+e^2c\right)}-\frac{\ln\left(ax^2+bx+c\right)cd}{\left(2\,ad^2-2\,bde+2\,e^2c\right)a}+\frac{\ln\left(ax^2+bx+c\right)b^2d}{\left(2\,ad^2-2\,bde+2\,e^2c\right)a^2}-\frac{\ln\left(ax^2+bx+c\right)bce}{\left(2\,ad^2-2\,bde+2\,e^2c\right)a^2}+3\,\frac{}{\left(ad^2-b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+c/x^2+b/x)/(e*x+d),x)
```

```
[Out] x/a/e-1/e^2*d^3/(a*d^2-b*d*e+c*e^2)*ln(e*x+d)-1/2/(a*d^2-b*d*e+c*e^2)/a*ln(
a*x^2+b*x+c)*c*d+1/2/(a*d^2-b*d*e+c*e^2)/a^2*ln(a*x^2+b*x+c)*b^2*d-1/2/(a*d
^2-b*d*e+c*e^2)/a^2*ln(a*x^2+b*x+c)*b*c*e+3/(a*d^2-b*d*e+c*e^2)/a/(4*a*c-b^
2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b*c*d-2/(a*d^2-b*d*e+c*e^2)/a/
(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*c^2*e-1/(a*d^2-b*d*e+
c*e^2)/a^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^3*d+1/(a
*d^2-b*d*e+c*e^2)/a^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))
*b^2*c*e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+c/x^2+b/x)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 30.6945, size = 1237, normalized size = 7.03

$$\left[-\frac{2\left(a^2b^2-4\,a^3c\right)d^3\log\left(ex+d\right)-\left(\left(b^3-3\,abc\right)de^2-\left(b^2c-2\,ac^2\right)e^3\right)\sqrt{b^2-4\,ac}\log\left(\frac{2\,a^2x^2+2\,abx+b^2-2\,ac+\sqrt{b^2-4\,ac}(2\,ax+b)}{ax^2+bx+c}\right)-}{2\left(\left(a^3b^2-4\,a^4c\right)d^2e^2-\left(a\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+c/x^2+b/x)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a^2*b^2 - 4*a^3*c)*d^3*log(e*x + d) - ((b^3 - 3*a*b*c)*d*e^2 - (b
^2*c - 2*a*c^2)*e^3)*sqrt(b^2 - 4*a*c)*log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a
*c + sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x + c)) - 2*((a^2*b^2 - 4*a^
3*c)*d^2*e - (a*b^3 - 4*a^2*b*c)*d*e^2 + (a*b^2*c - 4*a^2*c^2)*e^3)*x - ((b
^4 - 5*a*b^2*c + 4*a^2*c^2)*d*e^2 - (b^3*c - 4*a*b*c^2)*e^3)*log(a*x^2 + b*
```

```
x + c))/((a^3*b^2 - 4*a^4*c)*d^2*e^2 - (a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^2*b
^2*c - 4*a^3*c^2)*e^4), -1/2*(2*(a^2*b^2 - 4*a^3*c)*d^3*log(e*x + d) - 2*((
b^3 - 3*a*b*c)*d*e^2 - (b^2*c - 2*a*c^2)*e^3)*sqrt(-b^2 + 4*a*c)*arctan(-sq
rt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)) - 2*((a^2*b^2 - 4*a^3*c)*d^2*e
- (a*b^3 - 4*a^2*b*c)*d*e^2 + (a*b^2*c - 4*a^2*c^2)*e^3)*x - ((b^4 - 5*a*b^
2*c + 4*a^2*c^2)*d*e^2 - (b^3*c - 4*a*b*c^2)*e^3)*log(a*x^2 + b*x + c))/((a
^3*b^2 - 4*a^4*c)*d^2*e^2 - (a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^2*b^2*c - 4*a^
3*c^2)*e^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+c/x**2+b/x)/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09625, size = 250, normalized size = 1.42

$$-\frac{d^3\log\left(|xe+d|\right)}{ad^2e^2-bde^3+ce^4}+\frac{xe^{(-1)}}{a}+\frac{\left(b^2d-acd-bce\right)\log\left(ax^2+bx+c\right)}{2\left(a^3d^2-a^2bde+a^2ce^2\right)}-\frac{\left(b^3d-3\,abcd-b^2ce+2\,ac^2e\right)\arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(a^3d^2-a^2bde+a^2ce^2\right)\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+c/x^2+b/x)/(e*x+d),x, algorithm="giac")
```

```
[Out] -d^3*log(abs(x*e + d))/(a*d^2*e^2 - b*d*e^3 + c*e^4) + x*e^(-1)/a + 1/2*(b^
2*d - a*c*d - b*c*e)*log(a*x^2 + b*x + c)/(a^3*d^2 - a^2*b*d*e + a^2*c*e^2)
 - (b^3*d - 3*a*b*c*d - b^2*c*e + 2*a*c^2*e)*arctan((2*a*x + b)/sqrt(-b^2 +
 4*a*c))/((a^3*d^2 - a^2*b*d*e + a^2*c*e^2)*sqrt(-b^2 + 4*a*c))
```

3.64 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx$

Optimal. Leaf size=149

$$-\frac{\left(-2acd+b^2d-bce\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}+\frac{d^2\log(d+ex)}{e\left(ad^2-bde+ce^2\right)}-\frac{(bd-ce)\log\left(ax^2+bx+c\right)}{2a\left(ad^2-e(bd-ce)\right)}$$

```
[Out] -(((b^2*d - 2*a*c*d - b*c*e)*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(a*Sqr
t[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e)))) + (d^2*Log[d + e*x])/(e*(a*d^2 - b
*d*e + c*e^2)) - ((b*d - c*e)*Log[c + b*x + a*x^2])/(2*a*(a*d^2 - e*(b*d -
c*e)))
```

Rubi [A] time = 0.210117, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1445, 1628, 634, 618, 206, 628}

$$-\frac{\left(-2acd+b^2d-bce\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}+\frac{d^2\log(d+ex)}{e\left(ad^2-bde+ce^2\right)}-\frac{(bd-ce)\log\left(ax^2+bx+c\right)}{2a\left(ad^2-e(bd-ce)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + c/x^2 + b/x)*(d + e*x)),x]
```

```
[Out] -(((b^2*d - 2*a*c*d - b*c*e)*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(a*Sqr
t[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e)))) + (d^2*Log[d + e*x])/(e*(a*d^2 - b
*d*e + c*e^2)) - ((b*d - c*e)*Log[c + b*x + a*x^2])/(2*a*(a*d^2 - e*(b*d -
c*e)))
```

Rule 1445

```
Int[((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_) + (e_.)*(x
_)^(n_.))^(q_.), x_Symbol] :> Int[((d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p)
/x^(2*n*p), x] /; FreeQ[{a, b, c, d, e, n, q}, x] && EqQ[mn, -n] && EqQ[mn2
, 2*mn] && IntegerQ[p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
```

```
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx = \int \frac{x^2}{(d+ex)\left(c+bx+ax^2\right)}\,dx$$
$$= \int \left(\frac{d^2}{\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{-cd-(bd-ce)x}{\left(ad^2-e(bd-ce)\right)\left(c+bx+ax^2\right)}\right) dx$$
$$= \frac{d^2\log(d+ex)}{e\left(ad^2-bde+ce^2\right)} + \frac{\int \frac{-cd-(bd-ce)x}{c+bx+ax^2}\,dx}{ad^2-e(bd-ce)}$$
$$= \frac{d^2\log(d+ex)}{e\left(ad^2-bde+ce^2\right)} - \frac{(bd-ce)\int \frac{b+2ax}{c+bx+ax^2}\,dx}{2a\left(ad^2-e(bd-ce)\right)} + \frac{\left(b^2d-2acd-bce\right)\int \frac{1}{c+bx+ax^2}\,dx}{2a\left(ad^2-e(bd-ce)\right)}$$
$$= \frac{d^2\log(d+ex)}{e\left(ad^2-bde+ce^2\right)} - \frac{(bd-ce)\log\left(c+bx+ax^2\right)}{2a\left(ad^2-e(bd-ce)\right)} - \frac{\left(b^2d-2acd-bce\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x}\right.}{a\left(ad^2-e(bd-ce)\right)}$$
$$= -\frac{\left(b^2d-2acd-bce\right)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)} + \frac{d^2\log(d+ex)}{e\left(ad^2-bde+ce^2\right)} - \frac{(bd-ce)\log\left(c+bx+ax^2\right.}{2a\left(ad^2-e(bd-ce)\right)}$$

Mathematica [A] time = 0.127595, size = 132, normalized size = 0.89

$$-\frac{\sqrt{4ac-b^2}\left(e(bd-ce)\log(x(ax+b)+c)-2ad^2\log(d+ex)\right)+2e\left(2acd+b^2(-d)+bce\right)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{2ae\sqrt{4ac-b^2}\left(ad^2+e(ce-bd)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + c/x^2 + b/x)*(d + e*x)),x]
```

```
[Out] -(2*e*(-(b^2*d) + 2*a*c*d + b*c*e)*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]] +
 Sqrt[-b^2 + 4*a*c]*(-2*a*d^2*Log[d + e*x] + e*(b*d - c*e)*Log[c + x*(b + a
*x)]))/(2*a*Sqrt[-b^2 + 4*a*c]*e*(a*d^2 + e*(-(b*d) + c*e)))
```

Maple [A] time = 0.007, size = 275, normalized size = 1.9

$$\frac{d^2\ln(ex+d)}{e\left(ad^2-bde+e^2c\right)} - \frac{\ln\left(ax^2+bx+c\right)bd}{\left(2\,ad^2-2\,bde+2\,e^2c\right)a} + \frac{\ln\left(ax^2+bx+c\right)ce}{\left(2\,ad^2-2\,bde+2\,e^2c\right)a} - 2\,\frac{cd}{\left(ad^2-bde+e^2c\right)\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,a}{\sqrt{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)/(e*x+d),x)
```

```
[Out] d^2*ln(e*x+d)/e/(a*d^2-b*d*e+c*e^2)-1/2/(a*d^2-b*d*e+c*e^2)/a*ln(a*x^2+b*x+
c)*b*d+1/2/(a*d^2-b*d*e+c*e^2)/a*ln(a*x^2+b*x+c)*c*e-2/(a*d^2-b*d*e+c*e^2)/
(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*c*d+1/(a*d^2-b*d*e+c*
e^2)/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^2/a*d-1/(a*d^2
-b*d*e+c*e^2)/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b/a*c*e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 9.36673, size = 872, normalized size = 5.85

$$\left[\frac{2\left(ab^2-4a^2c\right)d^2\log(ex+d)+\left(bce^2-\left(b^2-2ac\right)de\right)\sqrt{b^2-4ac}\log\left(\frac{2a^2x^2+2abx+b^2-2ac+\sqrt{b^2-4ac}(2ax+b)}{ax^2+bx+c}\right)-\left(\left(b^3-4abc\right)de\right.}{2\left(\left(a^2b^2-4a^3c\right)d^2e-\left(ab^3-4a^2bc\right)de^2+\left(ab^2c-4a^2c^2\right)e^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a*b^2 - 4*a^2*c)*d^2*log(e*x + d) + (b*c*e^2 - (b^2 - 2*a*c)*d*e)*
sqrt(b^2 - 4*a*c)*log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c
)*(2*a*x + b))/(a*x^2 + b*x + c)) - ((b^3 - 4*a*b*c)*d*e - (b^2*c - 4*a*c^2
)*e^2)*log(a*x^2 + b*x + c))/((a^2*b^2 - 4*a^3*c)*d^2*e - (a*b^3 - 4*a^2*b*
c)*d*e^2 + (a*b^2*c - 4*a^2*c^2)*e^3), 1/2*(2*(a*b^2 - 4*a^2*c)*d^2*log(e*x
 + d) + 2*(b*c*e^2 - (b^2 - 2*a*c)*d*e)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^
2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)) - ((b^3 - 4*a*b*c)*d*e - (b^2*c - 4*a
*c^2)*e^2)*log(a*x^2 + b*x + c))/((a^2*b^2 - 4*a^3*c)*d^2*e - (a*b^3 - 4*a^
2*b*c)*d*e^2 + (a*b^2*c - 4*a^2*c^2)*e^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11749, size = 201, normalized size = 1.35

$$\frac{d^2\log\left(|xe+d|\right)}{ad^2e-bde^2+ce^3}-\frac{(bd-ce)\log\left(ax^2+bx+c\right)}{2\left(a^2d^2-abde+ace^2\right)}+\frac{\left(b^2d-2\,acd-bce\right)\arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(a^2d^2-abde+ace^2\right)\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/(e*x+d),x, algorithm="giac")
```

```
[Out] d^2*log(abs(x*e + d))/(a*d^2*e - b*d*e^2 + c*e^3) - 1/2*(b*d - c*e)*log(a*x
^2 + b*x + c)/(a^2*d^2 - a*b*d*e + a*c*e^2) + (b^2*d - 2*a*c*d - b*c*e)*arc
tan((2*a*x + b)/sqrt(-b^2 + 4*a*c))/((a^2*d^2 - a*b*d*e + a*c*e^2)*sqrt(-b^
2 + 4*a*c))
```

3.65 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x(d+ex)}\,dx$

Optimal. Leaf size=124

$$\frac{(bd-2ce)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}+\frac{d\log\left(ax^2+bx+c\right)}{2\left(ad^2-e(bd-ce)\right)}-\frac{d\log(d+ex)}{ad^2-e(bd-ce)}$$

```
[Out] ((b*d - 2*c*e)*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(
a*d^2 - e*(b*d - c*e))) - (d*Log[d + e*x])/(a*d^2 - e*(b*d - c*e)) + (d*Log
[c + b*x + a*x^2])/(2*(a*d^2 - e*(b*d - c*e)))
```

Rubi [A] time = 0.14495, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1569, 800, 634, 618, 206, 628}

$$\frac{(bd-2ce)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}+\frac{d\log\left(ax^2+bx+c\right)}{2\left(ad^2-e(bd-ce)\right)}-\frac{d\log(d+ex)}{ad^2-e(bd-ce)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + c/x^2 + b/x)*x*(d + e*x)),x]
```

```
[Out] ((b*d - 2*c*e)*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(
a*d^2 - e*(b*d - c*e))) - (d*Log[d + e*x])/(a*d^2 - e*(b*d - c*e)) + (d*Log
[c + b*x + a*x^2])/(2*(a*d^2 - e*(b*d - c*e)))
```

Rule 1569

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
```

```
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x(d+ex)}\,dx = \int \frac{x}{(d+ex)\left(c+bx+ax^2\right)}\,dx$$

$$= \int \left(\frac{de}{\left(-ad^2+e(bd-ce)\right)(d+ex)} + \frac{ce+adx}{\left(ad^2-e(bd-ce)\right)\left(c+bx+ax^2\right)}\right)dx$$

$$= -\frac{d\log(d+ex)}{ad^2-bde+ce^2} + \frac{\int \frac{ce+adx}{c+bx+ax^2}\,dx}{ad^2-e(bd-ce)}$$

$$= -\frac{d\log(d+ex)}{ad^2-bde+ce^2} + \frac{d\int \frac{b+2ax}{c+bx+ax^2}\,dx}{2\left(ad^2-bde+ce^2\right)} + \frac{(-bd+2ce)\int \frac{1}{c+bx+ax^2}\,dx}{2\left(ad^2-e(bd-ce)\right)}$$

$$= -\frac{d\log(d+ex)}{ad^2-bde+ce^2} + \frac{d\log\left(c+bx+ax^2\right)}{2\left(ad^2-bde+ce^2\right)} + \frac{(bd-2ce)\,\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2ax\right)}{ad^2-e(bd-ce)}$$

$$= \frac{(bd-2ce)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)} - \frac{d\log(d+ex)}{ad^2-bde+ce^2} + \frac{d\log\left(c+bx+ax^2\right)}{2\left(ad^2-bde+ce^2\right)}$$

Mathematica [A] time = 0.0789739, size = 107, normalized size = 0.86

$$\frac{d\sqrt{4ac-b^2}(2\log(d+ex)-\log(x(ax+b)+c))+2(bd-2ce)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\left(e(bd-ce)-ad^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + c/x^2 + b/x)*x*(d + e*x)),x]
```

```
[Out] (2*(b*d - 2*c*e)*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]] + Sqrt[-b^2 + 4*a*c
]*d*(2*Log[d + e*x] - Log[c + x*(b + a*x)]))/(2*Sqrt[-b^2 + 4*a*c]*(-(a*d^2
) + e*(b*d - c*e)))
```

Maple [A] time = 0.003, size = 169, normalized size = 1.4

$$-\frac{d\ln(ex+d)}{ad^2-bde+e^2c} + \frac{d\ln\left(ax^2+bx+c\right)}{2\,ad^2-2\,bde+2\,e^2c} - \frac{bd}{ad^2-bde+e^2c}\arctan\left((2\,ax+b)\frac{1}{\sqrt{4\,ac-b^2}}\right)\frac{1}{\sqrt{4\,ac-b^2}} + 2\,\frac{ce}{\left(ad^2-bde+e^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)/x/(e*x+d),x)
```

```
[Out] -d/(a*d^2-b*d*e+c*e^2)*ln(e*x+d)+1/2/(a*d^2-b*d*e+c*e^2)*d*ln(a*x^2+b*x+c)-
1/(a*d^2-b*d*e+c*e^2)/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))
*b*d+2/(a*d^2-b*d*e+c*e^2)/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(
1/2))*c*e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.2561, size = 694, normalized size = 5.6

$$\left[\frac{\left(b^2-4\,ac\right)d\log\left(ax^2+bx+c\right)-2\left(b^2-4\,ac\right)d\log\left(ex+d\right)-\sqrt{b^2-4\,ac}(bd-2\,ce)\log\left(\frac{2\,a^2x^2+2\,abx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,ax+b}{ax^2+bx+c}\right.}{2\left(\left(ab^2-4\,a^2c\right)d^2-\left(b^3-4\,abc\right)de+\left(b^2c-4\,ac^2\right)e^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/2*((b^2 - 4*a*c)*d*log(a*x^2 + b*x + c) - 2*(b^2 - 4*a*c)*d*log(e*x + d)
 - sqrt(b^2 - 4*a*c)*(b*d - 2*c*e)*log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c -
 sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x + c)))/((a*b^2 - 4*a^2*c)*d^2
- (b^3 - 4*a*b*c)*d*e + (b^2*c - 4*a*c^2)*e^2), 1/2*((b^2 - 4*a*c)*d*log(a*
x^2 + b*x + c) - 2*(b^2 - 4*a*c)*d*log(e*x + d) + 2*sqrt(-b^2 + 4*a*c)*(b*d
 - 2*c*e)*arctan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)))/((a*b^2 -
4*a^2*c)*d^2 - (b^3 - 4*a*b*c)*d*e + (b^2*c - 4*a*c^2)*e^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)/x/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.10904, size = 171, normalized size = 1.38

$$-\frac{de\log\left(|xe+d|\right)}{ad^2e-bde^2+ce^3}+\frac{d\log\left(ax^2+bx+c\right)}{2\left(ad^2-bde+ce^2\right)}-\frac{(bd-2\,ce)\arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(ad^2-bde+ce^2\right)\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x/(e*x+d),x, algorithm="giac")
```

```
[Out] -d*e*log(abs(x*e + d))/(a*d^2*e - b*d*e^2 + c*e^3) + 1/2*d*log(a*x^2 + b*x
+ c)/(a*d^2 - b*d*e + c*e^2) - (b*d - 2*c*e)*arctan((2*a*x + b)/sqrt(-b^2 +
 4*a*c))/((a*d^2 - b*d*e + c*e^2)*sqrt(-b^2 + 4*a*c))
```

3.66 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^2(d+ex)}\,dx$

Optimal. Leaf size=123

$$-\frac{(2ad-be)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}-\frac{e\log\left(ax^2+bx+c\right)}{2\left(ad^2-bde+ce^2\right)}+\frac{e\log(d+ex)}{ad^2-bde+ce^2}$$

```
[Out] -(((2*a*d - b*e)*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]
*(a*d^2 - e*(b*d - c*e)))) + (e*Log[d + e*x])/(a*d^2 - b*d*e + c*e^2) - (e*
Log[c + b*x + a*x^2])/(2*(a*d^2 - b*d*e + c*e^2))
```

Rubi [A] time = 0.107172, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {1569, 705, 31, 634, 618, 206, 628}

$$-\frac{(2ad-be)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}-\frac{e\log\left(ax^2+bx+c\right)}{2\left(ad^2-bde+ce^2\right)}+\frac{e\log(d+ex)}{ad^2-bde+ce^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + c/x^2 + b/x)*x^2*(d + e*x)),x]
```


```
[Out] -(((2*a*d - b*e)*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]
*(a*d^2 - e*(b*d - c*e)))) + (e*Log[d + e*x])/(a*d^2 - b*d*e + c*e^2) - (e*
Log[c + b*x + a*x^2])/(2*(a*d^2 - b*d*e + c*e^2))
```


Rule 1569

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
```

```
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^2(d+ex)}\,dx = \int \frac{1}{(d+ex)\left(c+bx+ax^2\right)}\,dx$$
$$= \frac{e^2\int \frac{1}{d+ex}\,dx}{ad^2-bde+ce^2} + \frac{\int \frac{ad-be-aex}{c+bx+ax^2}\,dx}{ad^2-e(bd-ce)}$$
$$= \frac{e\log(d+ex)}{ad^2-bde+ce^2} - \frac{e\int \frac{b+2ax}{c+bx+ax^2}\,dx}{2\left(ad^2-bde+ce^2\right)} + \frac{(2ad-be)\int \frac{1}{c+bx+ax^2}\,dx}{2\left(ad^2-e(bd-ce)\right)}$$
$$= \frac{e\log(d+ex)}{ad^2-bde+ce^2} - \frac{e\log\left(c+bx+ax^2\right)}{2\left(ad^2-bde+ce^2\right)} - \frac{(2ad-be)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2ax\right)}{ad^2-e(bd-ce)}$$
$$= -\frac{(2ad-be)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)} + \frac{e\log(d+ex)}{ad^2-bde+ce^2} - \frac{e\log\left(c+bx+ax^2\right)}{2\left(ad^2-bde+ce^2\right)}$$

Mathematica [A] time = 0.077105, size = 105, normalized size = 0.85

$$\frac{e\sqrt{4ac-b^2}(\log(x(ax+b)+c)-2\log(d+ex))+(2be-4ad)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\left(e(bd-ce)-ad^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + c/x^2 + b/x)*x^2*(d + e*x)),x]
```

```
[Out] ((-4*a*d + 2*b*e)*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]] + Sqrt[-b^2 + 4*a*
c]*e*(-2*Log[d + e*x] + Log[c + x*(b + a*x)]))/(2*Sqrt[-b^2 + 4*a*c]*(-(a*d
^2) + e*(b*d - c*e)))
```

Maple [A] time = 0.006, size = 168, normalized size = 1.4

$$\frac{e\ln(ex+d)}{ad^2-bde+e^2c} - \frac{e\ln\left(ax^2+bx+c\right)}{2\,ad^2-2\,bde+2\,e^2c} + 2\,\frac{ad}{\left(ad^2-bde+e^2c\right)\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,ax+b}{\sqrt{4\,ac-b^2}}\right) - \frac{be}{ad^2-bde+e^2c}\text{arctan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)/x^2/(e*x+d),x)
```

```
[Out] e*ln(e*x+d)/(a*d^2-b*d*e+c*e^2)-1/2*e*ln(a*x^2+b*x+c)/(a*d^2-b*d*e+c*e^2)+2
/(a*d^2-b*d*e+c*e^2)/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*
a*d-1/(a*d^2-b*d*e+c*e^2)/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1
/2))*b*e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 3.29332, size = 697, normalized size = 5.67

$$\left[-\frac{(b^2-4ac)e\log\left(ax^2+bx+c\right)-2\left(b^2-4ac\right)e\log\left(ex+d\right)+\sqrt{b^2-4ac}(2ad-be)\log\left(\frac{2a^2x^2+2abx+b^2-2ac+\sqrt{b^2-4ac}(2ax+b)}{ax^2+bx+c}\right.}{2\left(\left(ab^2-4a^2c\right)d^2-\left(b^3-4abc\right)de+\left(b^2c-4ac^2\right)e^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] [-1/2*((b^2 - 4*a*c)*e*log(a*x^2 + b*x + c) - 2*(b^2 - 4*a*c)*e*log(e*x + d
) + sqrt(b^2 - 4*a*c)*(2*a*d - b*e)*log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c
+ sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x + c)))/((a*b^2 - 4*a^2*c)*d^2
 - (b^3 - 4*a*b*c)*d*e + (b^2*c - 4*a*c^2)*e^2), -1/2*((b^2 - 4*a*c)*e*log(
a*x^2 + b*x + c) - 2*(b^2 - 4*a*c)*e*log(e*x + d) + 2*sqrt(-b^2 + 4*a*c)*(2
*a*d - b*e)*arctan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)))/((a*b^2
- 4*a^2*c)*d^2 - (b^3 - 4*a*b*c)*d*e + (b^2*c - 4*a*c^2)*e^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)/x**2/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09606, size = 170, normalized size = 1.38

$$-\frac{e\log\left(ax^2+bx+c\right)}{2\left(ad^2-bde+ce^2\right)}+\frac{e^2\log\left(|xe+d|\right)}{ad^2e-bde^2+ce^3}+\frac{(2\,ad-be)\arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(ad^2-bde+ce^2\right)\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^2/(e*x+d),x, algorithm="giac")
```

```
[Out] -1/2*e*log(a*x^2 + b*x + c)/(a*d^2 - b*d*e + c*e^2) + e^2*log(abs(x*e + d))
/(a*d^2*e - b*d*e^2 + c*e^3) + (2*a*d - b*e)*arctan((2*a*x + b)/sqrt(-b^2 +
 4*a*c))/((a*d^2 - b*d*e + c*e^2)*sqrt(-b^2 + 4*a*c))
```

3.67 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^3(d+ex)}\,dx$

Optimal. Leaf size=158

$$\frac{\left(abd+2ace+b^2(-e)\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}-\frac{e^2\log(d+ex)}{d\left(ad^2-bde+ce^2\right)}-\frac{(ad-be)\log\left(ax^2+bx+c\right)}{2c\left(ad^2-e(bd-ce)\right)}+\frac{\log(x)}{cd}$$

```
[Out] ((a*b*d - b^2*e + 2*a*c*e)*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[
b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))) + Log[x]/(c*d) - (e^2*Log[d + e*x])/(
d*(a*d^2 - b*d*e + c*e^2)) - ((a*d - b*e)*Log[c + b*x + a*x^2])/(2*c*(a*d^2
 - e*(b*d - c*e)))
```

Rubi [A] time = 0.270758, antiderivative size = 159, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1569, 893, 634, 618, 206, 628}

$$\frac{\left(abd+2ace+b^2(-e)\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}-\frac{e^2\log(d+ex)}{d\left(ad^2-e(bd-ce)\right)}-\frac{(ad-be)\log\left(ax^2+bx+c\right)}{2c\left(ad^2-e(bd-ce)\right)}+\frac{\log(x)}{cd}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + c/x^2 + b/x)*x^3*(d + e*x)),x]
```

```
[Out] ((a*b*d - b^2*e + 2*a*c*e)*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[
b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))) + Log[x]/(c*d) - (e^2*Log[d + e*x])/(
d*(a*d^2 - e*(b*d - c*e))) - ((a*d - b*e)*Log[c + b*x + a*x^2])/(2*c*(a*d^2
 - e*(b*d - c*e)))
```

Rule 1569

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
```

```
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^3(d+ex)}\,dx = \int \frac{1}{x(d+ex)\left(c+bx+ax^2\right)}\,dx$$

$$= \int \left(\frac{1}{cdx} + \frac{e^3}{d\left(-ad^2+e(bd-ce)\right)(d+ex)} + \frac{b^2e-a(bd+ce)-a(ad-be)x}{c\left(ad^2-e(bd-ce)\right)\left(c+bx+ax^2\right)}\right)dx$$

$$= \frac{\log(x)}{cd} - \frac{e^2\log(d+ex)}{d\left(ad^2-e(bd-ce)\right)} + \frac{\int \frac{b^2e-a(bd+ce)-a(ad-be)x}{c+bx+ax^2}\,dx}{c\left(ad^2-bde+ce^2\right)}$$

$$= \frac{\log(x)}{cd} - \frac{e^2\log(d+ex)}{d\left(ad^2-e(bd-ce)\right)} + \frac{\left(-abd+b^2e-2ace\right)\int \frac{1}{c+bx+ax^2}\,dx}{2c\left(ad^2-bde+ce^2\right)} - \frac{(ad-be)\int \frac{b+2ax}{c+bx+ax^2}}{2c\left(ad^2-e(bd-ce\right.}$$

$$= \frac{\log(x)}{cd} - \frac{e^2\log(d+ex)}{d\left(ad^2-e(bd-ce)\right)} - \frac{(ad-be)\log\left(c+bx+ax^2\right)}{2c\left(ad^2-e(bd-ce)\right)} - \frac{\left(-abd+b^2e-2ace\right)\mathrm{Subs}}{c\left(ad^2-\right.}$$

$$= \frac{\left(abd-b^2e+2ace\right)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}\left(ad^2-bde+ce^2\right)} + \frac{\log(x)}{cd} - \frac{e^2\log(d+ex)}{d\left(ad^2-e(bd-ce)\right)} - \frac{(ad-be)\log\left(c+\right.}{2c\left(ad^2-e(bc\right.}$$

Mathematica [A] time = 0.188878, size = 152, normalized size = 0.96

$$-\frac{\sqrt{4ac-b^2}\left(-2\log(x)\left(ad^2+e(ce-bd)\right)+d(ad-be)\log(x(ax+b)+c)+2ce^2\log(d+ex)\right)+2d\left(abd+2ace+b^2(-e)\right)\mathrm{t}}{2cd\sqrt{4ac-b^2}\left(ad^2+e(ce-bd)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + c/x^2 + b/x)*x^3*(d + e*x)),x]
```

```
[Out] -(2*d*(a*b*d - b^2*e + 2*a*c*e)*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]] + Sq
rt[-b^2 + 4*a*c]*(-2*(a*d^2 + e*(-(b*d) + c*e))*Log[x] + 2*c*e^2*Log[d + e*
x] + d*(a*d - b*e)*Log[c + x*(b + a*x)]))/(2*c*Sqrt[-b^2 + 4*a*c]*d*(a*d^2
+ e*(-(b*d) + c*e)))
```

Maple [A] time = 0.008, size = 285, normalized size = 1.8

$$-\frac{e^2\ln(ex+d)}{d\left(ad^2-bde+e^2c\right)} - \frac{a\ln\left(ax^2+bx+c\right)d}{\left(2ad^2-2bde+2e^2c\right)c} + \frac{\ln\left(ax^2+bx+c\right)be}{\left(2ad^2-2bde+2e^2c\right)c} - \frac{abd}{\left(ad^2-bde+e^2c\right)c}\arctan\left((2ax+b)\frac{1}{\sqrt{4ac\ldots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)/x^3/(e*x+d),x)
```

```
[Out] -e^2*ln(e*x+d)/d/(a*d^2-b*d*e+c*e^2)-1/2/(a*d^2-b*d*e+c*e^2)/c*a*ln(a*x^2+b
*x+c)*d+1/2/(a*d^2-b*d*e+c*e^2)/c*ln(a*x^2+b*x+c)*b*e-1/(a*d^2-b*d*e+c*e^2)
/c/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a*b*d-2/(a*d^2-b*d
*e+c*e^2)/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a*e+1/(a*d^
2-b*d*e+c*e^2)/c/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^2*
e+ln(x)/c/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^3/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^3/(e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)/x**3/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1042, size = 221, normalized size = 1.4

$$-\frac{(ad-be)\log\left(ax^2+bx+c\right)}{2\left(acd^2-bcde+c^2e^2\right)}-\frac{e^3\log\left(|xe+d|\right)}{ad^3e-bd^2e^2+cde^3}-\frac{\left(abd-b^2e+2\,ace\right)\arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(acd^2-bcde+c^2e^2\right)\sqrt{-b^2+4\,ac}}+\frac{\log\left(|x|\right)}{cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^3/(e*x+d),x, algorithm="giac")
```

```
[Out] -1/2*(a*d - b*e)*log(a*x^2 + b*x + c)/(a*c*d^2 - b*c*d*e + c^2*e^2) - e^3*l
og(abs(x*e + d))/(a*d^3*e - b*d^2*e^2 + c*d*e^3) - (a*b*d - b^2*e + 2*a*c*e
)*arctan((2*a*x + b)/sqrt(-b^2 + 4*a*c))/((a*c*d^2 - b*c*d*e + c^2*e^2)*sqr
t(-b^2 + 4*a*c)) + log(abs(x))/(c*d)
```

3.68 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^4(d+ex)}\,dx$

Optimal. Leaf size=193

$$\frac{\left(2a^2cd-ab(bd+3ce)+b^3e\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}+\frac{\left(abd+ace+b^2(-e)\right)\log\left(ax^2+bx+c\right)}{2c^2\left(ad^2-e(bd-ce)\right)}+\frac{e^3\log(d+ex)}{d^2\left(ad^2-e(bd-ce)\right)}-\frac{\log}{}$$

```
[Out] -(1/(c*d*x)) + ((2*a^2*c*d + b^3*e - a*b*(b*d + 3*c*e))*ArcTanh[(b + 2*a*x)
/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))) - ((b*
d + c*e)*Log[x])/(c^2*d^2) + (e^3*Log[d + e*x])/(d^2*(a*d^2 - e*(b*d - c*e)
)) + ((a*b*d - b^2*e + a*c*e)*Log[c + b*x + a*x^2])/(2*c^2*(a*d^2 - e*(b*d
- c*e)))
```

Rubi [A] time = 0.343094, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1569, 893, 634, 618, 206, 628}

$$\frac{\left(2a^2cd-ab(bd+3ce)+b^3e\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}+\frac{\left(abd+ace+b^2(-e)\right)\log\left(ax^2+bx+c\right)}{2c^2\left(ad^2-e(bd-ce)\right)}+\frac{e^3\log(d+ex)}{d^2\left(ad^2-e(bd-ce)\right)}-\frac{\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + c/x^2 + b/x)*x^4*(d + e*x)),x]
```


```
[Out] -(1/(c*d*x)) + ((2*a^2*c*d + b^3*e - a*b*(b*d + 3*c*e))*ArcTanh[(b + 2*a*x)
/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))) - ((b*
d + c*e)*Log[x])/(c^2*d^2) + (e^3*Log[d + e*x])/(d^2*(a*d^2 - e*(b*d - c*e)
)) + ((a*b*d - b^2*e + a*c*e)*Log[c + b*x + a*x^2])/(2*c^2*(a*d^2 - e*(b*d
- c*e)))
```


Rule 1569

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^4(d+ex)}\,dx = \int \frac{1}{x^2(d+ex)\left(c+bx+ax^2\right)}\,dx$$

$$= \int \left(\frac{1}{cdx^2} + \frac{-bd-ce}{c^2d^2x} + \frac{e^4}{d^2\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{-a^2cd-b^3e+ab(bd+2ce)+a}{c^2\left(ad^2-e(bd-ce)\right)(c}\right.$$

$$= -\frac{1}{cdx} - \frac{(bd+ce)\log(x)}{c^2d^2} + \frac{e^3\log(d+ex)}{d^2\left(ad^2-e(bd-ce)\right)} + \frac{\int \frac{-a^2cd-b^3e+ab(bd+2ce)+a\left(abd-b^2e+ace\right)x}{c+bx+ax^2}\,d}{c^2\left(ad^2-e(bd-ce)\right)}$$

$$= -\frac{1}{cdx} - \frac{(bd+ce)\log(x)}{c^2d^2} + \frac{e^3\log(d+ex)}{d^2\left(ad^2-e(bd-ce)\right)} + \frac{\left(abd-b^2e+ace\right)\int \frac{b+2ax}{c+bx+ax^2}\,dx}{2c^2\left(ad^2-e(bd-ce)\right)} - \frac{(2}{}$$

$$= -\frac{1}{cdx} - \frac{(bd+ce)\log(x)}{c^2d^2} + \frac{e^3\log(d+ex)}{d^2\left(ad^2-e(bd-ce)\right)} + \frac{\left(abd-b^2e+ace\right)\log\left(c+bx+ax^2\right)}{2c^2\left(ad^2-e(bd-ce)\right)}$$

$$= -\frac{1}{cdx} + \frac{\left(2a^2cd+b^3e-ab(bd+3ce)\right)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)} - \frac{(bd+ce)\log(x)}{c^2d^2} + \frac{e^3\log(}{d^2\left(ad^2-\right.}$$

Mathematica [A] time = 0.177771, size = 194, normalized size = 1.01

$$\frac{\left(2a^2cd-ab(bd+3ce)+b^3e\right)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{c^2\sqrt{4ac-b^2}\left(e(bd-ce)-ad^2\right)} + \frac{\left(abd+ace+b^2(-e)\right)\log(x(ax+b)+c)}{2c^2\left(ad^2+e(ce-bd)\right)} + \frac{e^3\log(d+ex)}{ad^4+d^2e(ce-bd)} - \frac{\log(x)(}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + c/x^2 + b/x)*x^4*(d + e*x)),x]
```

```
[Out] -(1/(c*d*x)) + ((2*a^2*c*d + b^3*e - a*b*(b*d + 3*c*e))*ArcTan[(b + 2*a*x)/
Sqrt[-b^2 + 4*a*c]])/(c^2*Sqrt[-b^2 + 4*a*c]*(-(a*d^2) + e*(b*d - c*e))) -
((b*d + c*e)*Log[x])/(c^2*d^2) + (e^3*Log[d + e*x])/(a*d^4 + d^2*e*(-(b*d)
+ c*e)) + ((a*b*d - b^2*e + a*c*e)*Log[c + x*(b + a*x)])/(2*c^2*(a*d^2 + e*
(-(b*d) + c*e)))
```

Maple [B] time = 0.011, size = 412, normalized size = 2.1

$$\frac{e^3\ln(ex+d)}{d^2\left(ad^2-bde+e^2c\right)} + \frac{a\ln\left(ax^2+bx+c\right)bd}{\left(2ad^2-2bde+2e^2c\right)c^2} + \frac{a\ln\left(ax^2+bx+c\right)e}{\left(2ad^2-2bde+2e^2c\right)c} - \frac{\ln\left(ax^2+bx+c\right)b^2e}{\left(2ad^2-2bde+2e^2c\right)c^2} - 2\frac{}{\left(ad^2-bde\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)/x^4/(e*x+d),x)
```

```
[Out] e^3/d^2/(a*d^2-b*d*e+c*e^2)*ln(e*x+d)+1/2/(a*d^2-b*d*e+c*e^2)/c^2*a*ln(a*x^
2+b*x+c)*b*d+1/2/(a*d^2-b*d*e+c*e^2)/c*a*ln(a*x^2+b*x+c)*e-1/2/(a*d^2-b*d*e
+c*e^2)/c^2*ln(a*x^2+b*x+c)*b^2*e-2/(a*d^2-b*d*e+c*e^2)/c/(4*a*c-b^2)^(1/2)
*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a^2*d+1/(a*d^2-b*d*e+c*e^2)/c^2/(4*a*c
-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a*b^2*d+3/(a*d^2-b*d*e+c*e^
2)/c/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a*b*e-1/(a*d^2-b
*d*e+c*e^2)/c^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^3*e
-1/c/d/x-1/c^2/d*ln(x)*b-1/c/d^2*ln(x)*e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^4/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^4/(e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)/x**4/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.10823, size = 284, normalized size = 1.47

$$\frac{\left(abd - b^2e + ace\right)\log\left(ax^2 + bx + c\right)}{2\left(ac^2d^2 - bc^2de + c^3e^2\right)} + \frac{e^4\log\left(|xe + d|\right)}{ad^4e - bd^3e^2 + cd^2e^3} + \frac{\left(ab^2d - 2\,a^2cd - b^3e + 3\,abce\right)\arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(ac^2d^2 - bc^2de + c^3e^2\right)\sqrt{-b^2 + 4\,ac}} - \frac{(bd

3.69 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^5(d+ex)}\,dx$

Optimal. Leaf size=252

$$\frac{\left(a^2cd-ab(bd+2ce)+b^3e\right)\log\left(ax^2+bx+c\right)}{2c^3\left(ad^2-e(bd-ce)\right)}-\frac{\left(a^2c(3bd+2ce)-ab^2(bd+4ce)+b^4e\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}+\frac{\log(x)\left(-c\left(a\right.\right.}{}$$

```
[Out] -1/(2*c*d*x^2) + (b*d + c*e)/(c^2*d^2*x) - ((b^4*e + a^2*c*(3*b*d + 2*c*e)
- a*b^2*(b*d + 4*c*e))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(c^3*Sqrt[b^
2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))) + ((b^2*d^2 + b*c*d*e - c*(a*d^2 - c*e^
2))*Log[x])/(c^3*d^3) - (e^4*Log[d + e*x])/(d^3*(a*d^2 - e*(b*d - c*e))) +
((a^2*c*d + b^3*e - a*b*(b*d + 2*c*e))*Log[c + b*x + a*x^2])/(2*c^3*(a*d^2
- e*(b*d - c*e)))
```

Rubi [A] time = 0.428431, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1569, 893, 634, 618, 206, 628}

$$\frac{\left(a^2cd-ab(bd+2ce)+b^3e\right)\log\left(ax^2+bx+c\right)}{2c^3\left(ad^2-e(bd-ce)\right)}-\frac{\left(a^2c(3bd+2ce)-ab^2(bd+4ce)+b^4e\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}+\frac{\log(x)\left(-c\left(a\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + c/x^2 + b/x)*x^5*(d + e*x)),x]
```



```
[Out] -1/(2*c*d*x^2) + (b*d + c*e)/(c^2*d^2*x) - ((b^4*e + a^2*c*(3*b*d + 2*c*e)
- a*b^2*(b*d + 4*c*e))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(c^3*Sqrt[b^
2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))) + ((b^2*d^2 + b*c*d*e - c*(a*d^2 - c*e^
2))*Log[x])/(c^3*d^3) - (e^4*Log[d + e*x])/(d^3*(a*d^2 - e*(b*d - c*e))) +
((a^2*c*d + b^3*e - a*b*(b*d + 2*c*e))*Log[c + b*x + a*x^2])/(2*c^3*(a*d^2
- e*(b*d - c*e)))
```


Rule 1569

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^5(d+ex)}\,dx = \int \frac{1}{x^3(d+ex)\left(c+bx+ax^2\right)}\,dx$$

$$= \int \left(\frac{1}{cdx^3} + \frac{-bd-ce}{c^2d^2x^2} + \frac{b^2d^2+bcde-c\left(ad^2-ce^2\right)}{c^3d^3x} + \frac{e^5}{d^3\left(-ad^2+e(bd-ce)\right)(d+ex)} + \cdots\right.$$

$$= -\frac{1}{2cdx^2} + \frac{bd+ce}{c^2d^2x} + \frac{\left(b^2d^2+bcde-c\left(ad^2-ce^2\right)\right)\log(x)}{c^3d^3} - \frac{e^4\log(d+ex)}{d^3\left(ad^2-e(bd-ce)\right)} + \int \frac{b^4e \cdots}{\cdots}$$

$$= -\frac{1}{2cdx^2} + \frac{bd+ce}{c^2d^2x} + \frac{\left(b^2d^2+bcde-c\left(ad^2-ce^2\right)\right)\log(x)}{c^3d^3} - \frac{e^4\log(d+ex)}{d^3\left(ad^2-e(bd-ce)\right)} + \frac{\left(a^2c \cdots\right.}{\cdots}$$

$$= -\frac{1}{2cdx^2} + \frac{bd+ce}{c^2d^2x} + \frac{\left(b^2d^2+bcde-c\left(ad^2-ce^2\right)\right)\log(x)}{c^3d^3} - \frac{e^4\log(d+ex)}{d^3\left(ad^2-e(bd-ce)\right)} + \frac{\left(a^2c \cdots\right.}{\cdots}$$

$$= -\frac{1}{2cdx^2} + \frac{bd+ce}{c^2d^2x} - \frac{\left(b^4e+a^2c(3bd+2ce)-ab^2(bd+4ce)\right)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)} + \frac{\left(b^2d^2+\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.233504, size = 252, normalized size = 1.

$$\frac{\left(a^2cd-ab(bd+2ce)+b^3e\right)\log(x(ax+b)+c)}{2c^3\left(ad^2+e(ce-bd)\right)} - \frac{\left(a^2c(3bd+2ce)-ab^2(bd+4ce)+b^4e\right)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{c^3\sqrt{4ac-b^2}\left(e(bd-ce)-ad^2\right)} + \frac{\log(x)\left(c\left(ce^2\cdots\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + c/x^2 + b/x)*x^5*(d + e*x)),x]
```

```
[Out] -1/(2*c*d*x^2) + (b*d + c*e)/(c^2*d^2*x) - ((b^4*e + a^2*c*(3*b*d + 2*c*e)
- a*b^2*(b*d + 4*c*e))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/(c^3*Sqrt[-b
^2 + 4*a*c]*(-(a*d^2) + e*(b*d - c*e))) + ((b^2*d^2 + b*c*d*e + c*(-(a*d^2)
 + c*e^2))*Log[x])/(c^3*d^3) - (e^4*Log[d + e*x])/(a*d^5 + d^3*e*(-(b*d) +
c*e)) + ((a^2*c*d + b^3*e - a*b*(b*d + 2*c*e))*Log[c + x*(b + a*x)])/(2*c^3
*(a*d^2 + e*(-(b*d) + c*e)))
```

Maple [B] time = 0.012, size = 562, normalized size = 2.2

$$-\frac{e^4\ln(ex+d)}{d^3\left(ad^2-bde+e^2c\right)} + \frac{a^2\ln\left(ax^2+bx+c\right)d}{\left(2\,ad^2-2\,bde+2\,e^2c\right)c^2} - \frac{a\ln\left(ax^2+bx+c\right)b^2d}{\left(2\,ad^2-2\,bde+2\,e^2c\right)c^3} - \frac{a\ln\left(ax^2+bx+c\right)be}{\left(ad^2-bde+e^2c\right)c^2} + \frac{\ln\left(ax^2+bx+\cdots\right.}{\left(2\,ad^2-2\,bde+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)/x^5/(e*x+d),x)
```

```
[Out] -e^4/d^3/(a*d^2-b*d*e+c*e^2)*ln(e*x+d)+1/2/(a*d^2-b*d*e+c*e^2)/c^2*a^2*ln(a
*x^2+b*x+c)*d-1/2/(a*d^2-b*d*e+c*e^2)/c^3*a*ln(a*x^2+b*x+c)*b^2*d-1/(a*d^2-
b*d*e+c*e^2)/c^2*a*ln(a*x^2+b*x+c)*b*e+1/2/(a*d^2-b*d*e+c*e^2)/c^3*ln(a*x^2
+b*x+c)*b^3*e+3/(a*d^2-b*d*e+c*e^2)/c^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/
(4*a*c-b^2)^(1/2))*a^2*b*d+2/(a*d^2-b*d*e+c*e^2)/c/(4*a*c-b^2)^(1/2)*arctan
((2*a*x+b)/(4*a*c-b^2)^(1/2))*a^2*e-1/(a*d^2-b*d*e+c*e^2)/c^3/(4*a*c-b^2)^(
1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a*b^3*d-4/(a*d^2-b*d*e+c*e^2)/c^2/
(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a*b^2*e+1/(a*d^2-b*d*
e+c*e^2)/c^3/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^4*e-1/
2/c/d/x^2+1/c^2/d/x*b+1/c/d^2/x*e-1/c^2/d*ln(x)*a+1/c^3/d*ln(x)*b^2+1/c^2/d
^2*ln(x)*b*e+1/c/d^3*ln(x)*e^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^5/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^5/(e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)/x**5/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09894, size = 377, normalized size = 1.5

$$-\frac{\left(ab^2d-a^2cd-b^3e+2\,abce\right)\log\left(ax^2+bx+c\right)}{2\left(ac^3d^2-bc^3de+c^4e^2\right)}-\frac{e^5\log\left(|xe+d|\right)}{ad^5e-bd^4e^2+cd^3e^3}-\frac{\left(ab^3d-3\,a^2bcd-b^4e+4\,ab^2ce-2\,a^2c^2e\right)\mathrm{arc}}{\left(ac^3d^2-bc^3de+c^4e^2\right)\sqrt{-b^2+4\,}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^5/(e*x+d),x, algorithm="giac")
```

```
[Out] -1/2*(a*b^2*d - a^2*c*d - b^3*e + 2*a*b*c*e)*log(a*x^2 + b*x + c)/(a*c^3*d^
2 - b*c^3*d*e + c^4*e^2) - e^5*log(abs(x*e + d))/(a*d^5*e - b*d^4*e^2 + c*d
^3*e^3) - (a*b^3*d - 3*a^2*b*c*d - b^4*e + 4*a*b^2*c*e - 2*a^2*c^2*e)*arcta
n((2*a*x + b)/sqrt(-b^2 + 4*a*c))/((a*c^3*d^2 - b*c^3*d*e + c^4*e^2)*sqrt(-
b^2 + 4*a*c)) + (b^2*d^2 - a*c*d^2 + b*c*d*e + c^2*e^2)*log(abs(x))/(c^3*d^
3) - 1/2*(c^2*d^2 - 2*(b*c*d^2 + c^2*d*e)*x)/(c^3*d^3*x^2)
```

3.70 $\int \frac{x^3}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx$

Optimal. Leaf size=343

$$\frac{\left(-b^2c\left(3ad^2-ce^2\right)+4abc^2de+ac^2\left(ad^2-ce^2\right)-2b^3cde+b^4d^2\right)\log\left(ax^2+bx+c\right)}{2a^3\left(ad^2-e(bd-ce)\right)^2}+\frac{\left(-4a^2c^3de+8ab^2c^2de-b^3c\left(5ad^2\right.\right.}{a}$$

```
[Out] -(((2*a*d + b*e)*x)/(a^2*e^3)) + x^2/(2*a*e^2) + d^5/(e^4*(a*d^2 - e*(b*d -
 c*e))*(d + e*x)) + ((b^5*d^2 - 2*b^4*c*d*e + 8*a*b^2*c^2*d*e - 4*a^2*c^3*d
*e + a*b*c^2*(5*a*d^2 - 3*c*e^2) - b^3*c*(5*a*d^2 - c*e^2))*ArcTanh[(b + 2*
a*x)/Sqrt[b^2 - 4*a*c]])/(a^3*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))^2)
+ (d^4*(3*a*d^2 - e*(4*b*d - 5*c*e))*Log[d + e*x])/(e^4*(a*d^2 - e*(b*d - c
*e))^2) + ((b^4*d^2 - 2*b^3*c*d*e + 4*a*b*c^2*d*e + a*c^2*(a*d^2 - c*e^2) -
 b^2*c*(3*a*d^2 - c*e^2))*Log[c + b*x + a*x^2])/(2*a^3*(a*d^2 - e*(b*d - c*
e))^2)
```

Rubi [A] time = 0.907376, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1569, 1628, 634, 618, 206, 628}

$$\frac{\left(-b^2c\left(3ad^2-ce^2\right)+4abc^2de+ac^2\left(ad^2-ce^2\right)-2b^3cde+b^4d^2\right)\log\left(ax^2+bx+c\right)}{2a^3\left(ad^2-e(bd-ce)\right)^2}+\frac{\left(-4a^2c^3de+8ab^2c^2de-b^3c\left(5ad^2\right.\right.}{a}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```

```
[Out] -(((2*a*d + b*e)*x)/(a^2*e^3)) + x^2/(2*a*e^2) + d^5/(e^4*(a*d^2 - e*(b*d -
 c*e))*(d + e*x)) + ((b^5*d^2 - 2*b^4*c*d*e + 8*a*b^2*c^2*d*e - 4*a^2*c^3*d
*e + a*b*c^2*(5*a*d^2 - 3*c*e^2) - b^3*c*(5*a*d^2 - c*e^2))*ArcTanh[(b + 2*
a*x)/Sqrt[b^2 - 4*a*c]])/(a^3*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))^2)
+ (d^4*(3*a*d^2 - e*(4*b*d - 5*c*e))*Log[d + e*x])/(e^4*(a*d^2 - e*(b*d - c
*e))^2) + ((b^4*d^2 - 2*b^3*c*d*e + 4*a*b*c^2*d*e + a*c^2*(a*d^2 - c*e^2) -
 b^2*c*(3*a*d^2 - c*e^2))*Log[c + b*x + a*x^2])/(2*a^3*(a*d^2 - e*(b*d - c*
e))^2)
```

Rule 1569

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^3}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx = \int \frac{x^5}{(d+ex)^2\left(c+bx+ax^2\right)}\,dx$$

$$= \int \left(\frac{-2ad-be}{a^2e^3} + \frac{x}{ae^2} + \frac{d^5}{e^3\left(-ad^2+e(bd-ce)\right)(d+ex)^2} + \frac{d^4\left(3ad^2-e(4bd-5ce)\right)}{e^3\left(ad^2-e(bd-ce)\right)^2(d+ex)}\right.$$

$$= -\frac{(2ad+be)x}{a^2e^3} + \frac{x^2}{2ae^2} + \frac{d^5}{e^4\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{d^4\left(3ad^2-e(4bd-5ce)\right)\log(d+}{e^4\left(ad^2-e(bd-ce)\right)^2}$$

$$= -\frac{(2ad+be)x}{a^2e^3} + \frac{x^2}{2ae^2} + \frac{d^5}{e^4\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{d^4\left(3ad^2-e(4bd-5ce)\right)\log(d+}{e^4\left(ad^2-e(bd-ce)\right)^2}$$

$$= -\frac{(2ad+be)x}{a^2e^3} + \frac{x^2}{2ae^2} + \frac{d^5}{e^4\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{d^4\left(3ad^2-e(4bd-5ce)\right)\log(d+}{e^4\left(ad^2-e(bd-ce)\right)^2}$$

$$= -\frac{(2ad+be)x}{a^2e^3} + \frac{x^2}{2ae^2} + \frac{d^5}{e^4\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{\left(b^5d^2-2b^4cde+8ab^2c^2de-4a^2\right.}{}$$

Mathematica [A] time = 0.379752, size = 338, normalized size = 0.99

$$\frac{\left(b^2c\left(ce^2-3ad^2\right)+4abc^2de+ac^2\left(ad^2-ce^2\right)-2b^3cde+b^4d^2\right)\log(x(ax+b)+c)}{2a^3\left(ad^2+e(ce-bd)\right)^2} - \frac{\left(-4a^2c^3de+8ab^2c^2de+b^3c\left(ce^2-\right.\right.}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```

```
[Out] -((((2*a*d + b*e)*x)/(a^2*e^3)) + x^2/(2*a*e^2) + d^5/(e^4*(a*d^2 + e*(-(b*d
) + c*e))*(d + e*x)) - ((b^5*d^2 - 2*b^4*c*d*e + 8*a*b^2*c^2*d*e - 4*a^2*c^
3*d*e + a*b*c^2*(5*a*d^2 - 3*c*e^2) + b^3*c*(-5*a*d^2 + c*e^2))*ArcTan[(b +
 2*a*x)/Sqrt[-b^2 + 4*a*c]])/(a^3*Sqrt[-b^2 + 4*a*c]*(a*d^2 + e*(-(b*d) + c
*e))^2) + ((3*a*d^6 + d^4*e*(-4*b*d + 5*c*e))*Log[d + e*x])/(e^4*(a*d^2 + e
*(-(b*d) + c*e))^2) + ((b^4*d^2 - 2*b^3*c*d*e + 4*a*b*c^2*d*e + a*c^2*(a*d^
2 - c*e^2) + b^2*c*(-3*a*d^2 + c*e^2))*Log[c + x*(b + a*x)])/(2*a^3*(a*d^2
+ e*(-(b*d) + c*e))^2)
```

Maple [B] time = 0.012, size = 943, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+c/x^2+b/x)/(e*x+d)^2,x)
```

```
[Out] 3/e^4*d^6/(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d)*a-4/e^3*d^5/(a*d^2-b*d*e+c*e^2)^2
*ln(e*x+d)*b+5/e^2*d^4/(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d)*c+1/e^4*d^5/(a*d^2-b
*d*e+c*e^2)/(e*x+d)+1/2*x^2/a/e^2-2/a/e^3*x*d-1/a^2/e^2*b*x+1/2/(a*d^2-b*d*
e+c*e^2)^2/a*ln(a*x^2+b*x+c)*c^2*d^2-3/2/(a*d^2-b*d*e+c*e^2)^2/a^2*ln(a*x^2
+b*x+c)*b^2*c*d^2+2/(a*d^2-b*d*e+c*e^2)^2/a^2*ln(a*x^2+b*x+c)*b*c^2*d*e-1/2
/(a*d^2-b*d*e+c*e^2)^2/a^2*ln(a*x^2+b*x+c)*c^3*e^2+1/2/(a*d^2-b*d*e+c*e^2)^
2/a^3*ln(a*x^2+b*x+c)*b^4*d^2-1/(a*d^2-b*d*e+c*e^2)^2/a^3*ln(a*x^2+b*x+c)*b
^3*c*d*e+1/2/(a*d^2-b*d*e+c*e^2)^2/a^3*ln(a*x^2+b*x+c)*b^2*c^2*e^2-5/(a*d^2
-b*d*e+c*e^2)^2/a/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b*c
^2*d^2+4/(a*d^2-b*d*e+c*e^2)^2/a/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-
b^2)^(1/2))*c^3*d*e+5/(a*d^2-b*d*e+c*e^2)^2/a^2/(4*a*c-b^2)^(1/2)*arctan((2
*a*x+b)/(4*a*c-b^2)^(1/2))*b^3*c*d^2-8/(a*d^2-b*d*e+c*e^2)^2/a^2/(4*a*c-b^2
)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^2*c^2*d*e+3/(a*d^2-b*d*e+c*e^
2)^2/a^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b*c^3*e^2-1/
(a*d^2-b*d*e+c*e^2)^2/a^3/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1
/2))*b^5*d^2+2/(a*d^2-b*d*e+c*e^2)^2/a^3/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)
/(4*a*c-b^2)^(1/2))*b^4*c*d*e-1/(a*d^2-b*d*e+c*e^2)^2/a^3/(4*a*c-b^2)^(1/2)
*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^3*c^2*e^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+c/x**2+b/x)/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13396, size = 763, normalized size = 2.22

$$\frac{d^5e^4}{\left(ad^2e^8 - bde^9 + ce^{10}\right)(xe + d)} + \frac{\left(b^5d^2e^2 - 5\,ab^3cd^2e^2 + 5\,a^2bc^2d^2e^2 - 2\,b^4cde^3 + 8\,ab^2c^2de^3 - 4\,a^2c^3de^3 + b^3c^2e^4 - 3\,abc\right.}{\left(a^5d^4 - 2\,a^4bd^3e + a^3b^2d^2e^2 + 2\,a^4cd^2e^2 - 2\,a^3bcde^3 + a^3c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] d^5*e^4/((a*d^2*e^8 - b*d*e^9 + c*e^10)*(x*e + d)) + (b^5*d^2*e^2 - 5*a*b^3
*c*d^2*e^2 + 5*a^2*b*c^2*d^2*e^2 - 2*b^4*c*d*e^3 + 8*a*b^2*c^2*d*e^3 - 4*a^
2*c^3*d*e^3 + b^3*c^2*e^4 - 3*a*b*c^3*e^4)*arctan(-(2*a*d - 2*a*d^2/(x*e +
d) - b*e + 2*b*d*e/(x*e + d) - 2*c*e^2/(x*e + d))*e^(-1)/sqrt(-b^2 + 4*a*c)
)*e^(-2)/((a^5*d^4 - 2*a^4*b*d^3*e + a^3*b^2*d^2*e^2 + 2*a^4*c*d^2*e^2 - 2*
a^3*b*c*d*e^3 + a^3*c^2*e^4)*sqrt(-b^2 + 4*a*c)) + 1/2*(a^2 - 2*(3*a^2*d*e
+ a*b*e^2)*e^(-1)/(x*e + d))*(x*e + d)^2*e^(-4)/a^3 + 1/2*(b^4*d^2 - 3*a*b^
2*c*d^2 + a^2*c^2*d^2 - 2*b^3*c*d*e + 4*a*b*c^2*d*e + b^2*c^2*e^2 - a*c^3*e
^2)*log(-a + 2*a*d/(x*e + d) - a*d^2/(x*e + d)^2 - b*e/(x*e + d) + b*d*e/(x
*e + d)^2 - c*e^2/(x*e + d)^2)/(a^5*d^4 - 2*a^4*b*d^3*e + a^3*b^2*d^2*e^2 +
 2*a^4*c*d^2*e^2 - 2*a^3*b*c*d*e^3 + a^3*c^2*e^4) - (3*a^2*d^2 + 2*a*b*d*e
+ b^2*e^2 - a*c*e^2)*e^(-4)*log(abs(x*e + d)*e^(-1)/(x*e + d)^2)/a^3
```

3.71 $\int \frac{x^2}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2} \, dx$

Optimal. Leaf size=274

$$-\frac{\left(-b^2c\left(4ad^2-ce^2\right)+6abc^2de+2ac^2\left(ad^2-ce^2\right)-2b^3cde+b^4d^2\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}-\frac{(bd-ce)\left(-2acd+b^2d-bce\right)\log}{2a^2\left(ad^2-e(bd-ce)\right.}$$

```
[Out] x/(a*e^2) - d^4/(e^3*(a*d^2 - e*(b*d - c*e))*(d + e*x)) - ((b^4*d^2 - 2*b^3
*c*d*e + 6*a*b*c^2*d*e + 2*a*c^2*(a*d^2 - c*e^2) - b^2*c*(4*a*d^2 - c*e^2))
*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(a^2*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*
(b*d - c*e))^2) - (d^3*(2*a*d^2 - e*(3*b*d - 4*c*e))*Log[d + e*x])/(e^3*(a*
d^2 - e*(b*d - c*e))^2) - ((b*d - c*e)*(b^2*d - 2*a*c*d - b*c*e)*Log[c + b*
x + a*x^2])/(2*a^2*(a*d^2 - e*(b*d - c*e))^2)
```

Rubi [A] time = 0.56307, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1569, 1628, 634, 618, 206, 628}

$$-\frac{\left(-b^2c\left(4ad^2-ce^2\right)+6abc^2de+2ac^2\left(ad^2-ce^2\right)-2b^3cde+b^4d^2\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}-\frac{(bd-ce)\left(-2acd+b^2d-bce\right)\log}{2a^2\left(ad^2-e(bd-ce)\right.}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```



```
[Out] x/(a*e^2) - d^4/(e^3*(a*d^2 - e*(b*d - c*e))*(d + e*x)) - ((b^4*d^2 - 2*b^3
*c*d*e + 6*a*b*c^2*d*e + 2*a*c^2*(a*d^2 - c*e^2) - b^2*c*(4*a*d^2 - c*e^2))
*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(a^2*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*
(b*d - c*e))^2) - (d^3*(2*a*d^2 - e*(3*b*d - 4*c*e))*Log[d + e*x])/(e^3*(a*
d^2 - e*(b*d - c*e))^2) - ((b*d - c*e)*(b^2*d - 2*a*c*d - b*c*e)*Log[c + b*
x + a*x^2])/(2*a^2*(a*d^2 - e*(b*d - c*e))^2)
```


Rule 1569

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
```

```
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^2}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx = \int \frac{x^4}{(d+ex)^2\left(c+bx+ax^2\right)}\,dx$$

$$= \int \left(\frac{1}{ae^2} + \frac{d^4}{e^2\left(ad^2-e(bd-ce)\right)(d+ex)^2} + \frac{d^3\left(-2ad^2+e(3bd-4ce)\right)}{e^2\left(ad^2-e(bd-ce)\right)^2(d+ex)} + \frac{-c\left(b^2d^2-2b\right.}{}\right.$$

$$= \frac{x}{ae^2} - \frac{d^4}{e^3\left(ad^2-e(bd-ce)\right)(d+ex)} - \frac{d^3\left(2ad^2-e(3bd-4ce)\right)\log(d+ex)}{e^3\left(ad^2-e(bd-ce)\right)^2} + \int \frac{-c\left(b^2d^2-2bc\right.}{}$$

$$= \frac{x}{ae^2} - \frac{d^4}{e^3\left(ad^2-e(bd-ce)\right)(d+ex)} - \frac{d^3\left(2ad^2-e(3bd-4ce)\right)\log(d+ex)}{e^3\left(ad^2-e(bd-ce)\right)^2} - \frac{\left((bd-ce)\left(b^2\right.\right.}{2a}$$

$$= \frac{x}{ae^2} - \frac{d^4}{e^3\left(ad^2-e(bd-ce)\right)(d+ex)} - \frac{d^3\left(2ad^2-e(3bd-4ce)\right)\log(d+ex)}{e^3\left(ad^2-e(bd-ce)\right)^2} - \frac{(bd-ce)\left(b^2\right.}{2}$$

$$= \frac{x}{ae^2} - \frac{d^4}{e^3\left(ad^2-e(bd-ce)\right)(d+ex)} - \frac{\left(b^4d^2-2b^3cde+6abc^2de+2ac^2\left(ad^2-ce^2\right)-b^2c\left(\right.\right.}{a^2\sqrt{b^2-4ac}\left(ad^2-e(bd-\right.}$$

Mathematica [A] time = 0.315942, size = 269, normalized size = 0.98

$$\frac{\left(b^2c\left(ce^2-4ad^2\right)+6abc^2de+2ac^2\left(ad^2-ce^2\right)-2b^3cde+b^4d^2\right)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{a^2\sqrt{4ac-b^2}\left(ad^2+e(ce-bd)\right)^2} + \frac{(bd-ce)\left(2acd+b^2(-d)+bce\right)\log(x(}{2a^2\left(ad^2+e(ce-bd)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```

```
[Out] x/(a*e^2) - d^4/(e^3*(a*d^2 + e*(-(b*d) + c*e))*(d + e*x)) + ((b^4*d^2 - 2*
b^3*c*d*e + 6*a*b*c^2*d*e + 2*a*c^2*(a*d^2 - c*e^2) + b^2*c*(-4*a*d^2 + c*e
^2))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/(a^2*Sqrt[-b^2 + 4*a*c]*(a*d^2
 + e*(-(b*d) + c*e))^2) - ((2*a*d^5 + d^3*e*(-3*b*d + 4*c*e))*Log[d + e*x])
/(e^3*(a*d^2 + e*(-(b*d) + c*e))^2) + ((b*d - c*e)*(-(b^2*d) + 2*a*c*d + b*
c*e)*Log[c + x*(b + a*x)])/(2*a^2*(a*d^2 + e*(-(b*d) + c*e))^2)
```

Maple [B] time = 0.012, size = 765, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+c/x^2+b/x)/(e*x+d)^2,x)
```

```
[Out] -1/e^3*d^4/(a*d^2-b*d*e+c*e^2)/(e*x+d)-2/e^3*d^5/(a*d^2-b*d*e+c*e^2)^2*ln(e
*x+d)*a+3/e^2*d^4/(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d)*b-4/e*d^3/(a*d^2-b*d*e+c*
e^2)^2*ln(e*x+d)*c+x/a/e^2+1/(a*d^2-b*d*e+c*e^2)^2/a*ln(a*x^2+b*x+c)*b*c*d^
2-1/(a*d^2-b*d*e+c*e^2)^2/a*ln(a*x^2+b*x+c)*c^2*d*e-1/2/(a*d^2-b*d*e+c*e^2)
^2/a^2*ln(a*x^2+b*x+c)*b^3*d^2+1/(a*d^2-b*d*e+c*e^2)^2/a^2*ln(a*x^2+b*x+c)*
b^2*c*d*e-1/2/(a*d^2-b*d*e+c*e^2)^2/a^2*ln(a*x^2+b*x+c)*b*c^2*e^2+2/(a*d^2-
b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*c^2*d^
2-4/(a*d^2-b*d*e+c*e^2)^2/a/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^
(1/2))*b^2*c*d^2+6/(a*d^2-b*d*e+c*e^2)^2/a/(4*a*c-b^2)^(1/2)*arctan((2*a*x+
b)/(4*a*c-b^2)^(1/2))*b*c^2*d*e-2/(a*d^2-b*d*e+c*e^2)^2/a/(4*a*c-b^2)^(1/2)
*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*c^3*e^2+1/(a*d^2-b*d*e+c*e^2)^2/a^2/(4
*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^4*d^2-2/(a*d^2-b*d*e+
c*e^2)^2/a^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^3*c*d*
e+1/(a*d^2-b*d*e+c*e^2)^2/a^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2
)^(1/2))*b^2*c^2*e^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+c/x**2+b/x)/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13066, size = 643, normalized size = 2.35

$$-\frac{d^4e^3}{\left(ad^2e^6-bde^7+ce^8\right)(xe+d)}-\frac{\left(b^4d^2e^2-4\,ab^2cd^2e^2+2\,a^2c^2d^2e^2-2\,b^3cde^3+6\,abc^2de^3+b^2c^2e^4-2\,ac^3e^4\right)\arctan\left(-\frac{(2}{}\right.}{\left(a^4d^4-2\,a^3bd^3e+a^2b^2d^2e^2+2\,a^3cd^2e^2-2\,a^2bcde^3+a^2c^2e^4\right)\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -d^4*e^3/((a*d^2*e^6 - b*d*e^7 + c*e^8)*(x*e + d)) - (b^4*d^2*e^2 - 4*a*b^2
*c*d^2*e^2 + 2*a^2*c^2*d^2*e^2 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3 + b^2*c^2*
e^4 - 2*a*c^3*e^4)*arctan(-(2*a*d - 2*a*d^2/(x*e + d) - b*e + 2*b*d*e/(x*e
+ d) - 2*c*e^2/(x*e + d))*e^(-1)/sqrt(-b^2 + 4*a*c))*e^(-2)/((a^4*d^4 - 2*a
^3*b*d^3*e + a^2*b^2*d^2*e^2 + 2*a^3*c*d^2*e^2 - 2*a^2*b*c*d*e^3 + a^2*c^2*
e^4)*sqrt(-b^2 + 4*a*c)) + (x*e + d)*e^(-3)/a - 1/2*(b^3*d^2 - 2*a*b*c*d^2
- 2*b^2*c*d*e + 2*a*c^2*d*e + b*c^2*e^2)*log(-a + 2*a*d/(x*e + d) - a*d^2/(
x*e + d)^2 - b*e/(x*e + d) + b*d*e/(x*e + d)^2 - c*e^2/(x*e + d)^2)/(a^4*d^
4 - 2*a^3*b*d^3*e + a^2*b^2*d^2*e^2 + 2*a^3*c*d^2*e^2 - 2*a^2*b*c*d*e^3 + a
^2*c^2*e^4) + (2*a*d + b*e)*e^(-3)*log(abs(x*e + d)*e^(-1)/(x*e + d)^2)/a^2
```

3.72 $\int \frac{x}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx$

Optimal. Leaf size=246

$$\frac{\left(-bc\left(3ad^2-ce^2\right)+4ac^2de-2b^2cde+b^3d^2\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}+\frac{\left(-c\left(ad^2-ce^2\right)+b^2d^2-2bcde\right)\log\left(ax^2+bx+c\right)}{2a\left(ad^2-e(bd-ce)\right)^2}+$$

```
[Out] d^3/(e^2*(a*d^2 - e*(b*d - c*e))*(d + e*x)) + ((b^3*d^2 - 2*b^2*c*d*e + 4*a
*c^2*d*e - b*c*(3*a*d^2 - c*e^2))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(
a*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))^2) + (d^2*(a*d^2 - e*(2*b*d - 3
*c*e))*Log[d + e*x])/(e^2*(a*d^2 - e*(b*d - c*e))^2) + ((b^2*d^2 - 2*b*c*d*
e - c*(a*d^2 - c*e^2))*Log[c + b*x + a*x^2])/(2*a*(a*d^2 - e*(b*d - c*e))^2
)
```

Rubi [A] time = 0.395346, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {1569, 1628, 634, 618, 206, 628}

$$\frac{\left(-bc\left(3ad^2-ce^2\right)+4ac^2de-2b^2cde+b^3d^2\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}+\frac{\left(-c\left(ad^2-ce^2\right)+b^2d^2-2bcde\right)\log\left(ax^2+bx+c\right)}{2a\left(ad^2-e(bd-ce)\right)^2}+$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```


```
[Out] d^3/(e^2*(a*d^2 - e*(b*d - c*e))*(d + e*x)) + ((b^3*d^2 - 2*b^2*c*d*e + 4*a
*c^2*d*e - b*c*(3*a*d^2 - c*e^2))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(
a*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))^2) + (d^2*(a*d^2 - e*(2*b*d - 3
*c*e))*Log[d + e*x])/(e^2*(a*d^2 - e*(b*d - c*e))^2) + ((b^2*d^2 - 2*b*c*d*
e - c*(a*d^2 - c*e^2))*Log[c + b*x + a*x^2])/(2*a*(a*d^2 - e*(b*d - c*e))^2
)
```


Rule 1569

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx = \int \frac{x^3}{(d+ex)^2\left(c+bx+ax^2\right)}\,dx$$

$$= \int \left(\frac{d^3}{e\left(-ad^2+e(bd-ce)\right)(d+ex)^2} + \frac{d^2\left(ad^2-e(2bd-3ce)\right)}{e\left(ad^2-e(bd-ce)\right)^2(d+ex)} + \frac{cd(bd-2ce)+\left(b^2 \cdots\right.}{\left(ad^2-e(bd \cdots\right.} \right.$$

$$= \frac{d^3}{e^2\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{d^2\left(ad^2-e(2bd-3ce)\right)\log(d+ex)}{e^2\left(ad^2-e(bd-ce)\right)^2} + \frac{\int \frac{cd(bd-2ce)+\left(b^2d^2-2 \cdots\right.}{c+bx+ \cdots}}{\left(ad^2-e(b \cdots\right.}$$

$$= \frac{d^3}{e^2\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{d^2\left(ad^2-e(2bd-3ce)\right)\log(d+ex)}{e^2\left(ad^2-e(bd-ce)\right)^2} + \frac{\left(b^2d^2-2bcde-c \cdots\right.}{2a\left(ad^2 \cdots\right.}$$

$$= \frac{d^3}{e^2\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{d^2\left(ad^2-e(2bd-3ce)\right)\log(d+ex)}{e^2\left(ad^2-e(bd-ce)\right)^2} + \frac{\left(b^2d^2-2bcde-c \cdots\right.}{2a\left(a \cdots\right.}$$

$$= \frac{d^3}{e^2\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{\left(b^3d^2-2b^2cde+4ac^2de-bc\left(3ad^2-ce^2\right)\right)\tanh^{-1}\left(\frac{b+2 \cdots}{\sqrt{b^2- \cdots}}\right.}{a\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}$$

Mathematica [A] time = 0.248042, size = 207, normalized size = 0.84

$$\frac{-\frac{2\left(bc\left(ce^2-3ad^2\right)+4ac^2de-2b^2cde+b^3d^2\right)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{a\sqrt{4ac-b^2}} + \frac{\left(c\left(ce^2-ad^2\right)+b^2d^2-2bcde\right)\log(x(ax+b)+c)}{a} + \frac{2d^3\left(ad^2+e(ce-bd)\right)}{e^2(d+ex)} + \frac{2\log(d+ex)\left(ad^4+d^2e(3ce- \cdots\right.}{e^2}}{2\left(ad^2+e(ce-bd)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```

```
[Out] ((2*d^3*(a*d^2 + e*(-(b*d) + c*e)))/(e^2*(d + e*x)) - (2*(b^3*d^2 - 2*b^2*c
*d*e + 4*a*c^2*d*e + b*c*(-3*a*d^2 + c*e^2))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 +
 4*a*c]])/(a*Sqrt[-b^2 + 4*a*c]) + (2*(a*d^4 + d^2*e*(-2*b*d + 3*c*e))*Log[
d + e*x])/e^2 + ((b^2*d^2 - 2*b*c*d*e + c*(-(a*d^2) + c*e^2))*Log[c + x*(b
+ a*x)])/a)/(2*(a*d^2 + e*(-(b*d) + c*e))^2)
```

Maple [B] time = 0.007, size = 580, normalized size = 2.4

$$\frac{d^4 \ln(ex+d)\, a}{e^2 \left(ad^2 - bde + e^2c\right)^2} - 2\,\frac{d^3 \ln(ex+d)\, b}{\left(ad^2 - bde + e^2c\right)^2 e} + 3\,\frac{d^2 \ln(ex+d)\, c}{\left(ad^2 - bde + e^2c\right)^2} + \frac{d^3}{e^2 \left(ad^2 - bde + e^2c\right)(ex+d)} - \frac{\ln\left(ax^2 + bx + c\right) c}{2 \left(ad^2 - bde + e^2c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+c/x^2+b/x)/(e*x+d)^2,x)
```

```
[Out] d^4/(a*d^2-b*d*e+c*e^2)^2/e^2*ln(e*x+d)*a-2*d^3/(a*d^2-b*d*e+c*e^2)^2/e*ln(
e*x+d)*b+3*d^2/(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d)*c+d^3/e^2/(a*d^2-b*d*e+c*e^2
)/(e*x+d)-1/2/(a*d^2-b*d*e+c*e^2)^2*ln(a*x^2+b*x+c)*c*d^2+1/2/(a*d^2-b*d*e+
c*e^2)^2/a*ln(a*x^2+b*x+c)*b^2*d^2-1/(a*d^2-b*d*e+c*e^2)^2/a*ln(a*x^2+b*x+c
)*b*c*d*e+1/2/(a*d^2-b*d*e+c*e^2)^2/a*ln(a*x^2+b*x+c)*c^2*e^2+3/(a*d^2-b*d*
e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b*c*d^2-4/
(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))
*c^2*d*e-1/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-
b^2)^(1/2))*b^3/a*d^2+2/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a
*x+b)/(4*a*c-b^2)^(1/2))*b^2/a*c*d*e-1/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1
/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b/a*c^2*e^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 110.084, size = 3005, normalized size = 12.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a^2*b^2 - 4*a^3*c)*d^5 - 2*(a*b^3 - 4*a^2*b*c)*d^4*e + 2*(a*b^2*c
- 4*a^2*c^2)*d^3*e^2 + (b*c^2*d*e^4 + (b^3 - 3*a*b*c)*d^3*e^2 - 2*(b^2*c -
2*a*c^2)*d^2*e^3 + (b*c^2*e^5 + (b^3 - 3*a*b*c)*d^2*e^3 - 2*(b^2*c - 2*a*c^
2)*d*e^4)*x)*sqrt(b^2 - 4*a*c)*log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c + sqr
t(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x + c)) + ((b^4 - 5*a*b^2*c + 4*a^2*
c^2)*d^3*e^2 - 2*(b^3*c - 4*a*b*c^2)*d^2*e^3 + (b^2*c^2 - 4*a*c^3)*d*e^4 +
((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^2*e^3 - 2*(b^3*c - 4*a*b*c^2)*d*e^4 + (b^2
*c^2 - 4*a*c^3)*e^5)*x)*log(a*x^2 + b*x + c) + 2*((a^2*b^2 - 4*a^3*c)*d^5 -
 2*(a*b^3 - 4*a^2*b*c)*d^4*e + 3*(a*b^2*c - 4*a^2*c^2)*d^3*e^2 + ((a^2*b^2
- 4*a^3*c)*d^4*e - 2*(a*b^3 - 4*a^2*b*c)*d^3*e^2 + 3*(a*b^2*c - 4*a^2*c^2)*
d^2*e^3)*x)*log(e*x + d))/((a^3*b^2 - 4*a^4*c)*d^5*e^2 - 2*(a^2*b^3 - 4*a^3
*b*c)*d^4*e^3 + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^3*e^4 - 2*(a*b^3*c - 4*
a^2*b*c^2)*d^2*e^5 + (a*b^2*c^2 - 4*a^2*c^3)*d*e^6 + ((a^3*b^2 - 4*a^4*c)*d
^4*e^3 - 2*(a^2*b^3 - 4*a^3*b*c)*d^3*e^4 + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2
)*d^2*e^5 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^6 + (a*b^2*c^2 - 4*a^2*c^3)*e^7)*
x), 1/2*(2*(a^2*b^2 - 4*a^3*c)*d^5 - 2*(a*b^3 - 4*a^2*b*c)*d^4*e + 2*(a*b^2
*c - 4*a^2*c^2)*d^3*e^2 + 2*(b*c^2*d*e^4 + (b^3 - 3*a*b*c)*d^3*e^2 - 2*(b^2
*c - 2*a*c^2)*d^2*e^3 + (b*c^2*e^5 + (b^3 - 3*a*b*c)*d^2*e^3 - 2*(b^2*c - 2
*a*c^2)*d*e^4)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b)
/(b^2 - 4*a*c)) + ((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^3*e^2 - 2*(b^3*c - 4*a*b
*c^2)*d^2*e^3 + (b^2*c^2 - 4*a*c^3)*d*e^4 + ((b^4 - 5*a*b^2*c + 4*a^2*c^2)*
d^2*e^3 - 2*(b^3*c - 4*a*b*c^2)*d*e^4 + (b^2*c^2 - 4*a*c^3)*e^5)*x)*log(a*x
^2 + b*x + c) + 2*((a^2*b^2 - 4*a^3*c)*d^5 - 2*(a*b^3 - 4*a^2*b*c)*d^4*e +
3*(a*b^2*c - 4*a^2*c^2)*d^3*e^2 + ((a^2*b^2 - 4*a^3*c)*d^4*e - 2*(a*b^3 - 4
*a^2*b*c)*d^3*e^2 + 3*(a*b^2*c - 4*a^2*c^2)*d^2*e^3)*x)*log(e*x + d))/((a^3
*b^2 - 4*a^4*c)*d^5*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d^4*e^3 + (a*b^4 - 2*a^2*
b^2*c - 8*a^3*c^2)*d^3*e^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^2*e^5 + (a*b^2*c^2
 - 4*a^2*c^3)*d*e^6 + ((a^3*b^2 - 4*a^4*c)*d^4*e^3 - 2*(a^2*b^3 - 4*a^3*b*c
)*d^3*e^4 + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^5 - 2*(a*b^3*c - 4*a^2*
b*c^2)*d*e^6 + (a*b^2*c^2 - 4*a^2*c^3)*e^7)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+c/x**2+b/x)/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11668, size = 556, normalized size = 2.26

$$\frac{1}{2}\left(\frac{2\,d^3e^2}{\left(ad^2e^3-bde^4+ce^5\right)(xe+d)}+\frac{2\left(b^3d^2e^3-3\,abcd^2e^3-2\,b^2cde^4+4\,ac^2de^4+bc^2e^5\right)\arctan\left(-\frac{\left(2\,ad-\frac{2\,ad^2}{xe+d}-be+\frac{2\,bde}{xe+d}-\frac{2\,ce^2}{xe+d}\right)e^{(-}}{\sqrt{-b^2+4\,ac}}\right.}{\left(a^3d^4-2\,a^2bd^3e+ab^2d^2e^2+2\,a^2cd^2e^2-2\,abcde^3+ac^2e^4\right)\sqrt{-b^2+4\,ac}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*d^3*e^2/((a*d^2*e^3 - b*d*e^4 + c*e^5)*(x*e + d)) + 2*(b^3*d^2*e^3 -
 3*a*b*c*d^2*e^3 - 2*b^2*c*d*e^4 + 4*a*c^2*d*e^4 + b*c^2*e^5)*arctan(-(2*a*
d - 2*a*d^2/(x*e + d) - b*e + 2*b*d*e/(x*e + d) - 2*c*e^2/(x*e + d))*e^(-1)
/sqrt(-b^2 + 4*a*c))*e^(-2)/((a^3*d^4 - 2*a^2*b*d^3*e + a*b^2*d^2*e^2 + 2*a
^2*c*d^2*e^2 - 2*a*b*c*d*e^3 + a*c^2*e^4)*sqrt(-b^2 + 4*a*c)) + (b^2*d^2*e
- a*c*d^2*e - 2*b*c*d*e^2 + c^2*e^3)*log(-a + 2*a*d/(x*e + d) - a*d^2/(x*e
+ d)^2 - b*e/(x*e + d) + b*d*e/(x*e + d)^2 - c*e^2/(x*e + d)^2)/(a^3*d^4 -
2*a^2*b*d^3*e + a*b^2*d^2*e^2 + 2*a^2*c*d^2*e^2 - 2*a*b*c*d*e^3 + a*c^2*e^4
) - 2*e^(-1)*log(abs(x*e + d)*e^(-1)/(x*e + d)^2)/a)*e^(-1)
```

3.73 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx$

Optimal. Leaf size=194

$$-\frac{\left(-2c\left(ad^2-ce^2\right)+b^2d^2-2bcde\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}-\frac{d^2}{e(d+ex)\left(ad^2-bde+ce^2\right)}-\frac{d(bd-2ce)\log\left(ax^2+bx+c\right)}{2\left(ad^2-e(bd-ce)\right)^2}+\frac{d}{}$$

```
[Out] -(d^2/(e*(a*d^2 - b*d*e + c*e^2)*(d + e*x))) - ((b^2*d^2 - 2*b*c*d*e - 2*c*
(a*d^2 - c*e^2))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]
*(a*d^2 - e*(b*d - c*e))^2) + (d*(b*d - 2*c*e)*Log[d + e*x])/(a*d^2 - e*(b*
d - c*e))^2 - (d*(b*d - 2*c*e)*Log[c + b*x + a*x^2])/(2*(a*d^2 - e*(b*d - c
*e))^2)
```

Rubi [A] time = 0.306283, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1445, 1628, 634, 618, 206, 628}

$$-\frac{\left(-2c\left(ad^2-ce^2\right)+b^2d^2-2bcde\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}-\frac{d^2}{e(d+ex)\left(ad^2-bde+ce^2\right)}-\frac{d(bd-2ce)\log\left(ax^2+bx+c\right)}{2\left(ad^2-e(bd-ce)\right)^2}+\frac{d}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```


```
[Out] -(d^2/(e*(a*d^2 - b*d*e + c*e^2)*(d + e*x))) - ((b^2*d^2 - 2*b*c*d*e - 2*c*
(a*d^2 - c*e^2))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]
*(a*d^2 - e*(b*d - c*e))^2) + (d*(b*d - 2*c*e)*Log[d + e*x])/(a*d^2 - e*(b*
d - c*e))^2 - (d*(b*d - 2*c*e)*Log[c + b*x + a*x^2])/(2*(a*d^2 - e*(b*d - c
*e))^2)
```


Rule 1445

```
Int[((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_) + (e_.)*(x
_)^(n_.))^(q_.), x_Symbol] :> Int[((d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p)
/x^(2*n*p), x] /; FreeQ[{a, b, c, d, e, n, q}, x] && EqQ[mn, -n] && EqQ[mn2
, 2*mn] && IntegerQ[p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx = \int \frac{x^2}{(d+ex)^2\left(c+bx+ax^2\right)}\,dx$$

$$= \int \left(\frac{d^2}{\left(ad^2-e(bd-ce)\right)(d+ex)^2} + \frac{de(bd-2ce)}{\left(ad^2-e(bd-ce)\right)^2(d+ex)} + \frac{-c\left(ad^2-ce^2\right)-ad(bc}{\left(ad^2-e(bd-ce)\right)^2\left(c+\right.}\right.$$

$$= -\frac{d^2}{e\left(ad^2-bde+ce^2\right)(d+ex)} + \frac{d(bd-2ce)\log(d+ex)}{\left(ad^2-e(bd-ce)\right)^2} + \frac{\int \frac{-c\left(ad^2-ce^2\right)-ad(bd-2ce)x}{c+bx+ax^2}\,dx}{\left(ad^2-e(bd-ce)\right)^2}$$

$$= -\frac{d^2}{e\left(ad^2-bde+ce^2\right)(d+ex)} + \frac{d(bd-2ce)\log(d+ex)}{\left(ad^2-e(bd-ce)\right)^2} - \frac{(d(bd-2ce))\int \frac{b+2ax}{c+bx+ax^2}\,dx}{2\left(ad^2-e(bd-ce)\right)^2} + $$

$$= -\frac{d^2}{e\left(ad^2-bde+ce^2\right)(d+ex)} + \frac{d(bd-2ce)\log(d+ex)}{\left(ad^2-e(bd-ce)\right)^2} - \frac{d(bd-2ce)\log\left(c+bx+ax^2\right)}{2\left(ad^2-e(bd-ce)\right)^2}$$

$$= -\frac{d^2}{e\left(ad^2-bde+ce^2\right)(d+ex)} - \frac{\left(b^2d^2-2bcde-2c\left(ad^2-ce^2\right)\right)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2} + \frac{d(bd}{(a}$$

Mathematica [A] time = 0.239581, size = 159, normalized size = 0.82

$$\frac{\frac{2\left(2c\left(ce^2-ad^2\right)+b^2d^2-2bcde\right)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} - \frac{2d^2\left(ad^2+e(ce-bd)\right)}{e(d+ex)} - d(bd-2ce)\log(x(ax+b)+c) + 2d(bd-2ce)\log(d+ex)}{2\left(ad^2+e(ce-bd)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```

```
[Out] ((-2*d^2*(a*d^2 + e*(-(b*d) + c*e)))/(e*(d + e*x)) + (2*(b^2*d^2 - 2*b*c*d*
e + 2*c*(-(a*d^2) + c*e^2))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b
^2 + 4*a*c] + 2*d*(b*d - 2*c*e)*Log[d + e*x] - d*(b*d - 2*c*e)*Log[c + x*(b
 + a*x)])/(2*(a*d^2 + e*(-(b*d) + c*e))^2)
```

Maple [B] time = 0.008, size = 389, normalized size = 2.

$$-\frac{d^2}{\left(ad^2-bde+e^2c\right)e(ex+d)} + \frac{d^2\ln(ex+d)\,b}{\left(ad^2-bde+e^2c\right)^2} - 2\,\frac{d\ln(ex+d)\,ce}{\left(ad^2-bde+e^2c\right)^2} - \frac{\ln\left(ax^2+bx+c\right)bd^2}{2\left(ad^2-bde+e^2c\right)^2} + \frac{\ln\left(ax^2+bx+c\right)c}{\left(ad^2-bde+e^2c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)/(e*x+d)^2,x)
```

```
[Out] -d^2/e/(a*d^2-b*d*e+c*e^2)/(e*x+d)+d^2/(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d)*b-2*
d/(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d)*c*e-1/2/(a*d^2-b*d*e+c*e^2)^2*ln(a*x^2+b*
x+c)*b*d^2+1/(a*d^2-b*d*e+c*e^2)^2*ln(a*x^2+b*x+c)*c*d*e-2/(a*d^2-b*d*e+c*e
^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a*c*d^2+1/(a*d^
2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^2*
d^2-2/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^
(1/2))*b*c*d*e+2/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(
4*a*c-b^2)^(1/2))*c^2*e^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 36.9767, size = 2365, normalized size = 12.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a*b^2 - 4*a^2*c)*d^4 - 2*(b^3 - 4*a*b*c)*d^3*e + 2*(b^2*c - 4*a*c
^2)*d^2*e^2 + (2*b*c*d^2*e^2 - 2*c^2*d*e^3 - (b^2 - 2*a*c)*d^3*e + (2*b*c*d
*e^3 - 2*c^2*e^4 - (b^2 - 2*a*c)*d^2*e^2)*x)*sqrt(b^2 - 4*a*c)*log((2*a^2*x
^2 + 2*a*b*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x +
c)) + ((b^3 - 4*a*b*c)*d^3*e - 2*(b^2*c - 4*a*c^2)*d^2*e^2 + ((b^3 - 4*a*b*
c)*d^2*e^2 - 2*(b^2*c - 4*a*c^2)*d*e^3)*x)*log(a*x^2 + b*x + c) - 2*((b^3 -
 4*a*b*c)*d^3*e - 2*(b^2*c - 4*a*c^2)*d^2*e^2 + ((b^3 - 4*a*b*c)*d^2*e^2 -
2*(b^2*c - 4*a*c^2)*d*e^3)*x)*log(e*x + d))/((a^2*b^2 - 4*a^3*c)*d^5*e - 2*
```

```
(a*b^3 - 4*a^2*b*c)*d^4*e^2 + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^3*e^3 - 2*(b^
3*c - 4*a*b*c^2)*d^2*e^4 + (b^2*c^2 - 4*a*c^3)*d*e^5 + ((a^2*b^2 - 4*a^3*c)
*d^4*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d^3*e^3 + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^
2*e^4 - 2*(b^3*c - 4*a*b*c^2)*d*e^5 + (b^2*c^2 - 4*a*c^3)*e^6)*x), -1/2*(2*
(a*b^2 - 4*a^2*c)*d^4 - 2*(b^3 - 4*a*b*c)*d^3*e + 2*(b^2*c - 4*a*c^2)*d^2*e
^2 - 2*(2*b*c*d^2*e^2 - 2*c^2*d*e^3 - (b^2 - 2*a*c)*d^3*e + (2*b*c*d*e^3 -
2*c^2*e^4 - (b^2 - 2*a*c)*d^2*e^2)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2
+ 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)) + ((b^3 - 4*a*b*c)*d^3*e - 2*(b^2*c - 4
*a*c^2)*d^2*e^2 + ((b^3 - 4*a*b*c)*d^2*e^2 - 2*(b^2*c - 4*a*c^2)*d*e^3)*x)*
log(a*x^2 + b*x + c) - 2*((b^3 - 4*a*b*c)*d^3*e - 2*(b^2*c - 4*a*c^2)*d^2*e
^2 + ((b^3 - 4*a*b*c)*d^2*e^2 - 2*(b^2*c - 4*a*c^2)*d*e^3)*x)*log(e*x + d))
/((a^2*b^2 - 4*a^3*c)*d^5*e - 2*(a*b^3 - 4*a^2*b*c)*d^4*e^2 + (b^4 - 2*a*b^
2*c - 8*a^2*c^2)*d^3*e^3 - 2*(b^3*c - 4*a*b*c^2)*d^2*e^4 + (b^2*c^2 - 4*a*c
^3)*d*e^5 + ((a^2*b^2 - 4*a^3*c)*d^4*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d^3*e^3 +
(b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^4 - 2*(b^3*c - 4*a*b*c^2)*d*e^5 + (b^2*
c^2 - 4*a*c^3)*e^6)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1218, size = 455, normalized size = 2.35

$$-\frac{\left(b^2d^2e^2 - 2\,acd^2e^2 - 2\,bcde^3 + 2\,c^2e^4\right)\arctan\left(-\frac{\left(2\,ad - \frac{2\,ad^2}{xe+d} - be + \frac{2\,bde}{xe+d} - \frac{2\,ce^2}{xe+d}\right)e^{(-1)}}{\sqrt{-b^2+4\,ac}}\right)e^{(-2)}}{\left(a^2d^4 - 2\,abd^3e + b^2d^2e^2 + 2\,acd^2e^2 - 2\,bcde^3 + c^2e^4\right)\sqrt{-b^2 + 4\,ac}} - \frac{d^2e}{\left(ad^2e^2 - bde^3 + ce^4\right)(xe + d)} - \frac{(bd^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -(b^2*d^2*e^2 - 2*a*c*d^2*e^2 - 2*b*c*d*e^3 + 2*c^2*e^4)*arctan(-(2*a*d - 2
*a*d^2/(x*e + d) - b*e + 2*b*d*e/(x*e + d) - 2*c*e^2/(x*e + d))*e^(-1)/sqrt
```

```
(-b^2 + 4*a*c))*e^(-2)/((a^2*d^4 - 2*a*b*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^
2 - 2*b*c*d*e^3 + c^2*e^4)*sqrt(-b^2 + 4*a*c)) - d^2*e/((a*d^2*e^2 - b*d*e^
3 + c*e^4)*(x*e + d)) - 1/2*(b*d^2 - 2*c*d*e)*log(-a + 2*a*d/(x*e + d) - a*
d^2/(x*e + d)^2 - b*e/(x*e + d) + b*d*e/(x*e + d)^2 - c*e^2/(x*e + d)^2)/(a
^2*d^4 - 2*a*b*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 - 2*b*c*d*e^3 + c^2*e^4)
```

3.74 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x(d+ex)^2}\,dx$

Optimal. Leaf size=183

$$\frac{\left(ad(bd-4ce)+bce^2\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}+\frac{\left(ad^2-ce^2\right)\log\left(ax^2+bx+c\right)}{2\left(ad^2-e(bd-ce)\right)^2}+\frac{d}{(d+ex)\left(ad^2-bde+ce^2\right)}-\frac{\left(ad^2-ce^2\right)\log(}{\left(ad^2-e(bd-\right.}$$

```
[Out] d/((a*d^2 - b*d*e + c*e^2)*(d + e*x)) + ((b*c*e^2 + a*d*(b*d - 4*c*e))*ArcT
anh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*
e))^2) - ((a*d^2 - c*e^2)*Log[d + e*x])/(a*d^2 - e*(b*d - c*e))^2 + ((a*d^2
 - c*e^2)*Log[c + b*x + a*x^2])/(2*(a*d^2 - e*(b*d - c*e))^2)
```

Rubi [A] time = 0.236279, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1569, 800, 634, 618, 206, 628}

$$\frac{\left(ad(bd-4ce)+bce^2\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}+\frac{\left(ad^2-ce^2\right)\log\left(ax^2+bx+c\right)}{2\left(ad^2-e(bd-ce)\right)^2}+\frac{d}{(d+ex)\left(ad^2-bde+ce^2\right)}-\frac{\left(ad^2-ce^2\right)\log(}{\left(ad^2-e(bd-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + c/x^2 + b/x)*x*(d + e*x)^2),x]
```

```
[Out] d/((a*d^2 - b*d*e + c*e^2)*(d + e*x)) + ((b*c*e^2 + a*d*(b*d - 4*c*e))*ArcT
anh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*
e))^2) - ((a*d^2 - c*e^2)*Log[d + e*x])/(a*d^2 - e*(b*d - c*e))^2 + ((a*d^2
 - c*e^2)*Log[c + b*x + a*x^2])/(2*(a*d^2 - e*(b*d - c*e))^2)
```

Rule 1569

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
```

```
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x(d+ex)^2}\,dx = \int \frac{x}{(d+ex)^2\left(c+bx+ax^2\right)}\,dx$$

$$= \int \left(\frac{de}{\left(-ad^2+e(bd-ce)\right)(d+ex)^2} + \frac{e\left(-ad^2+ce^2\right)}{\left(ad^2-e(bd-ce)\right)^2(d+ex)} + \frac{ce(2ad-be)+a\left(a\right.}{\left(ad^2-e(bd-ce)\right)^2\left(\right.}\right.$$

$$= \frac{d}{\left(ad^2-bde+ce^2\right)(d+ex)} - \frac{\left(ad^2-ce^2\right)\log(d+ex)}{\left(ad^2-e(bd-ce)\right)^2} + \frac{\int \frac{ce(2ad-be)+a\left(ad^2-ce^2\right)x}{c+bx+ax^2}\,dx}{\left(ad^2-e(bd-ce)\right)^2}$$

$$= \frac{d}{\left(ad^2-bde+ce^2\right)(d+ex)} - \frac{\left(ad^2-ce^2\right)\log(d+ex)}{\left(ad^2-e(bd-ce)\right)^2} + \frac{\left(ad^2-ce^2\right)\int \frac{b+2ax}{c+bx+ax^2}\,dx}{2\left(ad^2-e(bd-ce)\right)^2} - \frac{\left(bce\right.}{}$$

$$= \frac{d}{\left(ad^2-bde+ce^2\right)(d+ex)} - \frac{\left(ad^2-ce^2\right)\log(d+ex)}{\left(ad^2-e(bd-ce)\right)^2} + \frac{\left(ad^2-ce^2\right)\log\left(c+bx+ax^2\right)}{2\left(ad^2-e(bd-ce)\right)^2} +$$

$$= \frac{d}{\left(ad^2-bde+ce^2\right)(d+ex)} + \frac{\left(bce^2+ad(bd-4ce)\right)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2} - \frac{\left(ad^2-ce^2\right)\log(}{\left(ad^2-e(bd-\right.}$$

Mathematica [A] time = 0.262249, size = 148, normalized size = 0.81

$$\frac{-\frac{2\left(ad(bd-4ce)+bce^2\right)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} + \left(ad^2-ce^2\right)\log(x(ax+b)+c) + \frac{2d\left(ad^2+e(ce-bd)\right)}{d+ex} + \left(2ce^2-2ad^2\right)\log(d+ex)}{2\left(ad^2+e(ce-bd)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + c/x^2 + b/x)*x*(d + e*x)^2),x]
```

```
[Out] ((2*d*(a*d^2 + e*(-(b*d) + c*e)))/(d + e*x) - (2*(b*c*e^2 + a*d*(b*d - 4*c*
e))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (-2*a*d^2
+ 2*c*e^2)*Log[d + e*x] + (a*d^2 - c*e^2)*Log[c + x*(b + a*x)])/(2*(a*d^2 +
 e*(-(b*d) + c*e))^2)
```

Maple [A] time = 0.007, size = 328, normalized size = 1.8

$$\frac{d}{\left(ad^2-bde+e^2c\right)(ex+d)} - \frac{\ln(ex+d)\,ad^2}{\left(ad^2-bde+e^2c\right)^2} + \frac{\ln(ex+d)\,e^2c}{\left(ad^2-bde+e^2c\right)^2} + \frac{a\ln\left(ax^2+bx+c\right)d^2}{2\left(ad^2-bde+e^2c\right)^2} - \frac{\ln\left(ax^2+bx+c\right)ce^2}{2\left(ad^2-bde+e^2c\right)^2} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)/x/(e*x+d)^2,x)
```

```
[Out] d/(a*d^2-b*d*e+c*e^2)/(e*x+d)-1/(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d)*a*d^2+1/(a*
d^2-b*d*e+c*e^2)^2*ln(e*x+d)*e^2*c+1/2/(a*d^2-b*d*e+c*e^2)^2*a*ln(a*x^2+b*x
+c)*d^2-1/2/(a*d^2-b*d*e+c*e^2)^2*ln(a*x^2+b*x+c)*c*e^2-1/(a*d^2-b*d*e+c*e^
2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a*b*d^2+4/(a*d^2
-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a*c*d
*e-1/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(
1/2))*b*c*e^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 31.5593, size = 2217, normalized size = 12.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a*b^2 - 4*a^2*c)*d^3 - 2*(b^3 - 4*a*b*c)*d^2*e + 2*(b^2*c - 4*a*c^
2)*d*e^2 + (a*b*d^3 - 4*a*c*d^2*e + b*c*d*e^2 + (a*b*d^2*e - 4*a*c*d*e^2 +
b*c*e^3)*x)*sqrt(b^2 - 4*a*c)*log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c + sqrt
(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x + c)) + ((a*b^2 - 4*a^2*c)*d^3 - (b
^2*c - 4*a*c^2)*d*e^2 + ((a*b^2 - 4*a^2*c)*d^2*e - (b^2*c - 4*a*c^2)*e^3)*x
)*log(a*x^2 + b*x + c) - 2*((a*b^2 - 4*a^2*c)*d^3 - (b^2*c - 4*a*c^2)*d*e^2
 + ((a*b^2 - 4*a^2*c)*d^2*e - (b^2*c - 4*a*c^2)*e^3)*x)*log(e*x + d))/((a^2
*b^2 - 4*a^3*c)*d^5 - 2*(a*b^3 - 4*a^2*b*c)*d^4*e + (b^4 - 2*a*b^2*c - 8*a^
2*c^2)*d^3*e^2 - 2*(b^3*c - 4*a*b*c^2)*d^2*e^3 + (b^2*c^2 - 4*a*c^3)*d*e^4
```

```
+ ((a^2*b^2 - 4*a^3*c)*d^4*e - 2*(a*b^3 - 4*a^2*b*c)*d^3*e^2 + (b^4 - 2*a*b
^2*c - 8*a^2*c^2)*d^2*e^3 - 2*(b^3*c - 4*a*b*c^2)*d*e^4 + (b^2*c^2 - 4*a*c^
3)*e^5)*x), 1/2*(2*(a*b^2 - 4*a^2*c)*d^3 - 2*(b^3 - 4*a*b*c)*d^2*e + 2*(b^2
*c - 4*a*c^2)*d*e^2 + 2*(a*b*d^3 - 4*a*c*d^2*e + b*c*d*e^2 + (a*b*d^2*e - 4
*a*c*d*e^2 + b*c*e^3)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*a
*x + b)/(b^2 - 4*a*c)) + ((a*b^2 - 4*a^2*c)*d^3 - (b^2*c - 4*a*c^2)*d*e^2 +
 ((a*b^2 - 4*a^2*c)*d^2*e - (b^2*c - 4*a*c^2)*e^3)*x)*log(a*x^2 + b*x + c)
- 2*((a*b^2 - 4*a^2*c)*d^3 - (b^2*c - 4*a*c^2)*d*e^2 + ((a*b^2 - 4*a^2*c)*d
^2*e - (b^2*c - 4*a*c^2)*e^3)*x)*log(e*x + d))/((a^2*b^2 - 4*a^3*c)*d^5 - 2
*(a*b^3 - 4*a^2*b*c)*d^4*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^3*e^2 - 2*(b^3
*c - 4*a*b*c^2)*d^2*e^3 + (b^2*c^2 - 4*a*c^3)*d*e^4 + ((a^2*b^2 - 4*a^3*c)*
d^4*e - 2*(a*b^3 - 4*a^2*b*c)*d^3*e^2 + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e
^3 - 2*(b^3*c - 4*a*b*c^2)*d*e^4 + (b^2*c^2 - 4*a*c^3)*e^5)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)/x/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13598, size = 441, normalized size = 2.41

$$\frac{1}{2}\left(\frac{2\left(abd^2e-4\,acde^2+bce^3\right)\arctan\left(-\frac{\left(2\,ad-\frac{2\,ad^2}{xe+d}-be+\frac{2\,bde}{xe+d}-\frac{2\,ce^2}{xe+d}\right)e^{(-1)}}{\sqrt{-b^2+4\,ac}}\right)e^{(-2)}}{\left(a^2d^4-2\,abd^3e+b^2d^2e^2+2\,acd^2e^2-2\,bcde^3+c^2e^4\right)\sqrt{-b^2+4\,ac}}+\frac{\left(ad^2-ce^2\right)\log\left(-a+\frac{2\,ad}{xe+d}-\frac{ad^2}{(xe+d)^2}-\frac{be}{xe+d}+\right.}{a^2d^4e-2\,abd^3e^2+b^2d^2e^3+2\,acd^2e^3-2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*(a*b*d^2*e - 4*a*c*d*e^2 + b*c*e^3)*arctan(-(2*a*d - 2*a*d^2/(x*e +
d) - b*e + 2*b*d*e/(x*e + d) - 2*c*e^2/(x*e + d))*e^(-1)/sqrt(-b^2 + 4*a*c)
```

```
)*e^(-2)/((a^2*d^4 - 2*a*b*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 - 2*b*c*d*e^
3 + c^2*e^4)*sqrt(-b^2 + 4*a*c)) + (a*d^2 - c*e^2)*log(-a + 2*a*d/(x*e + d)
 - a*d^2/(x*e + d)^2 - b*e/(x*e + d) + b*d*e/(x*e + d)^2 - c*e^2/(x*e + d)^
2)/(a^2*d^4*e - 2*a*b*d^3*e^2 + b^2*d^2*e^3 + 2*a*c*d^2*e^3 - 2*b*c*d*e^4 +
 c^2*e^5) + 2*d*e/((a*d^2*e^2 - b*d*e^3 + c*e^4)*(x*e + d)))*e
```

3.75 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^2(d+ex)^2}\,dx$

Optimal. Leaf size=189

$$-\frac{\left(2a^2d^2-2ae(bd+ce)+b^2e^2\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}-\frac{e}{(d+ex)\left(ad^2-bde+ce^2\right)}-\frac{e(2ad-be)\log\left(ax^2+bx+c\right)}{2\left(ad^2-e(bd-ce)\right)^2}+\frac{e(2ad}{(ad^2}$$

```
[Out] -(e/((a*d^2 - b*d*e + c*e^2)*(d + e*x))) - ((2*a^2*d^2 + b^2*e^2 - 2*a*e*(b
*d + c*e))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(a*d^
2 - e*(b*d - c*e))^2) + (e*(2*a*d - b*e)*Log[d + e*x])/(a*d^2 - e*(b*d - c*
e))^2 - (e*(2*a*d - b*e)*Log[c + b*x + a*x^2])/(2*(a*d^2 - e*(b*d - c*e))^2
)
```

Rubi [A] time = 0.305199, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {1569, 709, 800, 634, 618, 206, 628}

$$-\frac{\left(2a^2d^2-2ae(bd+ce)+b^2e^2\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}-\frac{e}{(d+ex)\left(ad^2-bde+ce^2\right)}-\frac{e(2ad-be)\log\left(ax^2+bx+c\right)}{2\left(ad^2-e(bd-ce)\right)^2}+\frac{e(2ad}{(ad^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + c/x^2 + b/x)*x^2*(d + e*x)^2),x]
```

```
[Out] -(e/((a*d^2 - b*d*e + c*e^2)*(d + e*x))) - ((2*a^2*d^2 + b^2*e^2 - 2*a*e*(b
*d + c*e))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(a*d^
2 - e*(b*d - c*e))^2) + (e*(2*a*d - b*e)*Log[d + e*x])/(a*d^2 - e*(b*d - c*
e))^2 - (e*(2*a*d - b*e)*Log[c + b*x + a*x^2])/(2*(a*d^2 - e*(b*d - c*e))^2
)
```

Rule 1569

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^2(d+ex)^2}\,dx = \int \frac{1}{(d+ex)^2\left(c+bx+ax^2\right)}\,dx$$

$$= -\frac{e}{\left(ad^2-bde+ce^2\right)(d+ex)} + \frac{\int \frac{ad-be-aex}{(d+ex)\left(c+bx+ax^2\right)}\,dx}{ad^2-bde+ce^2}$$

$$= -\frac{e}{\left(ad^2-bde+ce^2\right)(d+ex)} + \frac{\int \left(\frac{e^2(2ad-be)}{\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{a^2d^2+b^2e^2-ae(2bd+ce)-ae(2ad-be)x}{\left(ad^2-e(bd-ce)\right)\left(c+bx+ax^2\right)}\right) dx}{ad^2-bde+ce^2}$$

$$= -\frac{e}{\left(ad^2-bde+ce^2\right)(d+ex)} + \frac{e(2ad-be)\log(d+ex)}{\left(ad^2-e(bd-ce)\right)^2} + \frac{\int \frac{a^2d^2+b^2e^2-ae(2bd+ce)-ae(2ad-be)x}{c+bx+ax^2}}{\left(ad^2-e(bd-ce)\right)^2}$$

$$= -\frac{e}{\left(ad^2-bde+ce^2\right)(d+ex)} + \frac{e(2ad-be)\log(d+ex)}{\left(ad^2-e(bd-ce)\right)^2} - \frac{(e(2ad-be))\int \frac{b+2ax}{c+bx+ax^2}\,dx}{2\left(ad^2-e(bd-ce)\right)^2} +$$

$$= -\frac{e}{\left(ad^2-bde+ce^2\right)(d+ex)} + \frac{e(2ad-be)\log(d+ex)}{\left(ad^2-e(bd-ce)\right)^2} - \frac{e(2ad-be)\log\left(c+bx+ax^2\right)}{2\left(ad^2-e(bd-ce)\right)^2}$$

$$= -\frac{e}{\left(ad^2-bde+ce^2\right)(d+ex)} - \frac{\left(2a^2d^2+b^2e^2-2ae(bd+ce)\right)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2} + \frac{e(2ac}{(ac}$$

Mathematica [A] time = 0.226302, size = 151, normalized size = 0.8

$$\frac{\frac{2\left(2a^2d^2-2ae(bd+ce)+b^2e^2\right)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}} - \frac{2e\left(ad^2+e(ce-bd)\right)}{d+ex} + e(be-2ad)\log(x(ax+b)+c) - 2e(be-2ad)\log(d+ex)}{2\left(ad^2+e(ce-bd)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + c/x^2 + b/x)*x^2*(d + e*x)^2),x]
```

```
[Out] ((-2*e*(a*d^2 + e*(-(b*d) + c*e)))/(d + e*x) + (2*(2*a^2*d^2 + b^2*e^2 - 2*
a*e*(b*d + c*e))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]
 - 2*e*(-2*a*d + b*e)*Log[d + e*x] + e*(-2*a*d + b*e)*Log[c + x*(b + a*x)])
/(2*(a*d^2 + e*(-(b*d) + c*e))^2)
```

Maple [B] time = 0.009, size = 386, normalized size = 2.

$$-\frac{e}{\left(ad^2-bde+e^2c\right)(ex+d)}+2\frac{e\ln(ex+d)ad}{\left(ad^2-bde+e^2c\right)^2}-\frac{e^2\ln(ex+d)b}{\left(ad^2-bde+e^2c\right)^2}-\frac{a\ln\left(ax^2+bx+c\right)de}{\left(ad^2-bde+e^2c\right)^2}+\frac{\ln\left(ax^2+bx+c\right)be^2}{2\left(ad^2-bde+e^2c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)/x^2/(e*x+d)^2,x)
```

```
[Out] -e/(a*d^2-b*d*e+c*e^2)/(e*x+d)+2*e/(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d)*a*d-e^2/
(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d)*b-1/(a*d^2-b*d*e+c*e^2)^2*a*ln(a*x^2+b*x+c)
*d*e+1/2/(a*d^2-b*d*e+c*e^2)^2*ln(a*x^2+b*x+c)*b*e^2+2/(a*d^2-b*d*e+c*e^2)^
2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a^2*d^2-2/(a*d^2-b*
d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a*b*d*e-
2/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2
))*a*c*e^2+1/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*
c-b^2)^(1/2))*b^2*e^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^2/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 17.4993, size = 2295, normalized size = 12.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^2/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a*b^2 - 4*a^2*c)*d^2*e - 2*(b^3 - 4*a*b*c)*d*e^2 + 2*(b^2*c - 4*a
*c^2)*e^3 + (2*a^2*d^3 - 2*a*b*d^2*e + (b^2 - 2*a*c)*d*e^2 + (2*a^2*d^2*e -
```

```
 2*a*b*d*e^2 + (b^2 - 2*a*c)*e^3)*x)*sqrt(b^2 - 4*a*c)*log((2*a^2*x^2 + 2*a
*b*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x + c)) + (2
*(a*b^2 - 4*a^2*c)*d^2*e - (b^3 - 4*a*b*c)*d*e^2 + (2*(a*b^2 - 4*a^2*c)*d*e
^2 - (b^3 - 4*a*b*c)*e^3)*x)*log(a*x^2 + b*x + c) - 2*(2*(a*b^2 - 4*a^2*c)*
d^2*e - (b^3 - 4*a*b*c)*d*e^2 + (2*(a*b^2 - 4*a^2*c)*d*e^2 - (b^3 - 4*a*b*c
)*e^3)*x)*log(e*x + d))/((a^2*b^2 - 4*a^3*c)*d^5 - 2*(a*b^3 - 4*a^2*b*c)*d^
4*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^3*e^2 - 2*(b^3*c - 4*a*b*c^2)*d^2*e^3
 + (b^2*c^2 - 4*a*c^3)*d*e^4 + ((a^2*b^2 - 4*a^3*c)*d^4*e - 2*(a*b^3 - 4*a^
2*b*c)*d^3*e^2 + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^3 - 2*(b^3*c - 4*a*b*c
^2)*d*e^4 + (b^2*c^2 - 4*a*c^3)*e^5)*x), -1/2*(2*(a*b^2 - 4*a^2*c)*d^2*e -
2*(b^3 - 4*a*b*c)*d*e^2 + 2*(b^2*c - 4*a*c^2)*e^3 + 2*(2*a^2*d^3 - 2*a*b*d^
2*e + (b^2 - 2*a*c)*d*e^2 + (2*a^2*d^2*e - 2*a*b*d*e^2 + (b^2 - 2*a*c)*e^3)
*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)
) + (2*(a*b^2 - 4*a^2*c)*d^2*e - (b^3 - 4*a*b*c)*d*e^2 + (2*(a*b^2 - 4*a^2*
c)*d*e^2 - (b^3 - 4*a*b*c)*e^3)*x)*log(a*x^2 + b*x + c) - 2*(2*(a*b^2 - 4*a
^2*c)*d^2*e - (b^3 - 4*a*b*c)*d*e^2 + (2*(a*b^2 - 4*a^2*c)*d*e^2 - (b^3 - 4
*a*b*c)*e^3)*x)*log(e*x + d))/((a^2*b^2 - 4*a^3*c)*d^5 - 2*(a*b^3 - 4*a^2*b
*c)*d^4*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^3*e^2 - 2*(b^3*c - 4*a*b*c^2)*d
^2*e^3 + (b^2*c^2 - 4*a*c^3)*d*e^4 + ((a^2*b^2 - 4*a^3*c)*d^4*e - 2*(a*b^3
- 4*a^2*b*c)*d^3*e^2 + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^3 - 2*(b^3*c - 4
*a*b*c^2)*d*e^4 + (b^2*c^2 - 4*a*c^3)*e^5)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)/x**2/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11155, size = 447, normalized size = 2.37

$$-\frac{\left(2\,a^{2}d^{2}e^{2}-2\,abde^{3}+b^{2}e^{4}-2\,ace^{4}\right)\arctan\left(-\frac{\left(2\,ad-\frac{2\,ad^{2}}{xe+d}-be+\frac{2\,bde}{xe+d}-\frac{2\,ce^{2}}{xe+d}\right)e^{(-1)}}{\sqrt{-b^{2}+4\,ac}}\right)e^{(-2)}}{\left(a^{2}d^{4}-2\,abd^{3}e+b^{2}d^{2}e^{2}+2\,acd^{2}e^{2}-2\,bcde^{3}+c^{2}e^{4}\right)\sqrt{-b^{2}+4\,ac}}-\frac{\left(2\,ade-be^{2}\right)\log\left(-a+\frac{2\,ad}{xe+d}-\frac{ad^{2}}{(xe+d)^{2}}\right.}{2\left(a^{2}d^{4}-2\,abd^{3}e+b^{2}d^{2}e^{2}+2\,ac\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^2/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -(2*a^2*d^2*e^2 - 2*a*b*d*e^3 + b^2*e^4 - 2*a*c*e^4)*arctan(-(2*a*d - 2*a*d
^2/(x*e + d) - b*e + 2*b*d*e/(x*e + d) - 2*c*e^2/(x*e + d))*e^(-1)/sqrt(-b^
2 + 4*a*c))*e^(-2)/((a^2*d^4 - 2*a*b*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 -
2*b*c*d*e^3 + c^2*e^4)*sqrt(-b^2 + 4*a*c)) - 1/2*(2*a*d*e - b*e^2)*log(-a +
 2*a*d/(x*e + d) - a*d^2/(x*e + d)^2 - b*e/(x*e + d) + b*d*e/(x*e + d)^2 -
c*e^2/(x*e + d)^2)/(a^2*d^4 - 2*a*b*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 - 2
*b*c*d*e^3 + c^2*e^4) - e^3/((a*d^2*e^2 - b*d*e^3 + c*e^4)*(x*e + d))
```

3.76 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^3(d+ex)^2}\,dx$

Optimal. Leaf size=248

$$-\frac{\left(a^2d^2-ae(2bd+ce)+b^2e^2\right)\log\left(ax^2+bx+c\right)}{2c\left(ad^2-e(bd-ce)\right)^2}+\frac{\left(a^2d(bd+4ce)-abe(2bd+3ce)+b^3e^2\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}+\frac{\ldots}{d(d+e\ldots}$$

```
[Out] e^2/(d*(a*d^2 - b*d*e + c*e^2)*(d + e*x)) + ((b^3*e^2 - a*b*e*(2*b*d + 3*c*
e) + a^2*d*(b*d + 4*c*e))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b
^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))^2) + Log[x]/(c*d^2) - (e^2*(3*a*d^2 - e
*(2*b*d - c*e))*Log[d + e*x])/(d^2*(a*d^2 - e*(b*d - c*e))^2) - ((a^2*d^2 +
 b^2*e^2 - a*e*(2*b*d + c*e))*Log[c + b*x + a*x^2])/(2*c*(a*d^2 - e*(b*d -
c*e))^2)
```

Rubi [A] time = 0.408682, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1569, 893, 634, 618, 206, 628}

$$-\frac{\left(a^2d^2-ae(2bd+ce)+b^2e^2\right)\log\left(ax^2+bx+c\right)}{2c\left(ad^2-e(bd-ce)\right)^2}+\frac{\left(a^2d(bd+4ce)-abe(2bd+3ce)+b^3e^2\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}+\frac{\ldots}{d(d+e\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + c/x^2 + b/x)*x^3*(d + e*x)^2),x]
```

```
[Out] e^2/(d*(a*d^2 - e*(b*d - c*e))*(d + e*x)) + ((b^3*e^2 - a*b*e*(2*b*d + 3*c*
e) + a^2*d*(b*d + 4*c*e))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b
^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))^2) + Log[x]/(c*d^2) - (e^2*(3*a*d^2 - e
*(2*b*d - c*e))*Log[d + e*x])/(d^2*(a*d^2 - e*(b*d - c*e))^2) - ((a^2*d^2 +
 b^2*e^2 - a*e*(2*b*d + c*e))*Log[c + b*x + a*x^2])/(2*c*(a*d^2 - e*(b*d -
c*e))^2)
```

Rule 1569

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^3(d+ex)^2}\,dx = \int \frac{1}{x(d+ex)^2\left(c+bx+ax^2\right)}\,dx$$

$$= \int \left(\frac{1}{cd^2x} + \frac{e^3}{d\left(-ad^2+e(bd-ce)\right)(d+ex)^2} + \frac{e^3\left(-3ad^2+e(2bd-ce)\right)}{d^2\left(ad^2-e(bd-ce)\right)^2(d+ex)} + \frac{-(ad-\right.$$

$$= \frac{e^2}{d\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{\log(x)}{cd^2} - \frac{e^2\left(3ad^2-e(2bd-ce)\right)\log(d+ex)}{d^2\left(ad^2-e(bd-ce)\right)^2} + \int \frac{-(ad-b}{}$$

$$= \frac{e^2}{d\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{\log(x)}{cd^2} - \frac{e^2\left(3ad^2-e(2bd-ce)\right)\log(d+ex)}{d^2\left(ad^2-e(bd-ce)\right)^2} - \frac{\left(a^2d^2+\right.}{}$$

$$= \frac{e^2}{d\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{\log(x)}{cd^2} - \frac{e^2\left(3ad^2-e(2bd-ce)\right)\log(d+ex)}{d^2\left(ad^2-e(bd-ce)\right)^2} - \frac{\left(a^2d^2+\right.}{}$$

$$= \frac{e^2}{d\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{\left(b^3e^2-abe(2bd+3ce)+a^2d(bd+4ce)\right)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4a}}\right.}{c\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}$$

Mathematica [A] time = 0.284886, size = 246, normalized size = 0.99

$$\frac{\left(-a^2d^2+ae(2bd+ce)-b^2e^2\right)\log(x(ax+b)+c)}{2c\left(ad^2+e(ce-bd)\right)^2} - \frac{\left(a^2d(bd+4ce)-abe(2bd+3ce)+b^3e^2\right)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{c\sqrt{4ac-b^2}\left(ad^2+e(ce-bd)\right)^2} + \frac{}{d(d+ex)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + c/x^2 + b/x)*x^3*(d + e*x)^2),x]
```

```
[Out] e^2/(d*(a*d^2 + e*(-(b*d) + c*e))*(d + e*x)) - ((b^3*e^2 - a*b*e*(2*b*d + 3
*c*e) + a^2*d*(b*d + 4*c*e))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/(c*Sqr
t[-b^2 + 4*a*c]*(a*d^2 + e*(-(b*d) + c*e))^2) + Log[x]/(c*d^2) - (e^2*(3*a*
d^2 + e*(-2*b*d + c*e))*Log[d + e*x])/(a*d^3 + d*e*(-(b*d) + c*e))^2 + ((-(
a^2*d^2) - b^2*e^2 + a*e*(2*b*d + c*e))*Log[c + x*(b + a*x)])/(2*c*(a*d^2 +
 e*(-(b*d) + c*e))^2)
```

Maple [B] time = 0.013, size = 589, normalized size = 2.4

$$\frac{e^{2}}{d\left(a d^{2}-b d e+e^{2} c\right)(e x+d)}-3 \frac{e^{2} \ln (e x+d) a}{\left(a d^{2}-b d e+e^{2} c\right)^{2}}+2 \frac{e^{3} \ln (e x+d) b}{d\left(a d^{2}-b d e+e^{2} c\right)^{2}}-\frac{e^{4} \ln (e x+d) c}{\left(a d^{2}-b d e+e^{2} c\right)^{2} d^{2}}-\frac{a^{2} \ln \left(a x^{2}+b x+c\right.}{2\left(a d^{2}-b d e+e^{2} c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)/x^3/(e*x+d)^2,x)
```

```
[Out] e^2/d/(a*d^2-b*d*e+c*e^2)/(e*x+d)-3*e^2/(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d)*a+2
*e^3/d/(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d)*b-e^4/d^2/(a*d^2-b*d*e+c*e^2)^2*ln(e
*x+d)*c-1/2/(a*d^2-b*d*e+c*e^2)^2/c*a^2*ln(a*x^2+b*x+c)*d^2+1/(a*d^2-b*d*e+
c*e^2)^2/c*a*ln(a*x^2+b*x+c)*b*d*e+1/2/(a*d^2-b*d*e+c*e^2)^2*a*ln(a*x^2+b*x
+c)*e^2-1/2/(a*d^2-b*d*e+c*e^2)^2/c*ln(a*x^2+b*x+c)*b^2*e^2-1/(a*d^2-b*d*e+
c*e^2)^2/c/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a^2*b*d^2-
4/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2
))*a^2*d*e+2/(a*d^2-b*d*e+c*e^2)^2/c/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*
a*c-b^2)^(1/2))*a*b^2*d*e+3/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arctan(
(2*a*x+b)/(4*a*c-b^2)^(1/2))*a*b*e^2-1/(a*d^2-b*d*e+c*e^2)^2/c/(4*a*c-b^2)^
(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*b^3*e^2+ln(x)/c/d^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^3/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^3/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)/x**3/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13596, size = 528, normalized size = 2.13

$$-\frac{\left(a^2bd^2e^2-2\,ab^2de^3+4\,a^2cde^3+b^3e^4-3\,abce^4\right)\arctan\left(\frac{\left(2\,ad-\frac{2\,ad^2}{xe+d}-be+\frac{2\,bde}{xe+d}-\frac{2\,ce^2}{xe+d}\right)e^{(-1)}}{\sqrt{-b^2+4\,ac}}\right)e^{(-2)}}{\left(a^2cd^4-2\,abcd^3e+b^2cd^2e^2+2\,ac^2d^2e^2-2\,bc^2de^3+c^3e^4\right)\sqrt{-b^2+4\,ac}}-\frac{\left(a^2d^2-2\,abde+b^2e^2-ac\right.}{2\left(a^2cd^4-2\,abcd\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^3/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -(a^2*b*d^2*e^2 - 2*a*b^2*d*e^3 + 4*a^2*c*d*e^3 + b^3*e^4 - 3*a*b*c*e^4)*ar
ctan((2*a*d - 2*a*d^2/(x*e + d) - b*e + 2*b*d*e/(x*e + d) - 2*c*e^2/(x*e +
d))*e^(-1)/sqrt(-b^2 + 4*a*c))*e^(-2)/((a^2*c*d^4 - 2*a*b*c*d^3*e + b^2*c*d
^2*e^2 + 2*a*c^2*d^2*e^2 - 2*b*c^2*d*e^3 + c^3*e^4)*sqrt(-b^2 + 4*a*c)) - 1
/2*(a^2*d^2 - 2*a*b*d*e + b^2*e^2 - a*c*e^2)*log(a - 2*a*d/(x*e + d) + a*d^
2/(x*e + d)^2 + b*e/(x*e + d) - b*d*e/(x*e + d)^2 + c*e^2/(x*e + d)^2)/(a^2
*c*d^4 - 2*a*b*c*d^3*e + b^2*c*d^2*e^2 + 2*a*c^2*d^2*e^2 - 2*b*c^2*d*e^3 +
c^3*e^4) + e^5/((a*d^3*e^3 - b*d^2*e^4 + c*d*e^5)*(x*e + d)) + log(abs(-d/(
x*e + d) + 1))/(c*d^2)
```

3.77 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^4(d+ex)^2}\,dx$

Optimal. Leaf size=291

$$\frac{\left(-a^2\left(b^2d^2+6bcde+2c^2e^2\right)+2a^3cd^2+2ab^2e(bd+2ce)+b^4\left(-e^2\right)\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}+\frac{(ad-be)\left(abd+2ace+b^2(-e)\right)\mathrm{l}}{2c^2\left(ad^2-e(bd-c\right.}$$

```
[Out] -(1/(c*d^2*x)) - e^3/(d^2*(a*d^2 - e*(b*d - c*e))*(d + e*x)) + ((2*a^3*c*d^
2 - b^4*e^2 + 2*a*b^2*e*(b*d + 2*c*e) - a^2*(b^2*d^2 + 6*b*c*d*e + 2*c^2*e^
2))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]*(a*d^2 -
 e*(b*d - c*e))^2) - ((b*d + 2*c*e)*Log[x])/(c^2*d^3) + (e^3*(4*a*d^2 - e*(
3*b*d - 2*c*e))*Log[d + e*x])/(d^3*(a*d^2 - e*(b*d - c*e))^2) + ((a*d - b*e
)*(a*b*d - b^2*e + 2*a*c*e)*Log[c + b*x + a*x^2])/(2*c^2*(a*d^2 - e*(b*d -
c*e))^2)
```

Rubi [A] time = 0.563207, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1569, 893, 634, 618, 206, 628}

$$\frac{\left(-a^2\left(b^2d^2+6bcde+2c^2e^2\right)+2a^3cd^2+2ab^2e(bd+2ce)+b^4\left(-e^2\right)\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}+\frac{(ad-be)\left(abd+2ace+b^2(-e)\right)\mathrm{l}}{2c^2\left(ad^2-e(bd-c\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + c/x^2 + b/x)*x^4*(d + e*x)^2),x]
```

```
[Out] -(1/(c*d^2*x)) - e^3/(d^2*(a*d^2 - e*(b*d - c*e))*(d + e*x)) + ((2*a^3*c*d^
2 - b^4*e^2 + 2*a*b^2*e*(b*d + 2*c*e) - a^2*(b^2*d^2 + 6*b*c*d*e + 2*c^2*e^
2))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]*(a*d^2 -
 e*(b*d - c*e))^2) - ((b*d + 2*c*e)*Log[x])/(c^2*d^3) + (e^3*(4*a*d^2 - e*(
3*b*d - 2*c*e))*Log[d + e*x])/(d^3*(a*d^2 - e*(b*d - c*e))^2) + ((a*d - b*e
)*(a*b*d - b^2*e + 2*a*c*e)*Log[c + b*x + a*x^2])/(2*c^2*(a*d^2 - e*(b*d -
c*e))^2)
```

Rule 1569

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
```

```
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^4(d+ex)^2}\,dx = \int \frac{1}{x^2(d+ex)^2\left(c+bx+ax^2\right)}\,dx$$

$$= \int \left(\frac{1}{cd^2x^2} + \frac{-bd-2ce}{c^2d^3x} + \frac{e^4}{d^2\left(ad^2-e(bd-ce)\right)(d+ex)^2} + \frac{e^4\left(4ad^2-e(3bd-2ce)\right)}{d^3\left(ad^2-e(bd-ce)\right)^2(d+e\right.$$

$$= -\frac{1}{cd^2x} - \frac{e^3}{d^2\left(ad^2-e(bd-ce)\right)(d+ex)} - \frac{(bd+2ce)\log(x)}{c^2d^3} + \frac{e^3\left(4ad^2-e(3bd-2ce)\right)\mathrm{lo}}{d^3\left(ad^2-e(bd-ce\right.}$$

$$= -\frac{1}{cd^2x} - \frac{e^3}{d^2\left(ad^2-e(bd-ce)\right)(d+ex)} - \frac{(bd+2ce)\log(x)}{c^2d^3} + \frac{e^3\left(4ad^2-e(3bd-2ce)\right)\mathrm{lo}}{d^3\left(ad^2-e(bd-ce\right.}$$

$$= -\frac{1}{cd^2x} - \frac{e^3}{d^2\left(ad^2-e(bd-ce)\right)(d+ex)} - \frac{(bd+2ce)\log(x)}{c^2d^3} + \frac{e^3\left(4ad^2-e(3bd-2ce)\right)\mathrm{lo}}{d^3\left(ad^2-e(bd-ce\right.}$$

$$= -\frac{1}{cd^2x} - \frac{e^3}{d^2\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{\left(2a^3cd^2-b^4e^2+2ab^2e(bd+2ce)-a^2\left(b^2d^2+\right.\right.}{c^2\sqrt{b^2-4ac}\left(ad^2-e(\right.}$$

Mathematica [A] time = 0.380898, size = 287, normalized size = 0.99

$$\frac{\left(a^2\left(b^2d^2+6bcde+2c^2e^2\right)-2a^3cd^2-2ab^2e(bd+2ce)+b^4e^2\right)\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{c^2\sqrt{4ac-b^2}\left(ad^2+e(ce-bd)\right)^2} + \frac{(ad-be)\left(abd+2ace+b^2(-e)\right)\log(x(a}{2c^2\left(ad^2+e(ce-bd)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + c/x^2 + b/x)*x^4*(d + e*x)^2),x]
```

```
[Out] -(1/(c*d^2*x)) - e^3/(d^2*(a*d^2 + e*(-(b*d) + c*e))*(d + e*x)) + ((-2*a^3*
c*d^2 + b^4*e^2 - 2*a*b^2*e*(b*d + 2*c*e) + a^2*(b^2*d^2 + 6*b*c*d*e + 2*c^
2*e^2))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/(c^2*Sqrt[-b^2 + 4*a*c]*(a*
d^2 + e*(-(b*d) + c*e))^2) - ((b*d + 2*c*e)*Log[x])/(c^2*d^3) + (e^3*(4*a*d
^2 + e*(-3*b*d + 2*c*e))*Log[d + e*x])/(d^3*(a*d^2 + e*(-(b*d) + c*e))^2) +
 ((a*d - b*e)*(a*b*d - b^2*e + 2*a*c*e)*Log[c + x*(b + a*x)])/(2*c^2*(a*d^2
 + e*(-(b*d) + c*e))^2)
```

Maple [B] time = 0.016, size = 791, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)/x^4/(e*x+d)^2,x)
```

```
[Out] -e^3/d^2/(a*d^2-b*d*e+c*e^2)/(e*x+d)+4*e^3/d/(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d
)*a-3*e^4/d^2/(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d)*b+2*e^5/d^3/(a*d^2-b*d*e+c*e^
2)^2*ln(e*x+d)*c+1/2/(a*d^2-b*d*e+c*e^2)^2/c^2*a^2*ln(a*x^2+b*x+c)*b*d^2+1/
(a*d^2-b*d*e+c*e^2)^2/c*a^2*ln(a*x^2+b*x+c)*d*e-1/(a*d^2-b*d*e+c*e^2)^2/c^2
*a*ln(a*x^2+b*x+c)*b^2*d*e-1/(a*d^2-b*d*e+c*e^2)^2/c*a*ln(a*x^2+b*x+c)*b*e^
2+1/2/(a*d^2-b*d*e+c*e^2)^2/c^2*ln(a*x^2+b*x+c)*b^3*e^2-2/(a*d^2-b*d*e+c*e^
2)^2/c/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a^3*d^2+1/(a*d
^2-b*d*e+c*e^2)^2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))
*a^2*b^2*d^2+6/(a*d^2-b*d*e+c*e^2)^2/c/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(
4*a*c-b^2)^(1/2))*a^2*b*d*e+2/(a*d^2-b*d*e+c*e^2)^2/(4*a*c-b^2)^(1/2)*arcta
n((2*a*x+b)/(4*a*c-b^2)^(1/2))*a^2*e^2-2/(a*d^2-b*d*e+c*e^2)^2/c^2/(4*a*c-b
^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a*b^3*d*e-4/(a*d^2-b*d*e+c*e^
2)^2/c/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a*b^2*e^2+1/(a
*d^2-b*d*e+c*e^2)^2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2
))*b^4*e^2-1/c/d^2/x-1/d^2/c^2*ln(x)*b-2/d^3/c*ln(x)*e
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^4/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^4/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)/x**4/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.10928, size = 657, normalized size = 2.26

$$-\frac{\left(a^2b^2d^2e^2-2\,a^3cd^2e^2-2\,ab^3de^3+6\,a^2bcde^3+b^4e^4-4\,ab^2ce^4+2\,a^2c^2e^4\right)\arctan\left(-\frac{\left(2\,ad-\frac{2\,ad^2}{xe+d}-be+\frac{2\,bde}{xe+d}-\frac{2\,ce^2}{xe+d}\right)e^{(-1)}}{\sqrt{-b^2+4\,ac}}\right)e^{(-2)}}{\left(a^2c^2d^4-2\,abc^2d^3e+b^2c^2d^2e^2+2\,ac^3d^2e^2-2\,bc^3de^3+c^4e^4\right)\sqrt{-b^2+4\,ac}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^4/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -(a^2*b^2*d^2*e^2 - 2*a^3*c*d^2*e^2 - 2*a*b^3*d*e^3 + 6*a^2*b*c*d*e^3 + b^4
*e^4 - 4*a*b^2*c*e^4 + 2*a^2*c^2*e^4)*arctan(-(2*a*d - 2*a*d^2/(x*e + d) -
b*e + 2*b*d*e/(x*e + d) - 2*c*e^2/(x*e + d))*e^(-1)/sqrt(-b^2 + 4*a*c))*e^(
-2)/((a^2*c^2*d^4 - 2*a*b*c^2*d^3*e + b^2*c^2*d^2*e^2 + 2*a*c^3*d^2*e^2 - 2
*b*c^3*d*e^3 + c^4*e^4)*sqrt(-b^2 + 4*a*c)) + 1/2*(a^2*b*d^2 - 2*a*b^2*d*e
+ 2*a^2*c*d*e + b^3*e^2 - 2*a*b*c*e^2)*log(-a + 2*a*d/(x*e + d) - a*d^2/(x*
e + d)^2 - b*e/(x*e + d) + b*d*e/(x*e + d)^2 - c*e^2/(x*e + d)^2)/(a^2*c^2*
d^4 - 2*a*b*c^2*d^3*e + b^2*c^2*d^2*e^2 + 2*a*c^3*d^2*e^2 - 2*b*c^3*d*e^3 +
 c^4*e^4) - e^7/((a*d^4*e^4 - b*d^3*e^5 + c*d^2*e^6)*(x*e + d)) - (b*d*e +
2*c*e^2)*e^(-1)*log(abs(-d/(x*e + d) + 1))/(c^2*d^3) + e/(c*d^3*(d/(x*e + d
) - 1))
```

3.78 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^5(d+ex)^2}\,dx$

Optimal. Leaf size=372

$$\frac{\left(-a^2\left(b^2d^2+4bcde+c^2e^2\right)+a^3cd^2+ab^2e(2bd+3ce)+b^4\left(-e^2\right)\right)\log\left(ax^2+bx+c\right)}{2c^3\left(ad^2-e(bd-ce)\right)^2}+\frac{\left(a^2b\left(b^2d^2+8bcde+5c^2e^2\right)-a^3\right.}{c^3}$$

```
[Out] -1/(2*c*d^2*x^2) + (b*d + 2*c*e)/(c^2*d^3*x) + e^4/(d^3*(a*d^2 - e*(b*d - c
*e))*(d + e*x)) + ((b^5*e^2 - a^3*c*d*(3*b*d + 4*c*e) - a*b^3*e*(2*b*d + 5*
c*e) + a^2*b*(b^2*d^2 + 8*b*c*d*e + 5*c^2*e^2))*ArcTanh[(b + 2*a*x)/Sqrt[b^
2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))^2) + ((b^2*d^2
+ 2*b*c*d*e - c*(a*d^2 - 3*c*e^2))*Log[x])/(c^3*d^4) - (e^4*(5*a*d^2 - e*(4
*b*d - 3*c*e))*Log[d + e*x])/(d^4*(a*d^2 - e*(b*d - c*e))^2) + ((a^3*c*d^2
- b^4*e^2 + a*b^2*e*(2*b*d + 3*c*e) - a^2*(b^2*d^2 + 4*b*c*d*e + c^2*e^2))*
Log[c + b*x + a*x^2])/(2*c^3*(a*d^2 - e*(b*d - c*e))^2)
```

Rubi [A] time = 0.85104, antiderivative size = 372, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1569, 893, 634, 618, 206, 628}

$$\frac{\left(-a^2\left(b^2d^2+4bcde+c^2e^2\right)+a^3cd^2+ab^2e(2bd+3ce)+b^4\left(-e^2\right)\right)\log\left(ax^2+bx+c\right)}{2c^3\left(ad^2-e(bd-ce)\right)^2}+\frac{\left(a^2b\left(b^2d^2+8bcde+5c^2e^2\right)-a^3\right.}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + c/x^2 + b/x)*x^5*(d + e*x)^2),x]
```


```
[Out] -1/(2*c*d^2*x^2) + (b*d + 2*c*e)/(c^2*d^3*x) + e^4/(d^3*(a*d^2 - e*(b*d - c
*e))*(d + e*x)) + ((b^5*e^2 - a^3*c*d*(3*b*d + 4*c*e) - a*b^3*e*(2*b*d + 5*
c*e) + a^2*b*(b^2*d^2 + 8*b*c*d*e + 5*c^2*e^2))*ArcTanh[(b + 2*a*x)/Sqrt[b^
2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))^2) + ((b^2*d^2
+ 2*b*c*d*e - c*(a*d^2 - 3*c*e^2))*Log[x])/(c^3*d^4) - (e^4*(5*a*d^2 - e*(4
*b*d - 3*c*e))*Log[d + e*x])/(d^4*(a*d^2 - e*(b*d - c*e))^2) + ((a^3*c*d^2
- b^4*e^2 + a*b^2*e*(2*b*d + 3*c*e) - a^2*(b^2*d^2 + 4*b*c*d*e + c^2*e^2))*
Log[c + b*x + a*x^2])/(2*c^3*(a*d^2 - e*(b*d - c*e))^2)
```


Rule 1569

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^5(d+ex)^2}\,dx = \int \frac{1}{x^3(d+ex)^2\left(c+bx+ax^2\right)}\,dx$$

$$= \int \left(\frac{1}{cd^2x^3} + \frac{-bd-2ce}{c^2d^3x^2} + \frac{b^2d^2+2bcde-c\left(ad^2-3ce^2\right)}{c^3d^4x} + \frac{e^5}{d^3\left(-ad^2+e(bd-ce)\right)(d+\right.$$

$$= -\frac{1}{2cd^2x^2} + \frac{bd+2ce}{c^2d^3x} + \frac{e^4}{d^3\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{\left(b^2d^2+2bcde-c\left(ad^2-3ce^2\right)\right.}{c^3d^4}$$

$$= -\frac{1}{2cd^2x^2} + \frac{bd+2ce}{c^2d^3x} + \frac{e^4}{d^3\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{\left(b^2d^2+2bcde-c\left(ad^2-3ce^2\right)\right.}{c^3d^4}$$

$$= -\frac{1}{2cd^2x^2} + \frac{bd+2ce}{c^2d^3x} + \frac{e^4}{d^3\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{\left(b^2d^2+2bcde-c\left(ad^2-3ce^2\right)\right.}{c^3d^4}$$

$$= -\frac{1}{2cd^2x^2} + \frac{bd+2ce}{c^2d^3x} + \frac{e^4}{d^3\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{\left(b^5e^2-a^3cd(3bd+4ce)-ab^3e\right.}{c}$$

Mathematica [A] time = 0.467986, size = 370, normalized size = 0.99

$$-\frac{\left(a^2\left(b^2d^2+4bcde+c^2e^2\right)-a^3cd^2-ab^2e(2bd+3ce)+b^4e^2\right)\log(x(ax+b)+c)}{2c^3\left(ad^2+e(ce-bd)\right)^2} + \frac{\left(-a^2b\left(b^2d^2+8bcde+5c^2e^2\right)+a^3cd\right.}{c^3\sqrt{4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + c/x^2 + b/x)*x^5*(d + e*x)^2),x]
```

```
[Out] -1/(2*c*d^2*x^2) + (b*d + 2*c*e)/(c^2*d^3*x) + e^4/(d^3*(a*d^2 + e*(-(b*d)
+ c*e))*(d + e*x)) + ((-(b^5*e^2) + a^3*c*d*(3*b*d + 4*c*e) + a*b^3*e*(2*b*
d + 5*c*e) - a^2*b*(b^2*d^2 + 8*b*c*d*e + 5*c^2*e^2))*ArcTan[(b + 2*a*x)/Sq
rt[-b^2 + 4*a*c]])/(c^3*Sqrt[-b^2 + 4*a*c]*(a*d^2 + e*(-(b*d) + c*e))^2) +
((b^2*d^2 + 2*b*c*d*e + c*(-(a*d^2) + 3*c*e^2))*Log[x])/(c^3*d^4) - (e^4*(5
*a*d^2 + e*(-4*b*d + 3*c*e))*Log[d + e*x])/(d^4*(a*d^2 + e*(-(b*d) + c*e))^
2) - ((-(a^3*c*d^2) + b^4*e^2 - a*b^2*e*(2*b*d + 3*c*e) + a^2*(b^2*d^2 + 4*
b*c*d*e + c^2*e^2))*Log[c + x*(b + a*x)])/(2*c^3*(a*d^2 + e*(-(b*d) + c*e))
^2)
```

Maple [B] time = 0.019, size = 993, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)/x^5/(e*x+d)^2,x)
```

```
[Out] -1/2/c/d^2/x^2-8/(a*d^2-b*d*e+c*e^2)^2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+
b)/(4*a*c-b^2)^(1/2))*a^2*b^2*d*e+2/(a*d^2-b*d*e+c*e^2)^2/c^3/(4*a*c-b^2)^(
1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a*b^4*d*e-1/2/(a*d^2-b*d*e+c*e^2)^
2/c*a^2*ln(a*x^2+b*x+c)*e^2+1/2/(a*d^2-b*d*e+c*e^2)^2/c^2*a^3*ln(a*x^2+b*x+
c)*d^2-5*e^4/d^2/(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d)*a-2/(a*d^2-b*d*e+c*e^2)^2/
c^2*a^2*ln(a*x^2+b*x+c)*b*d*e+1/(a*d^2-b*d*e+c*e^2)^2/c^3*a*ln(a*x^2+b*x+c)
*b^3*d*e+3/(a*d^2-b*d*e+c*e^2)^2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*
a*c-b^2)^(1/2))*a^3*b*d^2+4/(a*d^2-b*d*e+c*e^2)^2/c/(4*a*c-b^2)^(1/2)*arcta
n((2*a*x+b)/(4*a*c-b^2)^(1/2))*a^3*d*e-1/(a*d^2-b*d*e+c*e^2)^2/c^3/(4*a*c-b
^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a^2*b^3*d^2-5/(a*d^2-b*d*e+c*
e^2)^2/c/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))*a^2*b*e^2+5/
(a*d^2-b*d*e+c*e^2)^2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1
/2))*a*b^3*e^2+2/d^3/c^2*ln(x)*b*e-1/2/(a*d^2-b*d*e+c*e^2)^2/c^3*ln(a*x^2+b
*x+c)*b^4*e^2-3*e^6/d^4/(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d)*c+3/d^4/c*ln(x)*e^2
+e^4/d^3/(a*d^2-b*d*e+c*e^2)/(e*x+d)+1/c^2/d^2/x*b+2/c/d^3/x*e-1/d^2/c^2*ln
(x)*a+1/d^2/c^3*ln(x)*b^2+3/2/(a*d^2-b*d*e+c*e^2)^2/c^2*a*ln(a*x^2+b*x+c)*b
^2*e^2-1/(a*d^2-b*d*e+c*e^2)^2/c^3/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*
c-b^2)^(1/2))*b^5*e^2-1/2/(a*d^2-b*d*e+c*e^2)^2/c^3*a^2*ln(a*x^2+b*x+c)*b^2
*d^2+4*e^5/d^3/(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d)*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^5/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^5/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)/x**5/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11945, size = 792, normalized size = 2.13

$$\frac{\left(a^2b^3d^2e^2 - 3\,a^3bcd^2e^2 - 2\,ab^4de^3 + 8\,a^2b^2cde^3 - 4\,a^3c^2de^3 + b^5e^4 - 5\,ab^3ce^4 + 5\,a^2bc^2e^4\right)\arctan\left(-\frac{\left(2\,ad - \frac{2\,ad^2}{xe+d} - be + \frac{2\,bde}{xe+d} - \frac{2\,ce}{xe+}\right.}{\sqrt{-b^2+4\,ac}}\right.}{\left(a^2c^3d^4 - 2\,abc^3d^3e + b^2c^3d^2e^2 + 2\,ac^4d^2e^2 - 2\,bc^4de^3 + c^5e^4\right)\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^5/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] (a^2*b^3*d^2*e^2 - 3*a^3*b*c*d^2*e^2 - 2*a*b^4*d*e^3 + 8*a^2*b^2*c*d*e^3 -
4*a^3*c^2*d*e^3 + b^5*e^4 - 5*a*b^3*c*e^4 + 5*a^2*b*c^2*e^4)*arctan(-(2*a*d
 - 2*a*d^2/(x*e + d) - b*e + 2*b*d*e/(x*e + d) - 2*c*e^2/(x*e + d))*e^(-1)/
sqrt(-b^2 + 4*a*c))*e^(-2)/((a^2*c^3*d^4 - 2*a*b*c^3*d^3*e + b^2*c^3*d^2*e^
2 + 2*a*c^4*d^2*e^2 - 2*b*c^4*d*e^3 + c^5*e^4)*sqrt(-b^2 + 4*a*c)) - 1/2*(a
^2*b^2*d^2 - a^3*c*d^2 - 2*a*b^3*d*e + 4*a^2*b*c*d*e + b^4*e^2 - 3*a*b^2*c*
e^2 + a^2*c^2*e^2)*log(-a + 2*a*d/(x*e + d) - a*d^2/(x*e + d)^2 - b*e/(x*e
+ d) + b*d*e/(x*e + d)^2 - c*e^2/(x*e + d)^2)/(a^2*c^3*d^4 - 2*a*b*c^3*d^3*
e + b^2*c^3*d^2*e^2 + 2*a*c^4*d^2*e^2 - 2*b*c^4*d*e^3 + c^5*e^4) + e^9/((a*
d^5*e^5 - b*d^4*e^6 + c*d^3*e^7)*(x*e + d)) + (b^2*d^2*e - a*c*d^2*e + 2*b*
c*d*e^2 + 3*c^2*e^3)*e^(-1)*log(abs(-d/(x*e + d) + 1))/(c^3*d^4) + 1/2*(2*b
```

```
*c*d*e + 5*c^2*e^2 - 2*(b*c*d^2*e^2 + 3*c^2*d*e^3)*e^(-1)/(x*e + d))/(c^3*d
^4*(d/(x*e + d) - 1)^2)
```

3.79 $\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^4\sqrt{d+ex}\,dx$

Optimal. Leaf size=981

$$\frac{2}{11}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{d+ex}x^5+\frac{2(ad+be)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}(d+ex)^{7/2}x}{99ae^4}-\frac{2\left(29a^2d^2+8b^2e^2+ae(19bd-18ce)\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}(d}{693a^2e^4}$$

```
[Out] (-2*(187*a^4*d^4 + 64*b^4*e^4 + 4*a*b^2*e^3*(7*b*d - 69*c*e) - 4*a^3*d^2*e*
(2*b*d + 3*c*e) + 3*a^2*e^2*(3*b^2*d^2 - 29*b*c*d*e + 50*c^2*e^2))*Sqrt[a +
 c/x^2 + b/x]*x*Sqrt[d + e*x])/(3465*a^4*e^4) + (2*Sqrt[a + c/x^2 + b/x]*x^
5*Sqrt[d + e*x])/11 + (2*(233*a^3*d^3 + 48*b^3*e^3 + a*b*e^2*(67*b*d - 157*
c*e) + 4*a^2*d*e*(18*b*d - 37*c*e))*Sqrt[a + c/x^2 + b/x]*x*(d + e*x)^(3/2)
)/(3465*a^3*e^4) - (2*(29*a^2*d^2 + 8*b^2*e^2 + a*e*(19*b*d - 18*c*e))*Sqrt
[a + c/x^2 + b/x]*x*(d + e*x)^(5/2))/(693*a^2*e^4) + (2*(a*d + b*e)*Sqrt[a
+ c/x^2 + b/x]*x*(d + e*x)^(7/2))/(99*a*e^4) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(
128*a^5*d^5 + 128*b^5*e^5 - 4*a^4*d^3*e*(14*b*d - 27*c*e) - 8*a*b^3*e^4*(7*
b*d + 87*c*e) - a^2*b*e^3*(37*b^2*d^2 - 258*b*c*d*e - 771*c^2*e^2) - a^3*d*
e^2*(37*b^2*d^2 - 135*b*c*d*e + 156*c^2*e^2))*Sqrt[a + c/x^2 + b/x]*x*Sqrt[
d + e*x]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt
[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2
- 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3465*a^5*e^5*Sqrt[(a*(d
+ e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*(c + b*x + a*x^2)) - (2*Sqrt[2
]*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))*(128*a^4*d^4 - 64*b^4*e^4 - 4*a
*b^2*e^3*(7*b*d - 69*c*e) + 4*a^3*d^2*e*(2*b*d + 3*c*e) - 3*a^2*e^2*(3*b^2*
d^2 - 29*b*c*d*e + 50*c^2*e^2))*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(a*(d + e*x))/
(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*
a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a
*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)
])/(3465*a^5*e^5*Sqrt[d + e*x]*(c + b*x + a*x^2))
```

Rubi [A] time = 6.17238, antiderivative size = 981, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ =

0.241, Rules used = {1573, 918, 1653, 843, 718, 424, 419}

$$\frac{2}{11}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{d+ex}x^5+\frac{2(ad+be)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}(d+ex)^{7/2}x}{99ae^4}-\frac{2\left(29a^2d^2+8b^2e^2+ae(19bd-18ce)\right)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}(d}{693a^2e^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c/x^2 + b/x]*x^4*Sqrt[d + e*x],x]
```

```
[Out] (-2*(187*a^4*d^4 + 64*b^4*e^4 + 4*a*b^2*e^3*(7*b*d - 69*c*e) - 4*a^3*d^2*e*
(2*b*d + 3*c*e) + 3*a^2*e^2*(3*b^2*d^2 - 29*b*c*d*e + 50*c^2*e^2))*Sqrt[a +
 c/x^2 + b/x]*x*Sqrt[d + e*x])/(3465*a^4*e^4) + (2*Sqrt[a + c/x^2 + b/x]*x^
5*Sqrt[d + e*x])/11 + (2*(233*a^3*d^3 + 48*b^3*e^3 + a*b*e^2*(67*b*d - 157*
c*e) + 4*a^2*d*e*(18*b*d - 37*c*e))*Sqrt[a + c/x^2 + b/x]*x*(d + e*x)^(3/2)
)/(3465*a^3*e^4) - (2*(29*a^2*d^2 + 8*b^2*e^2 + a*e*(19*b*d - 18*c*e))*Sqrt
[a + c/x^2 + b/x]*x*(d + e*x)^(5/2))/(693*a^2*e^4) + (2*(a*d + b*e)*Sqrt[a
+ c/x^2 + b/x]*x*(d + e*x)^(7/2))/(99*a*e^4) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(
128*a^5*d^5 + 128*b^5*e^5 - 4*a^4*d^3*e*(14*b*d - 27*c*e) - 8*a*b^3*e^4*(7*
b*d + 87*c*e) - a^2*b*e^3*(37*b^2*d^2 - 258*b*c*d*e - 771*c^2*e^2) - a^3*d*
e^2*(37*b^2*d^2 - 135*b*c*d*e + 156*c^2*e^2))*Sqrt[a + c/x^2 + b/x]*x*Sqrt[
d + e*x]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt
[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2
- 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3465*a^5*e^5*Sqrt[(a*(d
+ e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*(c + b*x + a*x^2)) - (2*Sqrt[2
]*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))*(128*a^4*d^4 - 64*b^4*e^4 - 4*a
*b^2*e^3*(7*b*d - 69*c*e) + 4*a^3*d^2*e*(2*b*d + 3*c*e) - 3*a^2*e^2*(3*b^2*
d^2 - 29*b*c*d*e + 50*c^2*e^2))*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(a*(d + e*x))/
(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*
a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a
*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)
])/(3465*a^5*e^5*Sqrt[d + e*x]*(c + b*x + a*x^2))
```

Rule 1573

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_)
+ (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[(x^(2*n*FracPart[p])*(a + b/x^
n + c/x^(2*n))^FracPart[p])/(c + b*x^n + a*x^(2*n))^FracPart[p], Int[x^(m -
 2*n*p)*(d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p, x], x] /; FreeQ[{a, b, c,
d, e, m, n, p, q}, x] && EqQ[mn, -n] && EqQ[mn2, 2*mn] &&  !IntegerQ[p] &&
 !IntegerQ[q] && PosQ[n]
```

Rule 918

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[(2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*
Sqrt[a + b*x + c*x^2])/(e*(2*m + 5)), x] - Dist[1/(e*(2*m + 5)), Int[((d +
e*x)^m*Simp[b*d*f - 3*a*e*f + a*d*g + 2*(c*d*f - b*e*f + b*d*g - a*e*g)*x -
 (c*e*f - 3*c*d*g + b*e*g)*x^2, x])/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),
x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[e*f - d*g, 0] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[2*m] &&  !LtQ[m,
 -1]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^4\sqrt{d+ex}\,dx = \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int x^3\sqrt{d+ex}\sqrt{c+bx+ax^2}\,dx}{\sqrt{c+bx+ax^2}}$$

$$= \frac{2}{11}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^5\sqrt{d+ex} - \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int \frac{x^3\left(-3cd-2(bd+ce)x-(ad+be)x^2\right)}{\sqrt{d+ex}\sqrt{c+bx+ax^2}}\,dx}{11\sqrt{c+bx+ax^2}}$$

$$= \frac{2}{11}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^5\sqrt{d+ex} + \frac{2(ad+be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x(d+ex)^{7/2}}{99ae^4} - \frac{\left(2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int \cdots}{\cdots}$$

$$= \frac{2}{11}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^5\sqrt{d+ex} - \frac{2\left(29a^2d^2+8b^2e^2+ae(19bd-18ce)\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x(d+\cdots}{693a^2e^4}$$

$$= \frac{2}{11}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^5\sqrt{d+ex} + \frac{2\left(233a^3d^3+48b^3e^3+abe^2(67bd-157ce)+4a^2de(18bd\cdots\right.}{3465a^3e^4}$$

$$= -\frac{2\left(187a^4d^4+64b^4e^4+4ab^2e^3(7bd-69ce)-4a^3d^2e(2bd+3ce)+3a^2e^2\left(3b^2d^2-29b\cdots\right.\right.}{3465a^4e^4}$$

$$= -\frac{2\left(187a^4d^4+64b^4e^4+4ab^2e^3(7bd-69ce)-4a^3d^2e(2bd+3ce)+3a^2e^2\left(3b^2d^2-29b\cdots\right.\right.}{3465a^4e^4}$$

$$= -\frac{2\left(187a^4d^4+64b^4e^4+4ab^2e^3(7bd-69ce)-4a^3d^2e(2bd+3ce)+3a^2e^2\left(3b^2d^2-29b\cdots\right.\right.}{3465a^4e^4}$$

$$= -\frac{2\left(187a^4d^4+64b^4e^4+4ab^2e^3(7bd-69ce)-4a^3d^2e(2bd+3ce)+3a^2e^2\left(3b^2d^2-29b\cdots\right.\right.}{3465a^4e^4}$$

Mathematica [C] time = 14.3281, size = 10904, normalized size = 11.12

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + c/x^2 + b/x]*x^4*Sqrt[d + e*x],x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.148, size = 11938, normalized size = 12.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex + d}\sqrt{a + \frac{b}{x} + \frac{c}{x^2}}x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)*x^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{ex + d}x^4\sqrt{\frac{ax^2 + bx + c}{x^2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x + d)*x^4*sqrt((a*x^2 + b*x + c)/x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+c/x**2+b/x)**(1/2)*(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.80 $\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^3\sqrt{d+ex}\,dx$

Optimal. Leaf size=778

$$\frac{2\sqrt{2}x\sqrt{b^2-4ac}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}\left(6a^2cde^2+16a^3d^3-3abe^2(bd-9ce)-8b^3e^3\right)\left(ad^2-e(bd-ce)\right)\sqrt{\frac{a(d+ex)}{2ad-e\left(\sqrt{b^2-4a\right.}}}}{315a^4e^4\sqrt{d+ex}\left(ax^2+bx+c\right)}$$

```
[Out] (2*(19*a^3*d^3 - 6*a^2*c*d*e^2 + 8*b^3*e^3 + 3*a*b*e^2*(b*d - 9*c*e))*Sqrt[
a + c/x^2 + b/x]*x*Sqrt[d + e*x])/(315*a^3*e^3) + (2*Sqrt[a + c/x^2 + b/x]*
x^4*Sqrt[d + e*x])/9 - (4*(8*a^2*d^2 + 3*b^2*e^2 + a*e*(4*b*d - 7*c*e))*Sqr
t[a + c/x^2 + b/x]*x*(d + e*x)^(3/2))/(315*a^2*e^3) + (2*(a*d + b*e)*Sqrt[a
 + c/x^2 + b/x]*x*(d + e*x)^(5/2))/(63*a*e^3) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c
]*(8*a^4*d^4 + 8*b^4*e^4 - a^3*d^2*e*(4*b*d - 9*c*e) - 4*a*b^2*e^3*(b*d + 9
*c*e) - 3*a^2*e^2*(b^2*d^2 - 5*b*c*d*e - 7*c^2*e^2))*Sqrt[a + c/x^2 + b/x]*
x*Sqrt[d + e*x]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcS
in[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sq
rt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(315*a^4*e^4*Sqrt[
(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*(c + b*x + a*x^2)) + (2*
Sqrt[2]*Sqrt[b^2 - 4*a*c]*(16*a^3*d^3 + 6*a^2*c*d*e^2 - 8*b^3*e^3 - 3*a*b*e
^2*(b*d - 9*c*e))*(a*d^2 - e*(b*d - c*e))*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(a*(
d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^2))
/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt
[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 -
4*a*c])*e)])/(315*a^4*e^4*Sqrt[d + e*x]*(c + b*x + a*x^2))
```

Rubi [A] time = 2.3492, antiderivative size = 778, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1573, 918, 1653, 843, 718, 424, 419}

$$-\frac{2\sqrt{2}x\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}\left(-3a^2e^2\left(b^2d^2-5bcde-7c^2e^2\right)-a^3d^2e(4bd-9ce)+8a^4d^4-4ab^2e^3\right.}{315a^4e^4\left(ax^2+bx+c\right)\sqrt{\frac{a(d+ex)}{2ad-e\left(\sqrt{b^2-4ac}+b\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c/x^2 + b/x]*x^3*Sqrt[d + e*x],x]
```

```
[Out] (2*(19*a^3*d^3 - 6*a^2*c*d*e^2 + 8*b^3*e^3 + 3*a*b*e^2*(b*d - 9*c*e))*Sqrt[
a + c/x^2 + b/x]*x*Sqrt[d + e*x])/(315*a^3*e^3) + (2*Sqrt[a + c/x^2 + b/x]*
x^4*Sqrt[d + e*x])/9 - (4*(8*a^2*d^2 + 3*b^2*e^2 + a*e*(4*b*d - 7*c*e))*Sqr
t[a + c/x^2 + b/x]*x*(d + e*x)^(3/2))/(315*a^2*e^3) + (2*(a*d + b*e)*Sqrt[a
 + c/x^2 + b/x]*x*(d + e*x)^(5/2))/(63*a*e^3) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c
]*(8*a^4*d^4 + 8*b^4*e^4 - a^3*d^2*e*(4*b*d - 9*c*e) - 4*a*b^2*e^3*(b*d + 9
*c*e) - 3*a^2*e^2*(b^2*d^2 - 5*b*c*d*e - 7*c^2*e^2))*Sqrt[a + c/x^2 + b/x]*
x*Sqrt[d + e*x]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcS
in[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sq
rt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(315*a^4*e^4*Sqrt[
(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*(c + b*x + a*x^2)) + (2*
Sqrt[2]*Sqrt[b^2 - 4*a*c]*(16*a^3*d^3 + 6*a^2*c*d*e^2 - 8*b^3*e^3 - 3*a*b*e
^2*(b*d - 9*c*e))*(a*d^2 - e*(b*d - c*e))*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(a*(
d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^2))
/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt
[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 -
4*a*c])*e)])/(315*a^4*e^4*Sqrt[d + e*x]*(c + b*x + a*x^2))
```

Rule 1573

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_)
+ (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[(x^(2*n*FracPart[p])*(a + b/x^
n + c/x^(2*n))^FracPart[p])/(c + b*x^n + a*x^(2*n))^FracPart[p], Int[x^(m -
 2*n*p)*(d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p, x], x] /; FreeQ[{a, b, c,
d, e, m, n, p, q}, x] && EqQ[mn, -n] && EqQ[mn2, 2*mn] &&  !IntegerQ[p] &&
 !IntegerQ[q] && PosQ[n]
```

Rule 918

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[(2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*
Sqrt[a + b*x + c*x^2])/(e*(2*m + 5)), x] - Dist[1/(e*(2*m + 5)), Int[((d +
e*x)^m*Simp[b*d*f - 3*a*e*f + a*d*g + 2*(c*d*f - b*e*f + b*d*g - a*e*g)*x -
 (c*e*f - 3*c*d*g + b*e*g)*x^2, x])/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),
x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[e*f - d*g, 0] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[2*m] &&  !LtQ[m,
 -1]
```

Rule 1653

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
```

```
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^3\sqrt{d+ex}\,dx = \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int x^2\sqrt{d+ex}\sqrt{c+bx+ax^2}\,dx}{\sqrt{c+bx+ax^2}}$$

$$= \frac{2}{9}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^4\sqrt{d+ex} - \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int \frac{x^2\left(-3cd-2(bd+ce)x-(ad+be)x^2\right)}{\sqrt{d+ex}\sqrt{c+bx+ax^2}}\,dx}{9\sqrt{c+bx+ax^2}}$$

$$= \frac{2}{9}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^4\sqrt{d+ex} + \frac{2(ad+be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x(d+ex)^{5/2}}{63ae^3} - \frac{\left(2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int}{}$$

$$= \frac{2}{9}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^4\sqrt{d+ex} - \frac{4\left(8a^2d^2+3b^2e^2+ae(4bd-7ce)\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x(d+ex)^{3/}}{315a^2e^3}$$

$$= \frac{2\left(19a^3d^3-6a^2cde^2+8b^3e^3+3abe^2(bd-9ce)\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}}{315a^3e^3} + \frac{2}{9}\sqrt{a+\frac{c}{x^2}+}$$

$$= \frac{2\left(19a^3d^3-6a^2cde^2+8b^3e^3+3abe^2(bd-9ce)\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}}{315a^3e^3} + \frac{2}{9}\sqrt{a+\frac{c}{x^2}+}$$

$$= \frac{2\left(19a^3d^3-6a^2cde^2+8b^3e^3+3abe^2(bd-9ce)\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}}{315a^3e^3} + \frac{2}{9}\sqrt{a+\frac{c}{x^2}+}$$

$$= \frac{2\left(19a^3d^3-6a^2cde^2+8b^3e^3+3abe^2(bd-9ce)\right)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}}{315a^3e^3} + \frac{2}{9}\sqrt{a+\frac{c}{x^2}+}$$

Mathematica [C] time = 13.7562, size = 7531, normalized size = 9.68

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + c/x^2 + b/x]*x^3*Sqrt[d + e*x],x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.062, size = 9182, normalized size = 11.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex+d}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)*x^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{ex+d}x^3\sqrt{\frac{ax^2+bx+c}{x^2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x + d)*x^3*sqrt((a*x^2 + b*x + c)/x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+c/x**2+b/x)**(1/2)*(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.81 $\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^2\sqrt{d+ex}\,dx$

Optimal. Leaf size=636

$$-\frac{2\sqrt{2}x\sqrt{b^2-4ac}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{-\frac{a\left(ax^2+bx+c\right)}{b^2-4ac}}\left(8a^2d^2-ae(bd-10ce)-4b^2e^2\right)\left(ad^2-e(bd-ce)\right)\sqrt{\frac{a(d+ex)}{2ad-e\left(\sqrt{b^2-4ac}+b\right)}}\text{Elliptic}}{105a^3e^3\sqrt{d+ex}\left(ax^2+bx+c\right)}$$

```
[Out] (-2*Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x]*(4*a^2*d^2 + 4*b^2*e^2 - a*e*(2*b
*d - 5*c*e) - 3*a*e*(a*d - 4*b*e)*x))/(105*a^2*e^2) + (2*Sqrt[a + c/x^2 + b
/x]*x*Sqrt[d + e*x]*(c + b*x + a*x^2))/(7*a) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(
8*a^3*d^3 + 8*b^3*e^3 - a^2*d*e*(5*b*d - 16*c*e) - a*b*e^2*(5*b*d + 29*c*e)
)*Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 -
 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 -
4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])
*e)])/(105*a^3*e^3*Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*
(c + b*x + a*x^2)) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(8*a^2*d^2 - 4*b^2*e^2 -
a*e*(b*d - 10*c*e))*(a*d^2 - e*(b*d - c*e))*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(a
*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^2
))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sq
rt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2
- 4*a*c])*e)])/(105*a^3*e^3*Sqrt[d + e*x]*(c + b*x + a*x^2))
```

Rubi [A] time = 0.993044, antiderivative size = 636, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1573, 832, 814, 843, 718, 424, 419}

$$-\frac{2x\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\left(4a^2d^2-ae(2bd-5ce)-3aex(ad-4be)+4b^2e^2\right)}{105a^2e^2}-\frac{2\sqrt{2}x\sqrt{b^2-4ac}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{-\frac{a\left(ax^2+bx+c\right)}{b^2-4ac}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c/x^2 + b/x]*x^2*Sqrt[d + e*x],x]
```

```
[Out] (-2*Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x]*(4*a^2*d^2 + 4*b^2*e^2 - a*e*(2*b
*d - 5*c*e) - 3*a*e*(a*d - 4*b*e)*x))/(105*a^2*e^2) + (2*Sqrt[a + c/x^2 + b
/x]*x*Sqrt[d + e*x]*(c + b*x + a*x^2))/(7*a) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(
8*a^3*d^3 + 8*b^3*e^3 - a^2*d*e*(5*b*d - 16*c*e) - a*b*e^2*(5*b*d + 29*c*e)
)*Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 -
 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 -
4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])
*e)])/(105*a^3*e^3*Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*
(c + b*x + a*x^2)) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(8*a^2*d^2 - 4*b^2*e^2 -
a*e*(b*d - 10*c*e))*(a*d^2 - e*(b*d - c*e))*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(a
*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^2
))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sq
rt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2
- 4*a*c])*e)])/(105*a^3*e^3*Sqrt[d + e*x]*(c + b*x + a*x^2))
```

Rule 1573

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_)
+ (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[(x^(2*n*FracPart[p])*(a + b/x^
n + c/x^(2*n))^FracPart[p])/(c + b*x^n + a*x^(2*n))^FracPart[p], Int[x^(m -
 2*n*p)*(d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p, x], x] /; FreeQ[{a, b, c,
d, e, m, n, p, q}, x] && EqQ[mn, -n] && EqQ[mn2, 2*mn] &&  !IntegerQ[p] &&
 !IntegerQ[q] && PosQ[n]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 814

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2
) - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*(a + b*x + c*x^2)^p)
/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
```

```
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x^2\sqrt{d+ex}\,dx = \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int x\sqrt{d+ex}\sqrt{c+bx+ax^2}\,dx}{\sqrt{c+bx+ax^2}}$$

$$= \frac{2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\left(c+bx+ax^2\right)}{7a} + \frac{\left(2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int \frac{\left(\frac{1}{2}(-3bd-ce)+\frac{1}{2}(ad-4be)x\right)\sqrt{c+b}}{\sqrt{d+ex}}}{7a\sqrt{c+bx+ax^2}}$$

$$= -\frac{2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\left(4a^2d^2+4b^2e^2-ae(2bd-5ce)-3ae(ad-4be)x\right)}{105a^2e^2} + \frac{2\sqrt{a+}}{}$$

$$= -\frac{2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\left(4a^2d^2+4b^2e^2-ae(2bd-5ce)-3ae(ad-4be)x\right)}{105a^2e^2} + \frac{2\sqrt{a+}}{}$$

$$= -\frac{2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\left(4a^2d^2+4b^2e^2-ae(2bd-5ce)-3ae(ad-4be)x\right)}{105a^2e^2} + \frac{2\sqrt{a+}}{}$$

$$= -\frac{2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\left(4a^2d^2+4b^2e^2-ae(2bd-5ce)-3ae(ad-4be)x\right)}{105a^2e^2} + \frac{2\sqrt{a+}}{}$$

Mathematica [C] time = 13.0944, size = 5350, normalized size = 8.41

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + c/x^2 + b/x]*x^2*Sqrt[d + e*x],x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.049, size = 6302, normalized size = 9.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex+d}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)*x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{ex+d}x^2\sqrt{\frac{ax^2+bx+c}{x^2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x + d)*x^2*sqrt((a*x^2 + b*x + c)/x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+c/x**2+b/x)**(1/2)*(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.82 $\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{d+ex}\,dx$

Optimal. Leaf size=550

$$\frac{2\sqrt{2}x\sqrt{b^2-4ac}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}(2ad-be)\left(ad^2-e(bd-ce)\right)\sqrt{\frac{a(d+ex)}{2ad-e\left(\sqrt{b^2-4ac}+b\right)}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{\frac{\sqrt{b^2-4ac}+2ax+b}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right.\right.}{15a^2e^2\sqrt{d+ex}\left(ax^2+bx+c\right)}$$

```
[Out] (-2*(2*a*d - b*e)*Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x])/(15*a*e) + (2*Sqrt
[a + c/x^2 + b/x]*x*(d + e*x)^(3/2))/(5*e) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(
a^2*d^2 + b^2*e^2 - a*e*(b*d + 3*c*e))*Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x
]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + S
qrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c
]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(15*a^2*e^2*Sqrt[(a*(d + e*x))/(
2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*(c + b*x + a*x^2)) + (2*Sqrt[2]*Sqrt[b^
2 - 4*a*c]*(2*a*d - b*e)*(a*d^2 - e*(b*d - c*e))*Sqrt[a + c/x^2 + b/x]*x*Sq
rt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x +
a*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*
x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt
[b^2 - 4*a*c])*e)])/(15*a^2*e^2*Sqrt[d + e*x]*(c + b*x + a*x^2))
```

Rubi [A] time = 0.655454, antiderivative size = 550, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {1573, 734, 832, 843, 718, 424, 419}

$$\frac{2\sqrt{2}x\sqrt{b^2-4ac}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}(2ad-be)\left(ad^2-e(bd-ce)\right)\sqrt{\frac{a(d+ex)}{2ad-e\left(\sqrt{b^2-4ac}+b\right)}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2}{2ad-}\right.}{15a^2e^2\sqrt{d+ex}\left(ax^2+bx+c\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x],x]
```

```
[Out] (-2*(2*a*d - b*e)*Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x])/(15*a*e) + (2*Sqrt
[a + c/x^2 + b/x]*x*(d + e*x)^(3/2))/(5*e) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(
```

```
a^2*d^2 + b^2*e^2 - a*e*(b*d + 3*c*e))*Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x
]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + S
qrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c
]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(15*a^2*e^2*Sqrt[(a*(d + e*x))/(
2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*(c + b*x + a*x^2)) + (2*Sqrt[2]*Sqrt[b^
2 - 4*a*c]*(2*a*d - b*e)*(a*d^2 - e*(b*d - c*e))*Sqrt[a + c/x^2 + b/x]*x*Sq
rt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x +
a*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*
x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt
[b^2 - 4*a*c])*e)])/(15*a^2*e^2*Sqrt[d + e*x]*(c + b*x + a*x^2))
```

Rule 1573

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_)
+ (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[(x^(2*n*FracPart[p])*(a + b/x^
n + c/x^(2*n))^FracPart[p])/(c + b*x^n + a*x^(2*n))^FracPart[p], Int[x^(m -
 2*n*p)*(d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p, x], x] /; FreeQ[{a, b, c,
d, e, m, n, p, q}, x] && EqQ[mn, -n] && EqQ[mn2, 2*mn] &&  !IntegerQ[p] &&
 !IntegerQ[q] && PosQ[n]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + b*x + c*x^2)^(p +
1))/(c*(m + 2*p + 2)), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
```

```
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\,dx = \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int \sqrt{d+ex}\sqrt{c+bx+ax^2}\,dx}{\sqrt{c+bx+ax^2}}$$

$$= \frac{2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x(d+ex)^{3/2}}{5e} - \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int \frac{\sqrt{d+ex}(bd-2ce+(2ad-be)x)}{\sqrt{c+bx+ax^2}}\,dx}{5e\sqrt{c+bx+ax^2}}$$

$$= -\frac{2(2ad-be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}}{15ae} + \frac{2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x(d+ex)^{3/2}}{5e} - \frac{\left(2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int}{}$$

$$= -\frac{2(2ad-be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}}{15ae} + \frac{2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x(d+ex)^{3/2}}{5e} + \frac{\left((2ad-be)\left(ad^2-e\right.\right.}{}$$

$$= -\frac{2(2ad-be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}}{15ae} + \frac{2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x(d+ex)^{3/2}}{5e} - \frac{\left(2\sqrt{2}\sqrt{b^2-4ac}\left(a^2d\right.\right.}{}$$

$$= -\frac{2(2ad-be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}}{15ae} + \frac{2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x(d+ex)^{3/2}}{5e} - \frac{2\sqrt{2}\sqrt{b^2-4ac}\left(a^2d^2\right.}{}$$

Mathematica [C] time = 12.1613, size = 693, normalized size = 1.26

$$\frac{x\sqrt{a+\frac{bx+c}{x^2}}\left(\frac{i(d+ex)\sqrt{1-\frac{2\left(ad^2+e(ce-bd)\right)}{(d+ex)\left(\sqrt{e^2\left(b^2-4ac\right)}+2ad-be\right)}}\sqrt{\frac{4\left(ad^2+e(ce-bd)\right)}{(d+ex)\left(\sqrt{e^2\left(b^2-4ac\right)}-2ad+be\right)}+2}\left(a^2d\left(8ce^2-d\sqrt{e^2\left(b^2-4ac\right)}\right)+ae\left(bd\sqrt{e^2\left(b^2-4ac\right)}+3ce\sqrt{e^2\left(b^2-4ac\right)}-2b^2\right.\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x],x]
```

```
[Out] (x*Sqrt[a + (c + b*x)/x^2]*((4*e^2*(-(a^2*d^2) - b^2*e^2 + a*e*(b*d + 3*c*e
)))/Sqrt[d + e*x] + 2*a*e^2*Sqrt[d + e*x]*(b*e + a*(d + 3*e*x)) + (I*(d + e
*x)*Sqrt[1 - (2*(a*d^2 + e*(-(b*d) + c*e)))/((2*a*d - b*e + Sqrt[(b^2 - 4*a
*c)*e^2])*(d + e*x))]*Sqrt[2 + (4*(a*d^2 + e*(-(b*d) + c*e)))/((-2*a*d + b*
e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*((2*a*d - b*e + Sqrt[(b^2 - 4*a*c)
*e^2])*(a^2*d^2 + b^2*e^2 - a*e*(b*d + 3*c*e))*EllipticE[I*ArcSinh[(Sqrt[2]
*Sqrt[(a*d^2 - b*d*e + c*e^2)/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sq
rt[d + e*x]], -((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*a*d - b*e + Sqr
t[(b^2 - 4*a*c)*e^2]))] + (b^2*e^2*(b*e - Sqrt[(b^2 - 4*a*c)*e^2]) + a^2*d*
(8*c*e^2 - d*Sqrt[(b^2 - 4*a*c)*e^2]) + a*e*(-2*b^2*d*e - 4*b*c*e^2 + b*d*S
qrt[(b^2 - 4*a*c)*e^2] + 3*c*e*Sqrt[(b^2 - 4*a*c)*e^2]))*EllipticF[I*ArcSin
h[(Sqrt[2]*Sqrt[(a*d^2 - b*d*e + c*e^2)/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*
e^2])])/Sqrt[d + e*x]], -((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*a*d -
 b*e + Sqrt[(b^2 - 4*a*c)*e^2]))]))/(Sqrt[(a*d^2 + e*(-(b*d) + c*e))/(-2*a*
d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*(c + x*(b + a*x)))))/(15*a^2*e^3)
```

Maple [B] time = 0.045, size = 4361, normalized size = 7.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x)
```

```
[Out] -1/15*((a*x^2+b*x+c)/x^2)^(1/2)*x*(e*x+d)^(1/2)*(-10*x^2*a^2*b*d*e^3-2*x*a^
2*b*d^2*e^2-8*x*a^2*c*d*e^3-2*x*a*b^2*d*e^3-2*x*a*b*c*e^4-2*a*b*c*d*e^3-8*x
^3*a^3*d*e^3-8*x^3*a^2*b*e^4-2*x^2*a^3*d^2*e^2-6*x^2*a^2*c*e^4-2*x^2*a*b^2*
e^4-2*a^2*c*d^2*e^2-4*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))
^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(
1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2
)*EllipticE(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(
e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*b^
2*c*e^4-12*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(
-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+
2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticF
(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(e*(-4*a*c+b
^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*a*c^2*e^4-3*2
^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a
*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*
c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticF(2^(1/2)*(-a
*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*
a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*b^3*d*e^3+3*2^(1/2)*(-a*(
```

e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2
)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2)
)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticF(2^(1/2)*(-a*(e*x+d)/(e*
(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*
(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*b^2*c*e^4+12*2^(1/2)*(-a*(e*x+d)/(e*(
-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4
*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*
c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticE(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2
)^(1/2)-2*a*d+b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2
)^(1/2)+2*a*d-b*e))^(1/2))*a*c^2*e^4+4*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^
(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1
/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)
-2*a*d+b*e))^(1/2)*EllipticE(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*
d+b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*
d-b*e))^(1/2))*b^3*d*e^3-4*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+
b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*
e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))
^(1/2)*EllipticE(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2
),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2
))*a^3*d^4-3*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e
*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(
b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*Ellipti
cF(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(e*(-4*a*c
+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*(-4*a*c+b^2
)^(1/2)*a*b*d^2*e^2+2*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))
^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(
1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2
)*EllipticF(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(
e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*(-
4*a*c+b^2)^(1/2)*a*c*d*e^3+12*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a
*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d
-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*
e))^(1/2)*EllipticF(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(
1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(
1/2))*a*b*c*d*e^3-8*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(
1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/
2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*
EllipticE(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(e*
(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*a*b*
c*d*e^3+2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*
a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a
*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticF(2^
(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(e*(-4*a*c+b^2)
^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*(-4*a*c+b^2)^(1/
2)*b^2*d*e^3+2*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*

```
(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e
*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*Ellip
ticF(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(e*(-4*a
*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*(-4*a*c+b
^2)^(1/2)*a^2*d^3*e+8*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))
^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(
1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2
)*EllipticE(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(
e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*a^
2*b*d^3*e+8*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*
(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b
+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*Elliptic
E(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(e*(-4*a*c+
b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*a^2*c*d^2*e^
2-8*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+
(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(
-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticE(2^(1/2
)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1/
2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*a*b^2*d^2*e^2-2^(1/2
)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^
2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2
)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticF(2^(1/2)*(-a*(e*x
+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b
*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*(-4*a*c+b^2)^(1/2)*b*c*e^4-12*
2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*
a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a
*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticF(2^(1/2)*(-
a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2
*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*a^2*c*d^2*e^2+3*2^(1/2)*
(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)
^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^
(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticF(2^(1/2)*(-a*(e*x+d
)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e
)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*a*b^2*d^2*e^2-6*x^4*a^3*e^4)/a^2
/(a*e*x^3+a*d*x^2+b*e*x^2+b*d*x+c*e*x+c*d)/e^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex+d}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)*x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{ex+d}x\sqrt{\frac{ax^2+bx+c}{x^2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x + d)*x*sqrt((a*x^2 + b*x + c)/x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+c/x**2+b/x)**(1/2)*(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.83 $\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}\,dx$

Optimal. Leaf size=955

$$\frac{\sqrt{2}\sqrt{b^2-4ac}(ad+be)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{d+ex}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}\right)x}{3ae\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\left(ax^2+bx+c\right)}-\frac{2\sqrt{2}\sqrt{b^2-4ac}\cdots}{\cdots}$$

```
[Out] (2*Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x])/3 + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(a
*d + b*e)*Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x]*Sqrt[-((a*(c + b*x + a*x^2)
)/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqr
t[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 -
 4*a*c])*e)])/(3*a*e*Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)
]*(c + b*x + a*x^2)) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*d*(a*d + b*e)*Sqrt[a +
c/x^2 + b/x]*x*Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt
[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^
2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(
2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*a*e*Sqrt[d + e*x]*(c + b*x + a*x^2)
) + (4*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(b*d + c*e)*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(
a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^
2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/S
qrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2
 - 4*a*c])*e)])/(3*a*Sqrt[d + e*x]*(c + b*x + a*x^2)) - (Sqrt[2]*c*Sqrt[2*a
*d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[a + c/x^2 + b/x]*x*Sqrt[1 - (2*a*(d +
e*x))/(2*a*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*a*(d + e*x))/(2*a*d
- (b + Sqrt[b^2 - 4*a*c])*e)]*EllipticPi[(2*a*d - b*e + Sqrt[b^2 - 4*a*c]*e
)/(2*a*d), ArcSin[(Sqrt[2]*Sqrt[a]*Sqrt[d + e*x])/Sqrt[2*a*d - (b - Sqrt[b^
2 - 4*a*c])*e]], (b - Sqrt[b^2 - 4*a*c] - (2*a*d)/e)/(b + Sqrt[b^2 - 4*a*c]
 - (2*a*d)/e)])/(Sqrt[a]*(c + b*x + a*x^2))
```

Rubi [A] time = 3.31272, antiderivative size = 955, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$

= 0.423, Rules used = {1449, 918, 6742, 718, 419, 934, 169, 538, 537, 843, 424}

$$\frac{\sqrt{2}\sqrt{b^2-4ac}(ad+be)\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{d+ex}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}\right)x}{3ae\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\left(ax^2+bx+c\right)}-\frac{2\sqrt{2}\sqrt{b^2-4a}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x],x]
```

```
[Out] (2*Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x])/3 + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(a
*d + b*e)*Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x]*Sqrt[-((a*(c + b*x + a*x^2)
)/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqr
t[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 -
 4*a*c])*e)])/(3*a*e*Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)
]*(c + b*x + a*x^2)) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*d*(a*d + b*e)*Sqrt[a +
c/x^2 + b/x]*x*Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt
[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^
2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(
2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*a*e*Sqrt[d + e*x]*(c + b*x + a*x^2)
) + (4*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(b*d + c*e)*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(
a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^
2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/S
qrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2
 - 4*a*c])*e)])/(3*a*Sqrt[d + e*x]*(c + b*x + a*x^2)) - (Sqrt[2]*c*Sqrt[2*a
*d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[a + c/x^2 + b/x]*x*Sqrt[1 - (2*a*(d +
e*x))/(2*a*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*a*(d + e*x))/(2*a*d
- (b + Sqrt[b^2 - 4*a*c])*e)]*EllipticPi[(2*a*d - b*e + Sqrt[b^2 - 4*a*c]*e
)/(2*a*d), ArcSin[(Sqrt[2]*Sqrt[a]*Sqrt[d + e*x])/Sqrt[2*a*d - (b - Sqrt[b^
2 - 4*a*c])*e]], (b - Sqrt[b^2 - 4*a*c] - (2*a*d)/e)/(b + Sqrt[b^2 - 4*a*c]
 - (2*a*d)/e)])/(Sqrt[a]*(c + b*x + a*x^2))
```

Rule 1449

```
Int[((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_
)^(n_.))^(q_.), x_Symbol] :> Dist[(x^(2*n*FracPart[p])*(a + b/x^n + c/x^(2*
n))^FracPart[p])/(c + b*x^n + a*x^(2*n))^FracPart[p], Int[((d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p)/x^(2*n*p), x], x] /; FreeQ[{a, b, c, d, e, n, p, q
}, x] && EqQ[mn, -n] && EqQ[mn2, 2*mn] &&  !IntegerQ[p] &&  !IntegerQ[q] &&
 PosQ[n]
```

Rule 918

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[(2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*
Sqrt[a + b*x + c*x^2])/(e*(2*m + 5)), x] - Dist[1/(e*(2*m + 5)), Int[((d +
e*x)^m*Simp[b*d*f - 3*a*e*f + a*d*g + 2*(c*d*f - b*e*f + b*d*g - a*e*g)*x -
 (c*e*f - 3*c*d*g + b*e*g)*x^2, x])/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),
x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[e*f - d*g, 0] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[2*m] &&  !LtQ[m,
 -1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}\,dx = \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int \frac{\sqrt{d+ex}\sqrt{c+bx+ax^2}}{x}\,dx}{\sqrt{c+bx+ax^2}}$$

$$= \frac{2}{3}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex} - \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int \frac{-3cd-2(bd+ce)x-(ad+be)x^2}{x\sqrt{d+ex}\sqrt{c+bx+ax^2}}\,dx}{3\sqrt{c+bx+ax^2}}$$

$$= \frac{2}{3}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex} - \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int \left(-\frac{2(bd+ce)}{\sqrt{d+ex}\sqrt{c+bx+ax^2}} - \frac{3cd}{x\sqrt{d+ex}\sqrt{c+bx+ax^2}} - \frac{(a\ldots}{\sqrt{d+ex}\ldots}\right.}{3\sqrt{c+bx+ax^2}}$$

$$= \frac{2}{3}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex} + \frac{\left(cd\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int \frac{1}{x\sqrt{d+ex}\sqrt{c+bx+ax^2}}\,dx}{\sqrt{c+bx+ax^2}} - \frac{\left((-ad-be)\sqrt{a+\frac{c}{x^2}\ldots}\right.}{3\sqrt{c\ldots}}$$

$$= \frac{2}{3}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex} + \frac{\left(cd\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{b-\sqrt{b^2-4ac}+2ax}\sqrt{b+\sqrt{b^2-4ac}+2ax\ldots}\right.}{c+bx+ax^2}$$

$$= \frac{2}{3}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex} + \frac{4\sqrt{2}\sqrt{b^2-4ac}(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(c+\ldots}{b\ldots}}}{3a\sqrt{d+ex}\left(c+bx\ldots\right.}$$

$$= \frac{2}{3}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex} + \frac{\sqrt{2}\sqrt{b^2-4ac}(ad+be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\sqrt{-\frac{a\left(c+bx+ax^2\right)}{b^2-4ac}}E\left(\ldots\right.}{3ae\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\left(c+bx\ldots\right.}$$

$$= \frac{2}{3}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex} + \frac{\sqrt{2}\sqrt{b^2-4ac}(ad+be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\sqrt{-\frac{a\left(c+bx+ax^2\right)}{b^2-4ac}}E\left(\ldots\right.}{3ae\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\left(c+bx\ldots\right.}$$

$$= \frac{2}{3}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex} + \frac{\sqrt{2}\sqrt{b^2-4ac}(ad+be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\sqrt{-\frac{a\left(c+bx+ax^2\right)}{b^2-4ac}}E\left(\ldots\right.}{3ae\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\left(c+bx\ldots\right.}$$

<u>**Mathematica**</u> **[C]** time = 10.6806, size = 1258, normalized size = 1.32

$$x\sqrt{a+\frac{c+bx}{x^2}}\left(\frac{4(ad+be)\sqrt{\frac{ad^2+e(ce-bd)}{-2ad+be+\sqrt{(b^2-4ac)e^2}}}(c+x(b+ax))e^2}{(d+ex)^2}+\frac{6i\sqrt{2}ac\sqrt{\frac{-2ce^2+2adxe+b(d-ex)e+\sqrt{(b^2-4ac)e^2}(d+ex)}{\left(2ad-be+\sqrt{(b^2-4ac)e^2}\right)(d+ex)}}\sqrt{\frac{2ce^2-2adxe+b(ex-d)e+\sqrt{(b^2-4ac)e^2}(d+ex)}{\left(-2ad+be+\sqrt{(b^2-4ac)e^2}\right)(d+ex)}}\Pi\left(\frac{d}{-}\right.}{\sqrt{d+ex}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x],x]
```

```
[Out] (2*x*Sqrt[d + e*x]*Sqrt[a + (c + b*x)/x^2])/3 + (x*(d + e*x)^(3/2)*Sqrt[a +
 (c + b*x)/x^2]*((4*e^2*(a*d + b*e)*Sqrt[(a*d^2 + e*(-(b*d) + c*e))/(-2*a*d
 + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*(c + x*(b + a*x)))/(d + e*x)^2 - (I*Sqrt
[2]*(a*d + b*e)*(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*Sqrt[(-2*c*e^2 + 2*
a*d*e*x + b*e*(d - e*x) + Sqrt[(b^2 - 4*a*c)*e^2]*(d + e*x))/((2*a*d - b*e
+ Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[(2*c*e^2 - 2*a*d*e*x + b*e*(-d
+ e*x) + Sqrt[(b^2 - 4*a*c)*e^2]*(d + e*x))/((-2*a*d + b*e + Sqrt[(b^2 - 4*
a*c)*e^2])*(d + e*x))]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(a*d^2 - b*d*e + c
*e^2)/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*a*d
+ b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])
/Sqrt[d + e*x] + (I*Sqrt[2]*(b*e*(-(b*e) + Sqrt[(b^2 - 4*a*c)*e^2]) + a*(3*
b*d*e - 2*c*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2]))*Sqrt[(-2*c*e^2 + 2*a*d*e*x +
b*e*(d - e*x) + Sqrt[(b^2 - 4*a*c)*e^2]*(d + e*x))/((2*a*d - b*e + Sqrt[(b^
2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[(2*c*e^2 - 2*a*d*e*x + b*e*(-d + e*x) + S
qrt[(b^2 - 4*a*c)*e^2]*(d + e*x))/((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])
*(d + e*x))]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(a*d^2 - b*d*e + c*e^2)/(-2*
a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*a*d + b*e + Sq
rt[(b^2 - 4*a*c)*e^2])/(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt[d +
e*x] + ((6*I)*Sqrt[2]*a*c*e^2*Sqrt[(-2*c*e^2 + 2*a*d*e*x + b*e*(d - e*x) +
Sqrt[(b^2 - 4*a*c)*e^2]*(d + e*x))/((2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])
*(d + e*x))]*Sqrt[(2*c*e^2 - 2*a*d*e*x + b*e*(-d + e*x) + Sqrt[(b^2 - 4*a*c
)*e^2]*(d + e*x))/((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Ell
ipticPi[(d*(2*a*d - b*e - Sqrt[(b^2 - 4*a*c)*e^2]))/(2*(a*d^2 + e*(-(b*d) +
 c*e))), I*ArcSinh[(Sqrt[2]*Sqrt[(a*d^2 - b*d*e + c*e^2)/(-2*a*d + b*e + Sq
rt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*a*d + b*e + Sqrt[(b^2 - 4*a*
c)*e^2])/(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt[d + e*x]))/(6*a*e^
2*Sqrt[(a*d^2 + e*(-(b*d) + c*e))/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]
```

```
*(c + x*(b + a*x)))
```

Maple [B] time = 0.048, size = 3023, normalized size = 3.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x)
```

```
[Out] 1/3*((a*x^2+b*x+c)/x^2)^(1/2)*x*(e*x+d)^(1/2)*(2^(1/2)*(-a*(e*x+d)/(e*(-4*a
*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c
+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^
2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticF(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1
/2)-2*a*d+b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1
/2)+2*a*d-b*e))^(1/2))*(-4*a*c+b^2)^(1/2)*a*d^2*e-2^(1/2)*(-a*(e*x+d)/(e*(-
4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*
a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c
+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticF(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)
^(1/2)-2*a*d+b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)
^(1/2)+2*a*d-b*e))^(1/2))*(-4*a*c+b^2)^(1/2)*b*d*e^2-2*2^(1/2)*(-a*(e*x+d)/
(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e
*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-
4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticF(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c
+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c
+b^2)^(1/2)+2*a*d-b*e))^(1/2))*(-4*a*c+b^2)^(1/2)*c*e^3+3*2^(1/2)*(-a*(e*x+
d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)
/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e
*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticF(2^(1/2)*(-a*(e*x+d)/(e*(-4*
a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*
a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*a*b*d^2*e+6*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*
c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+
b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2
)^(1/2)-2*a*d+b*e))^(1/2)*EllipticF(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/
2)-2*a*d+b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/
2)+2*a*d-b*e))^(1/2))*a*c*d*e^2-3*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)
-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2
*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*
d+b*e))^(1/2)*EllipticF(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e
))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e
))^(1/2))*b^2*d*e^2-2*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))
^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(
```

```
1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2
)*EllipticE(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(
e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*a^
2*d^3-2*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*
a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a
*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticE(2^
(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(e*(-4*a*c+b^2)
^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*a*c*d*e^2+2*2^(1
/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+
b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b
^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticE(2^(1/2)*(-a*(e
*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d
+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*b^2*d*e^2-2*2^(1/2)*(-a*(e*x
+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b
)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(
e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticE(2^(1/2)*(-a*(e*x+d)/(e*(-4
*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4
*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*b*c*e^3+3*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c
+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b
^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)
^(1/2)-2*a*d+b*e))^(1/2)*EllipticPi(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/
2)-2*a*d+b*e))^(1/2),-1/2*(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/a/d,(-(e*(-4*a*c
+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*(-4*a*c+b^2
)^(1/2)*c*e^3-6*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)
*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(
e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*Elli
pticPi(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),-1/2*(e*
(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/a/d,(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4
*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*a*c*d*e^2+3*2^(1/2)*(-a*(e*x+d)/(e*(-4*a
*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c
+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^
2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticPi(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(
1/2)-2*a*d+b*e))^(1/2),-1/2*(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/a/d,(-(e*(-4*a
*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*b*c*e^3+2
*x^3*a^2*e^3+2*x^2*a^2*d*e^2+2*x^2*a*b*e^3+2*x*a*b*d*e^2+2*x*a*c*e^3+2*a*c*
d*e^2)/a/e^2/(a*e*x^3+a*d*x^2+b*e*x^2+b*d*x+c*e*x+c*d)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex+d}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x**2+b/x)**(1/2)*(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.84 $\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{x}\,dx$

Optimal. Leaf size=929

$$\frac{3\sqrt{b^2-4ac}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,x\sqrt{d+ex}\sqrt{-\frac{a\left(ax^2+bx+c\right)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}\,e}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{2}\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\left(ax^2+bx+c\right)}-\frac{3\sqrt{2}\sqrt{b^2-4ac}\,d\sqrt{a+\frac{b}{x}}}{}$$

```
[Out] -(Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x]) + (3*Sqrt[b^2 - 4*a*c]*Sqrt[a + c/x^
2 + b/x]*x*Sqrt[d + e*x]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*Ellip
ticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]
], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(Sqrt[2]*
Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*(c + b*x + a*x^2))
- (3*Sqrt[2]*Sqrt[b^2 - 4*a*c]*d*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(a*(d + e*x))
/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4
*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*
a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e
)])/(Sqrt[d + e*x]*(c + b*x + a*x^2)) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(a*d +
 b*e)*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4
*a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sq
rt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^
2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(a*Sqrt[d + e*x]*(c + b
*x + a*x^2)) - ((b*d + c*e)*Sqrt[2*a*d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[a
+ c/x^2 + b/x]*x*Sqrt[1 - (2*a*(d + e*x))/(2*a*d - (b - Sqrt[b^2 - 4*a*c])*
e)]*Sqrt[1 - (2*a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*EllipticP
i[(2*a*d - b*e + Sqrt[b^2 - 4*a*c]*e)/(2*a*d), ArcSin[(Sqrt[2]*Sqrt[a]*Sqrt
[d + e*x])/Sqrt[2*a*d - (b - Sqrt[b^2 - 4*a*c])*e]], (b - Sqrt[b^2 - 4*a*c]
 - (2*a*d)/e)/(b + Sqrt[b^2 - 4*a*c] - (2*a*d)/e)])/(Sqrt[2]*Sqrt[a]*d*(c +
 b*x + a*x^2))
```

Rubi [A] time = 2.72487, antiderivative size = 929, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$

= 0.379, Rules used = {1573, 916, 6742, 718, 419, 934, 169, 538, 537, 843, 424}

$$\frac{3\sqrt{b^2-4ac}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}x\sqrt{d+ex}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}e}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{2}\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\left(ax^2+bx+c\right)}-\frac{3\sqrt{2}\sqrt{b^2-4ac}d\sqrt{a+\frac{b}{x}+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x])/x,x]
```

```
[Out] -(Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x]) + (3*Sqrt[b^2 - 4*a*c]*Sqrt[a + c/x^
2 + b/x]*x*Sqrt[d + e*x]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*Ellip
ticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]
], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(Sqrt[2]*
Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*(c + b*x + a*x^2))
- (3*Sqrt[2]*Sqrt[b^2 - 4*a*c]*d*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(a*(d + e*x))
/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4
*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*
a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e
)])/(Sqrt[d + e*x]*(c + b*x + a*x^2)) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(a*d +
 b*e)*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4
*a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sq
rt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^
2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(a*Sqrt[d + e*x]*(c + b
*x + a*x^2)) - ((b*d + c*e)*Sqrt[2*a*d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[a
+ c/x^2 + b/x]*x*Sqrt[1 - (2*a*(d + e*x))/(2*a*d - (b - Sqrt[b^2 - 4*a*c])*
e)]*Sqrt[1 - (2*a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*EllipticP
i[(2*a*d - b*e + Sqrt[b^2 - 4*a*c]*e)/(2*a*d), ArcSin[(Sqrt[2]*Sqrt[a]*Sqrt
[d + e*x])/Sqrt[2*a*d - (b - Sqrt[b^2 - 4*a*c])*e]], (b - Sqrt[b^2 - 4*a*c]
 - (2*a*d)/e)/(b + Sqrt[b^2 - 4*a*c] - (2*a*d)/e)])/(Sqrt[2]*Sqrt[a]*d*(c +
 b*x + a*x^2))
```

Rule 1573

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_)
+ (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[(x^(2*n*FracPart[p])*(a + b/x^
n + c/x^(2*n))^FracPart[p])/(c + b*x^n + a*x^(2*n))^FracPart[p], Int[x^(m -
 2*n*p)*(d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p, x], x] /; FreeQ[{a, b, c,
d, e, m, n, p, q}, x] && EqQ[mn, -n] && EqQ[mn2, 2*mn] &&  !IntegerQ[p] &&
 !IntegerQ[q] && PosQ[n]
```

Rule 916

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[((d + e*x)^(m + 1)*Sqrt[f + g*x]*Sq
rt[a + b*x + c*x^2])/(e*(m + 1)), x] - Dist[1/(2*e*(m + 1)), Int[((d + e*x)
^(m + 1)*Simp[b*f + a*g + 2*(c*f + b*g)*x + 3*c*g*x^2, x])/(Sqrt[f + g*x]*S
qrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f
 - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Integ
erQ[2*m] && LtQ[m, -1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
```

```
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{x}\,dx = \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int \frac{\sqrt{d+ex}\sqrt{c+bx+ax^2}}{x^2}\,dx}{\sqrt{c+bx+ax^2}}$$

$$= -\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex} + \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int \frac{bd+ce+2(ad+be)x+3aex^2}{x\sqrt{d+ex}\sqrt{c+bx+ax^2}}\,dx}{2\sqrt{c+bx+ax^2}}$$

$$= -\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex} + \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int \left(\frac{2(ad+be)}{\sqrt{d+ex}\sqrt{c+bx+ax^2}} + \frac{bd+ce}{x\sqrt{d+ex}\sqrt{c+bx+ax^2}} + \frac{3a}{\sqrt{d+ex}\sqrt{c}} \right.}{2\sqrt{c+bx+ax^2}}$$

$$= -\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex} + \frac{\left(3ae\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int \frac{x}{\sqrt{d+ex}\sqrt{c+bx+ax^2}}\,dx}{2\sqrt{c+bx+ax^2}} + \frac{\left((ad+be)\sqrt{a+\frac{c}{x^2}+\right.}{\sqrt{c+}}$$

$$= -\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex} + \frac{\left((bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{b-\sqrt{b^2-4ac}+2ax}\sqrt{b+\sqrt{b^2-4ac}\right.}{2\left(c+bx+ax^2\right.}$$

$$= -\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex} + \frac{2\sqrt{2}\sqrt{b^2-4ac}(ad+be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\sqrt{-\frac{a(c+}{b}}}{a\sqrt{d+ex}\left(c+bx+\right.}$$

$$= -\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex} + \frac{3\sqrt{b^2-4ac}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\sqrt{-\frac{a\left(c+bx+ax^2\right)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{}}{}}}{}\right.\right.}{\sqrt{2}\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\left(c+bx+ax^2\right.}$$

$$= -\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex} + \frac{3\sqrt{b^2-4ac}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\sqrt{-\frac{a\left(c+bx+ax^2\right)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{}}{}}}{}\right.\right.}{\sqrt{2}\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\left(c+bx+ax^2\right.}$$

$$= -\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex} + \frac{3\sqrt{b^2-4ac}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{d+ex}\sqrt{-\frac{a\left(c+bx+ax^2\right)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{}}{}}}{}\right.\right.}{\sqrt{2}\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\left(c+bx+ax^2\right.}$$

Mathematica [C] time = 11.4859, size = 1372, normalized size = 1.48

$$\frac{x(d+ex)^{3/2}\sqrt{a+\frac{c+bx}{x^2}}\left(12d\sqrt{\frac{ad^2+e(ce-bd)}{-2ad+be+\sqrt{(b^2-4ac)e^2}}}\left(a\left(\frac{d}{d+ex}-1\right)^2+\frac{e\left(-\frac{db}{d+ex}+b+\frac{ce}{d+ex}\right)}{d+ex}\right)-\frac{3i\sqrt{2}d\left(2ad-be+\sqrt{(b^2-4ac)e^2}\right)\sqrt{\frac{-\frac{2ce^2}{d+ex}+b\left(\frac{2d}{d+ex}-1\right)e\cdots}{2ad-b\cdots}}\cdots}{\cdots}\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x])/x,x]
```

```
[Out] -(Sqrt[d + e*x]*Sqrt[a + (c + b*x)/x^2]) + (x*(d + e*x)^(3/2)*Sqrt[a + (c +
 b*x)/x^2]*(12*d*Sqrt[(a*d^2 + e*(-(b*d) + c*e))/(-2*a*d + b*e + Sqrt[(b^2
- 4*a*c)*e^2])]*(a*(-1 + d/(d + e*x))^2 + (e*(b - (b*d)/(d + e*x) + (c*e)/(
d + e*x)))/(d + e*x)) - ((3*I)*Sqrt[2]*d*(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*
e^2])*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] - (2*c*e^2)/(d + e*x) - 2*a*d*(-1 + d/(
d + e*x)) + b*e*(-1 + (2*d)/(d + e*x)))/(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e
^2])]*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] + (2*c*e^2)/(d + e*x) + 2*a*d*(-1 + d/(
d + e*x)) + b*(e - (2*d*e)/(d + e*x)))/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e
^2])]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(a*d^2 - b*d*e + c*e^2)/(-2*a*d + b
*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*a*d + b*e + Sqrt[(b^2
 - 4*a*c)*e^2])/(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt[d + e*x] +
(I*Sqrt[2]*(4*a*d^2 - b*d*e - 2*c*e^2 + 3*d*Sqrt[(b^2 - 4*a*c)*e^2])*Sqrt[(
Sqrt[(b^2 - 4*a*c)*e^2] - (2*c*e^2)/(d + e*x) - 2*a*d*(-1 + d/(d + e*x)) +
b*e*(-1 + (2*d)/(d + e*x)))/(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*Sqrt[(
Sqrt[(b^2 - 4*a*c)*e^2] + (2*c*e^2)/(d + e*x) + 2*a*d*(-1 + d/(d + e*x)) +
b*(e - (2*d*e)/(d + e*x)))/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*Ellipt
icF[I*ArcSinh[(Sqrt[2]*Sqrt[(a*d^2 - b*d*e + c*e^2)/(-2*a*d + b*e + Sqrt[(b
^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^
2])/(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt[d + e*x] + ((2*I)*Sqrt[
2]*e*(b*d + c*e)*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] - (2*c*e^2)/(d + e*x) - 2*a*
d*(-1 + d/(d + e*x)) + b*e*(-1 + (2*d)/(d + e*x)))/(2*a*d - b*e + Sqrt[(b^2
 - 4*a*c)*e^2])]*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] + (2*c*e^2)/(d + e*x) + 2*a*
d*(-1 + d/(d + e*x)) + b*(e - (2*d*e)/(d + e*x)))/(-2*a*d + b*e + Sqrt[(b^2
 - 4*a*c)*e^2])]*EllipticPi[(d*(2*a*d - b*e - Sqrt[(b^2 - 4*a*c)*e^2]))/(2*
(a*d^2 + e*(-(b*d) + c*e))), I*ArcSinh[(Sqrt[2]*Sqrt[(a*d^2 - b*d*e + c*e^2
)/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*a*d + b*
e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqr
```

```
t[d + e*x]))/(4*d*e*Sqrt[(a*d^2 + e*(-(b*d) + c*e))/(-2*a*d + b*e + Sqrt[(b
^2 - 4*a*c)*e^2])]*Sqrt[c + b*x + a*x^2]*Sqrt[((d + e*x)^2*(a*(-1 + d/(d +
e*x))^2 + (e*(b - (b*d)/(d + e*x) + (c*e)/(d + e*x)))/(d + e*x)))/e^2])
```

Maple [B] time = 0.045, size = 3553, normalized size = 3.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)/x,x)
```

```
[Out] 1/2*((a*x^2+b*x+c)/x^2)^(1/2)*(e*x+d)^(1/2)*(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c
+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b
^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)
^(1/2)-2*a*d+b*e))^(1/2)*EllipticF(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2
)-2*a*d+b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2
)+2*a*d-b*e))^(1/2))*(-4*a*c+b^2)^(1/2)*x*a*d^2*e-2*2^(1/2)*(-a*(e*x+d)/(e*
(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-
4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a
*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticF(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^
2)^(1/2)-2*a*d+b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^
2)^(1/2)+2*a*d-b*e))^(1/2))*(-4*a*c+b^2)^(1/2)*x*b*d*e^2+4*2^(1/2)*(-a*(e*x
+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b
)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(
e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticF(2^(1/2)*(-a*(e*x+d)/(e*(-4
*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4
*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*x*a^2*d^3-2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c
+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b
^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)
^(1/2)-2*a*d+b*e))^(1/2)*EllipticF(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2
)-2*a*d+b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2
)+2*a*d-b*e))^(1/2))*x*a*b*d^2*e+6*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2
)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+
2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a
*d+b*e))^(1/2)*EllipticF(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*
e))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*
e))^(1/2))*x*a*c*d*e^2-2*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*
e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e)
)^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(
1/2)*EllipticF(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),
(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))
```

```
*x*b^2*d*e^2-6*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*
(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e
*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*Ellip
ticE(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(e*(-4*a
*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*x*a^2*d^3
+6*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(
-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-
4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticE(2^(1/2)
*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1/2
)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*x*a*b*d^2*e-6*2^(1/2)
*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2
)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)
^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticE(2^(1/2)*(-a*(e*x+
d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*
e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*x*a*c*d*e^2+2^(1/2)*(-a*(e*x+d)
/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(
e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(
-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticPi(2^(1/2)*(-a*(e*x+d)/(e*(-4*a
*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),-1/2*(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/a/d,(
-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*
(-4*a*c+b^2)^(1/2)*x*b*d*e^2+2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*
d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-
b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e
))^(1/2)*EllipticPi(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(
1/2),-1/2*(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/a/d,(-(e*(-4*a*c+b^2)^(1/2)-2*a*
d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*(-4*a*c+b^2)^(1/2)*x*c*e^3-
2*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-
4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4
*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticPi(2^(1/2)
*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),-1/2*(e*(-4*a*c+b^2)^(
1/2)-2*a*d+b*e)/a/d,(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2
)+2*a*d-b*e))^(1/2))*x*a*b*d^2*e-2*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2
)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+
2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a
*d+b*e))^(1/2)*EllipticPi(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b
*e))^(1/2),-1/2*(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/a/d,(-(e*(-4*a*c+b^2)^(1/2
)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*x*a*c*d*e^2+2^(1/2)*(
-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^
(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(
1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticPi(2^(1/2)*(-a*(e*x+d
)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),-1/2*(e*(-4*a*c+b^2)^(1/2)-2*a*d+
b*e)/a/d,(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e
))^(1/2))*x*b^2*d*e^2+2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))
^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(
1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2
```

```
)*EllipticPi(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),-1
/2*(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/a/d,(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/
(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*x*b*c*e^3-2*x^3*a^2*d*e^2-2*x^2*a^
2*d^2*e-2*x^2*a*b*d*e^2-2*x*a*b*d^2*e-2*x*a*c*d*e^2-2*a*c*d^2*e)/(a*e*x^3+a
*d*x^2+b*e*x^2+b*d*x+c*e*x+c*d)/a/e/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)/x, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x**2+b/x)**(1/2)*(e*x+d)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(d + e*x)*sqrt(a + b/x + c/x**2)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)/x, x)
```

3.85 $\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{x^2}\,dx$

Optimal. Leaf size=1287

result too large to display

```
[Out] -((b*d + c*e)*Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x])/(4*c*d) - (Sqrt[a + c/x^
2 + b/x]*Sqrt[d + e*x])/(2*x) + (Sqrt[b^2 - 4*a*c]*(b*d + c*e)*Sqrt[a + c/x
^2 + b/x]*x*Sqrt[d + e*x]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*Elli
pticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2
]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(4*Sqrt[
2]*c*d*Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*(c + b*x + a
*x^2)) + (3*Sqrt[b^2 - 4*a*c]*e*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(a*(d + e*x))/
(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*
a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a
*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)
])/(Sqrt[2]*Sqrt[d + e*x]*(c + b*x + a*x^2)) - (Sqrt[b^2 - 4*a*c]*(b*d + c*
e)*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*
c])*e)]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[
(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 -
 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(2*Sqrt[2]*c*Sqrt[d + e*x]
*(c + b*x + a*x^2)) - ((a*d + b*e)*Sqrt[2*a*d - (b - Sqrt[b^2 - 4*a*c])*e]*
Sqrt[a + c/x^2 + b/x]*x*Sqrt[1 - (2*a*(d + e*x))/(2*a*d - (b - Sqrt[b^2 - 4
*a*c])*e)]*Sqrt[1 - (2*a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*El
lipticPi[(2*a*d - b*e + Sqrt[b^2 - 4*a*c]*e)/(2*a*d), ArcSin[(Sqrt[2]*Sqrt[
a]*Sqrt[d + e*x])/Sqrt[2*a*d - (b - Sqrt[b^2 - 4*a*c])*e]], (b - Sqrt[b^2 -
 4*a*c] - (2*a*d)/e)/(b + Sqrt[b^2 - 4*a*c] - (2*a*d)/e)])/(Sqrt[2]*Sqrt[a]
*d*(c + b*x + a*x^2)) + ((b*d + c*e)^2*Sqrt[2*a*d - (b - Sqrt[b^2 - 4*a*c])
*e]*Sqrt[a + c/x^2 + b/x]*x*Sqrt[1 - (2*a*(d + e*x))/(2*a*d - (b - Sqrt[b^2
 - 4*a*c])*e)]*Sqrt[1 - (2*a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)
]*EllipticPi[(2*a*d - b*e + Sqrt[b^2 - 4*a*c]*e)/(2*a*d), ArcSin[(Sqrt[2]*S
qrt[a]*Sqrt[d + e*x])/Sqrt[2*a*d - (b - Sqrt[b^2 - 4*a*c])*e]], (b - Sqrt[b
^2 - 4*a*c] - (2*a*d)/e)/(b + Sqrt[b^2 - 4*a*c] - (2*a*d)/e)])/(4*Sqrt[2]*S
qrt[a]*c*d^2*(c + b*x + a*x^2))
```

Rubi [A] time = 5.30239, antiderivative size = 1287, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 12, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$

= 0.414, Rules used = {1573, 916, 6742, 718, 419, 939, 934, 169, 538, 537, 843, 424}

$$\frac{\sqrt{2ad-\left(b-\sqrt{b^2-4ac}\right)e}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,x\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b-\sqrt{b^2-4ac}\right)e}}\sqrt{1-\frac{2a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\,\Pi\left(\frac{2ad-be+\sqrt{b^2-4ac}e}{2ad};\sin^{-1}\left(\frac{\sqrt{2}\sqrt{a}\sqrt{d+e}}{\sqrt{2ad-\left(b-\sqrt{b^2-}\right.}}\right.\right.}{4\sqrt{2}\sqrt{a}cd^2\left(ax^2+bx+c\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x])/x^2,x]
```

```
[Out] -((b*d + c*e)*Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x])/(4*c*d) - (Sqrt[a + c/x^
2 + b/x]*Sqrt[d + e*x])/(2*x) + (Sqrt[b^2 - 4*a*c]*(b*d + c*e)*Sqrt[a + c/x
^2 + b/x]*x*Sqrt[d + e*x]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*Elli
pticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2
]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(4*Sqrt[
2]*c*d*Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*(c + b*x + a
*x^2)) + (3*Sqrt[b^2 - 4*a*c]*e*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(a*(d + e*x))/
(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*
a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a
*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)
])/(Sqrt[2]*Sqrt[d + e*x]*(c + b*x + a*x^2)) - (Sqrt[b^2 - 4*a*c]*(b*d + c*
e)*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*
c])*e)]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[
(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 -
 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(2*Sqrt[2]*c*Sqrt[d + e*x]
*(c + b*x + a*x^2)) - ((a*d + b*e)*Sqrt[2*a*d - (b - Sqrt[b^2 - 4*a*c])*e]*
Sqrt[a + c/x^2 + b/x]*x*Sqrt[1 - (2*a*(d + e*x))/(2*a*d - (b - Sqrt[b^2 - 4
*a*c])*e)]*Sqrt[1 - (2*a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*El
lipticPi[(2*a*d - b*e + Sqrt[b^2 - 4*a*c]*e)/(2*a*d), ArcSin[(Sqrt[2]*Sqrt[
a]*Sqrt[d + e*x])/Sqrt[2*a*d - (b - Sqrt[b^2 - 4*a*c])*e]], (b - Sqrt[b^2 -
 4*a*c] - (2*a*d)/e)/(b + Sqrt[b^2 - 4*a*c] - (2*a*d)/e)])/(Sqrt[2]*Sqrt[a]
*d*(c + b*x + a*x^2)) + ((b*d + c*e)^2*Sqrt[2*a*d - (b - Sqrt[b^2 - 4*a*c])
*e]*Sqrt[a + c/x^2 + b/x]*x*Sqrt[1 - (2*a*(d + e*x))/(2*a*d - (b - Sqrt[b^2
 - 4*a*c])*e)]*Sqrt[1 - (2*a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)
]*EllipticPi[(2*a*d - b*e + Sqrt[b^2 - 4*a*c]*e)/(2*a*d), ArcSin[(Sqrt[2]*S
qrt[a]*Sqrt[d + e*x])/Sqrt[2*a*d - (b - Sqrt[b^2 - 4*a*c])*e]], (b - Sqrt[b
^2 - 4*a*c] - (2*a*d)/e)/(b + Sqrt[b^2 - 4*a*c] - (2*a*d)/e)])/(4*Sqrt[2]*S
qrt[a]*c*d^2*(c + b*x + a*x^2))
```

Rule 1573

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_)
+ (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[(x^(2*n*FracPart[p])*(a + b/x^
n + c/x^(2*n))^FracPart[p])/(c + b*x^n + a*x^(2*n))^FracPart[p], Int[x^(m -
 2*n*p)*(d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p, x], x] /; FreeQ[{a, b, c,
d, e, m, n, p, q}, x] && EqQ[mn, -n] && EqQ[mn2, 2*mn] &&  !IntegerQ[p] &&
 !IntegerQ[q] && PosQ[n]
```

Rule 916

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[((d + e*x)^(m + 1)*Sqrt[f + g*x]*Sq
rt[a + b*x + c*x^2])/(e*(m + 1)), x] - Dist[1/(2*e*(m + 1)), Int[((d + e*x)
^(m + 1)*Simp[b*f + a*g + 2*(c*f + b*g)*x + 3*c*g*x^2, x])/(Sqrt[f + g*x]*S
qrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f
 - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Integ
erQ[2*m] && LtQ[m, -1]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 718

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[(2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*Sqrt[-((c*(a + b*x + c*x^2)
)/(b^2 - 4*a*c))])/(c*Sqrt[a + b*x + c*x^2]*((2*c*(d + e*x))/(2*c*d - b*e -
 e*Rt[b^2 - 4*a*c, 2]))^m), Subst[Int[(1 + (2*e*Rt[b^2 - 4*a*c, 2]*x^2)/(2*
c*d - b*e - e*Rt[b^2 - 4*a*c, 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 939

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[(e^2*(d + e*x)^(m + 1)*Sqrt[f + g*
x]*Sqrt[a + b*x + c*x^2])/((m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2)), x]
```

```
 + Dist[1/(2*(m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x)^(
m + 1)*Simp[2*d*(c*e*f - c*d*g + b*e*g)*(m + 1) - e^2*(b*f + a*g)*(2*m + 3)
 + 2*e*(c*d*g*(m + 1) - e*(c*f + b*g)*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x
])/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f
, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 934

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[
b - q + 2*c*x]*Sqrt[b + q + 2*c*x])/Sqrt[a + b*x + c*x^2], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 169

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
```

```
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{x^2}\,dx = \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int \frac{\sqrt{d+ex}\sqrt{c+bx+ax^2}}{x^3}\,dx}{\sqrt{c+bx+ax^2}}$$

$$= -\frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{2x} + \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int \frac{bd+ce+2(ad+be)x+3aex^2}{x^2\sqrt{d+ex}\sqrt{c+bx+ax^2}}\,dx}{4\sqrt{c+bx+ax^2}}$$

$$= -\frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{2x} + \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int \left(\frac{3ae}{\sqrt{d+ex}\sqrt{c+bx+ax^2}} + \frac{bd+ce}{x^2\sqrt{d+ex}\sqrt{c+bx+ax^2}} + \frac{2(ad+\cdots}{x\sqrt{d+ex}\sqrt{c\cdots}}\cdots\right.}{4\sqrt{c+bx+ax^2}}$$

$$= -\frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{2x} + \frac{\left(3ae\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\right)\int \frac{1}{\sqrt{d+ex}\sqrt{c+bx+ax^2}}\,dx}{4\sqrt{c+bx+ax^2}} + \frac{\left((ad+be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}\cdots}\right.}{2\sqrt{c+\cdots}}$$

$$= -\frac{(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{4cd} - \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{2x} + \frac{\left((ad+be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{b-\sqrt{\cdots}}\right.}{\cdots}$$

$$= -\frac{(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{4cd} - \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{2x} + \frac{3\sqrt{b^2-4ac}e\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{\frac{\cdots}{2ad-\cdots}}}{\cdots}$$

$$= -\frac{(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{4cd} - \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{2x} + \frac{3\sqrt{b^2-4ac}e\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{\frac{\cdots}{2ad-\cdots}}}{\cdots}$$

$$= -\frac{(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{4cd} - \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{2x} + \frac{3\sqrt{b^2-4ac}e\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{\frac{\cdots}{2ad-\cdots}}}{\cdots}$$

$$= -\frac{(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{4cd} - \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{2x} + \frac{3\sqrt{b^2-4ac}e\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\sqrt{\frac{\cdots}{2ad-\cdots}}}{\cdots}$$

$$= -\frac{(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{4cd} - \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\sqrt{d+ex}}{2x} + \frac{\sqrt{b^2-4ac}(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}x\cdots}{4\sqrt{\cdots}}$$

Mathematica [C] time = 12.6526, size = 811, normalized size = 0.63

$$x\sqrt{a+\frac{c+bx}{x^2}}\left(-\frac{8cd^3}{x^2}-\frac{8ced^2}{x}-\frac{4(bd+ce)d^2}{x}-\frac{i(d+ex)^{3/2}\sqrt{1-\frac{2\left(ad^2+e(ce-bd)\right)}{\left(2ad-be+\sqrt{(b^2-4ac)e^2}\right)(d+ex)}}\sqrt{\frac{4\left(ad^2+e(ce-bd)\right)}{\left(-2ad+be+\sqrt{(b^2-4ac)e^2}\right)(d+ex)}+2}\left(d(bd+ce)\left(2ad-be+\sqrt{(b^2-4ac)e\ldots}\right.\right.}{\ldots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x])/x^2,x]
```

```
[Out] (x*Sqrt[a + (c + b*x)/x^2]*((-8*c*d^3)/x^2 - (8*c*d^2*e)/x - (4*d^2*(b*d +
c*e))/x - (I*(d + e*x)^(3/2)*Sqrt[1 - (2*(a*d^2 + e*(-(b*d) + c*e)))/((2*a*
d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[2 + (4*(a*d^2 + e*(-(b*
d) + c*e)))/((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*(d*(b*d +
 c*e)*(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*EllipticE[I*ArcSinh[(Sqrt[2]*
Sqrt[(a*d^2 - b*d*e + c*e^2)/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqr
t[d + e*x]], -((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*a*d - b*e + Sqrt
[(b^2 - 4*a*c)*e^2]))] - (b^2*d^2*e + b*d*(-5*c*e^2 + d*Sqrt[(b^2 - 4*a*c)*
e^2]) + c*e*(4*a*d^2 + 2*c*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2]))*EllipticF[I*Ar
cSinh[(Sqrt[2]*Sqrt[(a*d^2 - b*d*e + c*e^2)/(-2*a*d + b*e + Sqrt[(b^2 - 4*a
*c)*e^2])])/Sqrt[d + e*x]], -((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*a
*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))] + 2*e*(b^2*d^2 - 2*b*c*d*e + c*(-4*a*
d^2 + c*e^2))*EllipticPi[(d*(2*a*d - b*e - Sqrt[(b^2 - 4*a*c)*e^2]))/(2*(a*
d^2 + e*(-(b*d) + c*e))), I*ArcSinh[(Sqrt[2]*Sqrt[(a*d^2 - b*d*e + c*e^2)/(
-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*a*d + b*e +
 Sqrt[(b^2 - 4*a*c)*e^2])/(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))]))/(e*Sq
rt[(a*d^2 + e*(-(b*d) + c*e))/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*(c
+ x*(b + a*x)))))/(16*c*d^2*Sqrt[d + e*x])
```

Maple [B] time = 0.051, size = 4957, normalized size = 3.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)/x^2,x)
```

```
[Out] 1/8*((a*x^2+b*x+c)/x^2)^(1/2)*(e*x+d)^(1/2)*(-2*x^3*a*b*c*d*e^3-2*x^4*a^2*b
*d^2*e^2-2*x^4*a^2*c*d*e^3-2*x^3*a^2*b*d^3*e-6*x^3*a^2*c*d^2*e^2-2*x^3*a*b^
2*d^2*e^2-2*x^2*a*c^2*d*e^3-6*x*a*c^2*d^2*e^2-2*x^2*a*b^2*d^3*e-4*x^2*a^2*c
*d^3*e-4*a*c^2*d^3*e+2*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)
)^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^
(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/
2)*EllipticPi(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),-
1/2*(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/a/d,(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)
/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*(-4*a*c+b^2)^(1/2)*x^2*b*c*d*e^3-
5*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-
4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4
*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticF(2^(1/2)*
(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)
-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*(-4*a*c+b^2)^(1/2)*x^2
*a*c*d^2*e^2+2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e
*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(
b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*Ellipti
cF(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(e*(-4*a*c
+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*(-4*a*c+b^2
)^(1/2)*x^2*a*b*d^3*e-5*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e
))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))
^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1
/2)*EllipticF(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),(
-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*
x^2*a*b*c*d^2*e^2+4*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(
1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/
2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*
EllipticPi(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),-1/2
*(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/a/d,(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e
*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*(-4*a*c+b^2)^(1/2)*x^2*a*c*d^2*e^2+2
*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4
*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*
a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticF(2^(1/2)*(
-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-
2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*x^2*a*c^2*d*e^3+2*2^(1/
2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b
^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^
2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticPi(2^(1/2)*(-a*(e
*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),-1/2*(e*(-4*a*c+b^2)^(1/2)-2*
a*d+b*e)/a/d,(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d
-b*e))^(1/2))*x^2*a*c^2*d*e^3+2*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2
*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a
*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+
b*e))^(1/2)*EllipticPi(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)
)^(1/2),-1/2*(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/a/d,(-(e*(-4*a*c+b^2)^(1/2)-2
```

*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*x^2*b^2*c*d*e^3-2^(1/2)*
(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)
^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^
(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticPi(2^(1/2)*(-a*(e*x+
d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),-1/2*(e*(-4*a*c+b^2)^(1/2)-2*a*d
+b*e)/a/d,(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*
e))^(1/2))*(-4*a*c+b^2)^(1/2)*x^2*b^2*d^2*e^2-8*2^(1/2)*(-a*(e*x+d)/(e*(-4*
a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*
c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b
^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticPi(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^
(1/2)-2*a*d+b*e))^(1/2),-1/2*(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/a/d,(-(e*(-4*
a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*x^2*a^2*
c*d^3*e+2*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-
2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2
*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticPi
(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),-1/2*(e*(-4*a*
c+b^2)^(1/2)-2*a*d+b*e)/a/d,(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b
^2)^(1/2)+2*a*d-b*e))^(1/2))*x^2*a*b^2*d^3*e-2*2^(1/2)*(-a*(e*x+d)/(e*(-4*a
*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c
+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^
2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticE(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1
/2)-2*a*d+b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1
/2)+2*a*d-b*e))^(1/2))*x^2*a^2*c*d^3*e+2*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2
)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^
(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/
2)-2*a*d+b*e))^(1/2)*EllipticE(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*
a*d+b*e))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*
a*d-b*e))^(1/2))*x^2*a*b^2*d^3*e-2*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2
)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+
2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a
*d+b*e))^(1/2)*EllipticE(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*
e))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*
e))^(1/2))*x^2*a*c^2*d*e^3+12*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a
*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d
-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*
e))^(1/2)*EllipticF(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(
1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(
1/2))*x^2*a^2*c*d^3*e-2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))
^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(
1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2
)*EllipticF(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(
e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*x^
2*a*b^2*d^3*e-6*x*a*b*c*d^3*e-8*x^2*a*b*c*d^2*e^2-2^(1/2)*(-a*(e*x+d)/(e*(-
4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*
a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c

```
+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticPi(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2
)^(1/2)-2*a*d+b*e))^(1/2),-1/2*(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/a/d,(-(e*(-
4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*x^2*b*
c^2*e^4-2*2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-
2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2
*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticE(
2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),(-(e*(-4*a*c+b^
2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*x^2*a^2*b*d^4-
2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*
a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a
*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2)*EllipticPi(2^(1/2)*(
-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e))^(1/2),-1/2*(e*(-4*a*c+b^2)^(1/
2)-2*a*d+b*e)/a/d,(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+
2*a*d-b*e))^(1/2))*x^2*b^3*d^2*e^2-2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2
)-2*a*d+b*e))^(1/2)*(e*(-2*a*x+(-4*a*c+b^2)^(1/2)-b)/(e*(-4*a*c+b^2)^(1/2)+
2*a*d-b*e))^(1/2)*(e*(b+2*a*x+(-4*a*c+b^2)^(1/2))/(e*(-4*a*c+b^2)^(1/2)-2*a
*d+b*e))^(1/2)*EllipticPi(2^(1/2)*(-a*(e*x+d)/(e*(-4*a*c+b^2)^(1/2)-2*a*d+b
*e))^(1/2),-1/2*(e*(-4*a*c+b^2)^(1/2)-2*a*d+b*e)/a/d,(-(e*(-4*a*c+b^2)^(1/2
)-2*a*d+b*e)/(e*(-4*a*c+b^2)^(1/2)+2*a*d-b*e))^(1/2))*(-4*a*c+b^2)^(1/2)*x^
2*c^2*e^4)/x/a/e/(a*e*x^3+a*d*x^2+b*e*x^2+b*d*x+c*e*x+c*d)/c/d^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*x + d)*sqrt(a + b/x + c/x^2)/x^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x**2+b/x)**(1/2)*(e*x+d)**(1/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.86 $\int (fx)^m (d+ex^n)^q \left(a+cx^{2n}\right)^p \, dx$

Optimal. Leaf size=28

$$\text{Unintegrable}\left((fx)^m \left(a+cx^{2n}\right)^p (d+ex^n)^q, x\right)$$

```
[Out] Defer[Int][(f*x)^m*(d + e*x^n)^q*(a + c*x^(2*n))^p, x]
```

Rubi [A] time = 0.0186216, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (fx)^m (d+ex^n)^q \left(a+cx^{2n}\right)^p \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(f*x)^m*(d + e*x^n)^q*(a + c*x^(2*n))^p,x]
```

```
[Out] Defer[Int][(f*x)^m*(d + e*x^n)^q*(a + c*x^(2*n))^p, x]
```

Rubi steps

$$\int (fx)^m (d+ex^n)^q \left(a+cx^{2n}\right)^p \, dx = \int (fx)^m (d+ex^n)^q \left(a+cx^{2n}\right)^p \, dx$$

Mathematica [A] time = 0.19132, size = 0, normalized size = 0.

$$\int (fx)^m (d+ex^n)^q \left(a+cx^{2n}\right)^p \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f*x)^m*(d + e*x^n)^q*(a + c*x^(2*n))^p,x]
```

```
[Out] Integrate[(f*x)^m*(d + e*x^n)^q*(a + c*x^(2*n))^p, x]
```

Maple [A] time = 0.424, size = 0, normalized size = 0.

$$\int \left(fx\right)^{m} \left(d+e x^{n}\right)^{q} \left(a+c x^{2 n}\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d+e*x^n)^q*(a+c*x^(2*n))^p,x)
```

```
[Out] int((f*x)^m*(d+e*x^n)^q*(a+c*x^(2*n))^p,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(c x^{2 n}+a\right)^{p} \left(e x^{n}+d\right)^{q} \left(fx\right)^{m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)^q*(a+c*x^(2*n))^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + a)^p*(e*x^n + d)^q*(f*x)^m, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(c x^{2 n}+a\right)^{p} \left(e x^{n}+d\right)^{q} \left(fx\right)^{m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)^q*(a+c*x^(2*n))^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^(2*n) + a)^p*(e*x^n + d)^q*(f*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(d+e*x**n)**q*(a+c*x**(2*n))**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^{2n}+a\right)^{p}(ex^{n}+d)^{q}\left(fx\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)^q*(a+c*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + a)^p*(e*x^n + d)^q*(f*x)^m, x)
```

3.87 $\int (fx)^m (d+ex^n)^3 \left(a+cx^{2n}\right)^p \, dx$

Optimal. Leaf size=358

$$\frac{3d^2ex^{n+1}(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+n+1}{2n},-p;\frac{m+3n+1}{2n};-\frac{cx^{2n}}{a}\right)}{m+n+1}+\frac{d^3(fx)^{m+1}\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+1}{2n},\right.}{f(m+1)}$$

```
[Out] (d^3*(f*x)^(1 + m)*(a + c*x^(2*n))^p*Hypergeometric2F1[(1 + m)/(2*n), -p, 1
 + (1 + m)/(2*n), -((c*x^(2*n))/a)])/(f*(1 + m)*(1 + (c*x^(2*n))/a)^p) + (3
*d^2*e*x^(1 + n)*(f*x)^m*(a + c*x^(2*n))^p*Hypergeometric2F1[(1 + m + n)/(2
*n), -p, (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a)])/((1 + m + n)*(1 + (c*x^(2*
n))/a)^p) + (3*d*e^2*x^(1 + 2*n)*(f*x)^m*(a + c*x^(2*n))^p*Hypergeometric2F
1[(1 + m + 2*n)/(2*n), -p, (1 + m + 4*n)/(2*n), -((c*x^(2*n))/a)])/((1 + m
+ 2*n)*(1 + (c*x^(2*n))/a)^p) + (e^3*x^(1 + 3*n)*(f*x)^m*(a + c*x^(2*n))^p*
Hypergeometric2F1[(1 + m + 3*n)/(2*n), -p, (1 + m + 5*n)/(2*n), -((c*x^(2*n
))/a)])/((1 + m + 3*n)*(1 + (c*x^(2*n))/a)^p)
```

Rubi [A] time = 0.236908, antiderivative size = 358, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1561, 365, 364, 20}

$$\frac{3d^2ex^{n+1}(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+n+1}{2n},-p;\frac{m+3n+1}{2n};-\frac{cx^{2n}}{a}\right)}{m+n+1}+\frac{d^3(fx)^{m+1}\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+1}{2n},\right.}{f(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^n)^3*(a + c*x^(2*n))^p,x]
```


```
[Out] (d^3*(f*x)^(1 + m)*(a + c*x^(2*n))^p*Hypergeometric2F1[(1 + m)/(2*n), -p, 1
 + (1 + m)/(2*n), -((c*x^(2*n))/a)])/(f*(1 + m)*(1 + (c*x^(2*n))/a)^p) + (3
*d^2*e*x^(1 + n)*(f*x)^m*(a + c*x^(2*n))^p*Hypergeometric2F1[(1 + m + n)/(2
*n), -p, (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a)])/((1 + m + n)*(1 + (c*x^(2*
n))/a)^p) + (3*d*e^2*x^(1 + 2*n)*(f*x)^m*(a + c*x^(2*n))^p*Hypergeometric2F
1[(1 + m + 2*n)/(2*n), -p, (1 + m + 4*n)/(2*n), -((c*x^(2*n))/a)])/((1 + m
+ 2*n)*(1 + (c*x^(2*n))/a)^p) + (e^3*x^(1 + 3*n)*(f*x)^m*(a + c*x^(2*n))^p*
Hypergeometric2F1[(1 + m + 3*n)/(2*n), -p, (1 + m + 5*n)/(2*n), -((c*x^(2*n
))/a)])/((1 + m + 3*n)*(1 + (c*x^(2*n))/a)^p)
```


Rule 1561

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^n)^q*(a + c*
x^(2*n))^p, x], x] /; FreeQ[{a, c, d, e, f, m, n, p, q}, x] && EqQ[n2, 2*n]
 && (IGtQ[p, 0] || IGtQ[q, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rubi steps

$$\int (fx)^m (d+ex^n)^3 \left(a+cx^{2n}\right)^p dx = \int \left(d^3(fx)^m \left(a+cx^{2n}\right)^p + 3d^2ex^n(fx)^m \left(a+cx^{2n}\right)^p + 3de^2x^{2n}(fx)^m \left(a+cx^{2n}\right)^p + e^3 \right.$$

$$= d^3 \int (fx)^m \left(a+cx^{2n}\right)^p dx + \left(3d^2e\right) \int x^n(fx)^m \left(a+cx^{2n}\right)^p dx + \left(3de^2\right) \int x^{2n}(fx)^m$$

$$= \left(3d^2ex^{-m}(fx)^m\right) \int x^{m+n} \left(a+cx^{2n}\right)^p dx + \left(3de^2x^{-m}(fx)^m\right) \int x^{m+2n} \left(a+cx^{2n}\right)^p dx +$$

$$= \frac{d^3(fx)^{1+m} \left(a+cx^{2n}\right)^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} {}_2F_1\left(\frac{1+m}{2n}, -p; 1+\frac{1+m}{2n}; -\frac{cx^{2n}}{a}\right)}{f(1+m)} + \left(3d^2ex^{-m}(fx)^m \right.$$

$$= \frac{d^3(fx)^{1+m} \left(a+cx^{2n}\right)^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} {}_2F_1\left(\frac{1+m}{2n}, -p; 1+\frac{1+m}{2n}; -\frac{cx^{2n}}{a}\right)}{f(1+m)} + \frac{3d^2ex^{1+n}(fx)^m \left(\right.}{}$$

Mathematica [A] time = 0.313728, size = 249, normalized size = 0.7

$$x(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}\left(ex^n\left(\frac{3d^2\,{}_2F_1\left(\frac{m+n+1}{2n},-p;\frac{m+3n+1}{2n};-\frac{cx^{2n}}{a}\right)}{m+n+1}+ex^n\left(\frac{3d\,{}_2F_1\left(\frac{m+2n+1}{2n},-p;\frac{m+4n+1}{2n};-\frac{cx^{2n}}{a}\right)}{m+2n+1}+\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d + e*x^n)^3*(a + c*x^(2*n))^p,x]
```

```
[Out] (x*(f*x)^m*(a + c*x^(2*n))^p*((d^3*Hypergeometric2F1[(1 + m)/(2*n), -p, 1 +
 (1 + m)/(2*n), -((c*x^(2*n))/a)])/(1 + m) + e*x^n*((3*d^2*Hypergeometric2F
1[(1 + m + n)/(2*n), -p, (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a)])/(1 + m + n
) + e*x^n*((3*d*Hypergeometric2F1[(1 + m + 2*n)/(2*n), -p, (1 + m + 4*n)/(2
*n), -((c*x^(2*n))/a)])/(1 + m + 2*n) + (e*x^n*Hypergeometric2F1[(1 + m + 3
*n)/(2*n), -p, (1 + m + 5*n)/(2*n), -((c*x^(2*n))/a)])/(1 + m + 3*n)))))/(1
 + (c*x^(2*n))/a)^p
```

Maple [F] time = 0.074, size = 0, normalized size = 0.

$$\int \left(fx\right)^m (d+ex^n)^3\left(a+cx^{2\,n}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d+e*x^n)^3*(a+c*x^(2*n))^p,x)
```

```
[Out] int((f*x)^m*(d+e*x^n)^3*(a+c*x^(2*n))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex^n+d)^3\left(cx^{2\,n}+a\right)^p\left(fx\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)^3*(a+c*x^(2*n))^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x^n + d)^3*(c*x^(2*n) + a)^p*(f*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^3x^{3\,n}+3\,de^2x^{2\,n}+3\,d^2ex^n+d^3\right)\left(cx^{2\,n}+a\right)^p\left(fx\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)^3*(a+c*x^(2*n))^p,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^(3*n) + 3*d*e^2*x^(2*n) + 3*d^2*e*x^n + d^3)*(c*x^(2*n) + a
)^p*(f*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(d+e*x**n)**3*(a+c*x**(2*n))**p,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)^3*(a+c*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.88 $\int (fx)^m (d+ex^n)^2 \left(a+cx^{2n}\right)^p \, dx$

Optimal. Leaf size=262

$$\frac{d^2(fx)^{m+1}\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p} {}_2F_1\left(\frac{m+1}{2n},-p;\frac{m+1}{2n}+1;-\frac{cx^{2n}}{a}\right)}{f(m+1)}+\frac{2dex^{n+1}(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p} {}_2F_1\left(\frac{m+n+1}{2n},\cdot\right.}{m+n+1}$$

```
[Out] (d^2*(f*x)^(1 + m)*(a + c*x^(2*n))^p*Hypergeometric2F1[(1 + m)/(2*n), -p, 1
 + (1 + m)/(2*n), -((c*x^(2*n))/a)])/(f*(1 + m)*(1 + (c*x^(2*n))/a)^p) + (2
*d*e*x^(1 + n)*(f*x)^m*(a + c*x^(2*n))^p*Hypergeometric2F1[(1 + m + n)/(2*n
), -p, (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a)])/((1 + m + n)*(1 + (c*x^(2*n)
)/a)^p) + (e^2*x^(1 + 2*n)*(f*x)^m*(a + c*x^(2*n))^p*Hypergeometric2F1[(1 +
 m + 2*n)/(2*n), -p, (1 + m + 4*n)/(2*n), -((c*x^(2*n))/a)])/((1 + m + 2*n)
*(1 + (c*x^(2*n))/a)^p)
```

Rubi [A] time = 0.159533, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1561, 365, 364, 20}

$$\frac{d^2(fx)^{m+1}\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p} {}_2F_1\left(\frac{m+1}{2n},-p;\frac{m+1}{2n}+1;-\frac{cx^{2n}}{a}\right)}{f(m+1)}+\frac{2dex^{n+1}(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p} {}_2F_1\left(\frac{m+n+1}{2n},\cdot\right.}{m+n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^n)^2*(a + c*x^(2*n))^p,x]
```


```
[Out] (d^2*(f*x)^(1 + m)*(a + c*x^(2*n))^p*Hypergeometric2F1[(1 + m)/(2*n), -p, 1
 + (1 + m)/(2*n), -((c*x^(2*n))/a)])/(f*(1 + m)*(1 + (c*x^(2*n))/a)^p) + (2
*d*e*x^(1 + n)*(f*x)^m*(a + c*x^(2*n))^p*Hypergeometric2F1[(1 + m + n)/(2*n
), -p, (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a)])/((1 + m + n)*(1 + (c*x^(2*n)
)/a)^p) + (e^2*x^(1 + 2*n)*(f*x)^m*(a + c*x^(2*n))^p*Hypergeometric2F1[(1 +
 m + 2*n)/(2*n), -p, (1 + m + 4*n)/(2*n), -((c*x^(2*n))/a)])/((1 + m + 2*n)
*(1 + (c*x^(2*n))/a)^p)
```


Rule 1561

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^n)^q*(a + c*
x^(2*n))^p, x], x] /; FreeQ[{a, c, d, e, f, m, n, p, q}, x] && EqQ[n2, 2*n]
```

```
 && (IGtQ[p, 0] || IGtQ[q, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rubi steps

$$\begin{aligned}\int (fx)^m (d+ex^n)^2 \left(a+cx^{2n}\right)^p dx &= \int \left(d^2(fx)^m \left(a+cx^{2n}\right)^p + 2dex^n(fx)^m \left(a+cx^{2n}\right)^p + e^2x^{2n}(fx)^m \left(a+cx^{2n}\right)^p\right) dx \\ &= d^2 \int (fx)^m \left(a+cx^{2n}\right)^p dx + (2de)\int x^n(fx)^m \left(a+cx^{2n}\right)^p dx + e^2 \int x^{2n}(fx)^m \left(a+c\right. \\ &= \left(2dex^{-m}(fx)^m\right)\int x^{m+n}\left(a+cx^{2n}\right)^p dx + \left(e^2x^{-m}(fx)^m\right)\int x^{m+2n}\left(a+cx^{2n}\right)^p dx + \left(d\right. \\ &= \frac{d^2(fx)^{1+m}\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p} {}_2F_1\left(\frac{1+m}{2n},-p;1+\frac{1+m}{2n};-\frac{cx^{2n}}{a}\right)}{f(1+m)} + \left(2dex^{-m}(fx)^m \left(a\right.\right. \\ &= \frac{d^2(fx)^{1+m}\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p} {}_2F_1\left(\frac{1+m}{2n},-p;1+\frac{1+m}{2n};-\frac{cx^{2n}}{a}\right)}{f(1+m)} + \frac{2dex^{1+n}(fx)^m \left(a\right.}{}\end{aligned}$$

Mathematica [A] time = 0.168455, size = 189, normalized size = 0.72

$$x(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}\left(\frac{d^2\,{}_2F_1\left(\frac{m+1}{2n},-p;\frac{m+1}{2n}+1;-\frac{cx^{2n}}{a}\right)}{m+1}+ex^n\left(\frac{2d\,{}_2F_1\left(\frac{m+n+1}{2n},-p;\frac{m+3n+1}{2n};-\frac{cx^{2n}}{a}\right)}{m+n+1}+\frac{ex^n\,{}_2F_1\left(\right.}{}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d + e*x^n)^2*(a + c*x^(2*n))^p,x]
```

```
[Out] (x*(f*x)^m*(a + c*x^(2*n))^p*((d^2*Hypergeometric2F1[(1 + m)/(2*n), -p, 1 +
 (1 + m)/(2*n), -((c*x^(2*n))/a)])/(1 + m) + e*x^n*((2*d*Hypergeometric2F1[
(1 + m + n)/(2*n), -p, (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a)])/(1 + m + n)
+ (e*x^n*Hypergeometric2F1[(1 + m + 2*n)/(2*n), -p, (1 + m + 4*n)/(2*n), -(
(c*x^(2*n))/a)])/(1 + m + 2*n))))/(1 + (c*x^(2*n))/a)^p
```

Maple [F] time = 0.075, size = 0, normalized size = 0.

$$\int \left(fx\right)^{m} \left(d+ex^{n}\right)^{2} \left(a+cx^{2n}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d+e*x^n)^2*(a+c*x^(2*n))^p,x)
```

```
[Out] int((f*x)^m*(d+e*x^n)^2*(a+c*x^(2*n))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^{n}+d\right)^{2} \left(cx^{2n}+a\right)^{p} \left(fx\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)^2*(a+c*x^(2*n))^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x^n + d)^2*(c*x^(2*n) + a)^p*(f*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(e^{2} x^{2n}+2\, dex^{n}+d^{2}\right)\left(cx^{2n}+a\right)^{p} \left(fx\right)^{m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)^2*(a+c*x^(2*n))^p,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^(2*n) + 2*d*e*x^n + d^2)*(c*x^(2*n) + a)^p*(f*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(d+e*x**n)**2*(a+c*x**(2*n))**p,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)^2*(a+c*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.89 $\int (fx)^m \left(d+ex^n\right) \left(a+cx^{2n}\right)^p \, dx$

Optimal. Leaf size=166

$$\frac{d(fx)^{m+1}\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+1}{2n},-p;\frac{m+1}{2n}+1;-\frac{cx^{2n}}{a}\right)}{f(m+1)}+\frac{ex^{n+1}(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+n+1}{2n},-p;\right.}{m+n+1}$$

```
[Out] (d*(f*x)^(1 + m)*(a + c*x^(2*n))^p*Hypergeometric2F1[(1 + m)/(2*n), -p, 1 +
 (1 + m)/(2*n), -((c*x^(2*n))/a)])/(f*(1 + m)*(1 + (c*x^(2*n))/a)^p) + (e*x
^(1 + n)*(f*x)^m*(a + c*x^(2*n))^p*Hypergeometric2F1[(1 + m + n)/(2*n), -p,
 (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a)])/((1 + m + n)*(1 + (c*x^(2*n))/a)^p
)
```

Rubi [A] time = 0.0946427, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1561, 365, 364, 20}

$$\frac{d(fx)^{m+1}\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+1}{2n},-p;\frac{m+1}{2n}+1;-\frac{cx^{2n}}{a}\right)}{f(m+1)}+\frac{ex^{n+1}(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+n+1}{2n},-p;\right.}{m+n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^n)*(a + c*x^(2*n))^p,x]
```


```
[Out] (d*(f*x)^(1 + m)*(a + c*x^(2*n))^p*Hypergeometric2F1[(1 + m)/(2*n), -p, 1 +
 (1 + m)/(2*n), -((c*x^(2*n))/a)])/(f*(1 + m)*(1 + (c*x^(2*n))/a)^p) + (e*x
^(1 + n)*(f*x)^m*(a + c*x^(2*n))^p*Hypergeometric2F1[(1 + m + n)/(2*n), -p,
 (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a)])/((1 + m + n)*(1 + (c*x^(2*n))/a)^p
)
```


Rule 1561

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^n)^q*(a + c*
x^(2*n))^p, x], x] /; FreeQ[{a, c, d, e, f, m, n, p, q}, x] && EqQ[n2, 2*n]
 && (IGtQ[p, 0] || IGtQ[q, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
```

```
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rubi steps

$$\begin{aligned}\int (fx)^m\left(d+ex^n\right)\left(a+cx^{2n}\right)^p\,dx &= \int \left(d(fx)^m\left(a+cx^{2n}\right)^p+ex^n(fx)^m\left(a+cx^{2n}\right)^p\right)dx\\ &= d\int (fx)^m\left(a+cx^{2n}\right)^p\,dx+e\int x^n(fx)^m\left(a+cx^{2n}\right)^p\,dx\\ &= \left(ex^{-m}(fx)^m\right)\int x^{m+n}\left(a+cx^{2n}\right)^p\,dx+\left(d\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\right)\int (fx)^m\left(1+\frac{cx^{2}}{a}\right.\\ &= \frac{d(fx)^{1+m}\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}{}_2F_1\left(\frac{1+m}{2n},-p;1+\frac{1+m}{2n};-\frac{cx^{2n}}{a}\right)}{f(1+m)}+\left(ex^{-m}(fx)^m\left(a+c\right.\right.\\ &= \frac{d(fx)^{1+m}\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}{}_2F_1\left(\frac{1+m}{2n},-p;1+\frac{1+m}{2n};-\frac{cx^{2n}}{a}\right)}{f(1+m)}+\frac{ex^{1+n}(fx)^m\left(a+c\right.}{}\end{aligned}$$

Mathematica [A] time = 0.076134, size = 136, normalized size = 0.82

$$\frac{x(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}\left(d(m+n+1)\,{}_2F_1\left(\frac{m+1}{2n},-p;\frac{m+1}{2n}+1;-\frac{cx^{2n}}{a}\right)+e(m+1)x^n\,{}_2F_1\left(\frac{m+n+1}{2n},-p;\frac{m+3n+1}{2n};-\frac{cx^{2n}}{a}\right)\right)}{(m+1)(m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d + e*x^n)*(a + c*x^(2*n))^p,x]
```

```
[Out] (x*(f*x)^m*(a + c*x^(2*n))^p*(d*(1 + m + n)*Hypergeometric2F1[(1 + m)/(2*n)
, -p, 1 + (1 + m)/(2*n), -((c*x^(2*n))/a)] + e*(1 + m)*x^n*Hypergeometric2F
1[(1 + m + n)/(2*n), -p, (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a)]))/((1 + m)*
(1 + m + n)*(1 + (c*x^(2*n))/a)^p)
```

Maple [F] time = 0.073, size = 0, normalized size = 0.

$$\int \left(fx\right)^{m} \left(d+ex^{n}\right) \left(a+cx^{2n}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d+e*x^n)*(a+c*x^(2*n))^p,x)
```

```
[Out] int((f*x)^m*(d+e*x^n)*(a+c*x^(2*n))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(ex^{n}+d\right)\left(cx^{2n}+a\right)^{p} \left(fx\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)*(a+c*x^(2*n))^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x^n + d)*(c*x^(2*n) + a)^p*(f*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(ex^{n}+d\right)\left(cx^{2n}+a\right)^{p} \left(fx\right)^{m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)*(a+c*x^(2*n))^p,x, algorithm="fricas")
```

```
[Out] integral((e*x^n + d)*(c*x^(2*n) + a)^p*(f*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(d+e*x**n)*(a+c*x**(2*n))**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex^n + d)\left(cx^{2n} + a\right)^p \left(fx\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)*(a+c*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)*(c*x^(2*n) + a)^p*(f*x)^m, x)
```

3.90 $\int \frac{(fx)^m \left(a+cx^{2n}\right)^p}{d+ex^n} \, dx$

Optimal. Leaf size=194

$$\frac{x(fx)^m \left(a+cx^{2n}\right)^p \left(\frac{cx^{2n}}{a}+1\right)^{-p} F_1\left(\frac{m+1}{2n};-p,1;\frac{m+1}{2n}+1;-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d(m+1)} - \frac{ex^{n+1}(fx)^m \left(a+cx^{2n}\right)^p \left(\frac{cx^{2n}}{a}+1\right)^{-p} F_1\left(\frac{m+n+1}{2n};-\right.}{d^2(m+n+1)}$$

```
[Out] (x*(f*x)^m*(a + c*x^(2*n))^p*AppellF1[(1 + m)/(2*n), -p, 1, 1 + (1 + m)/(2*
n), -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d*(1 + m)*(1 + (c*x^(2*n))/a)^p)
 - (e*x^(1 + n)*(f*x)^m*(a + c*x^(2*n))^p*AppellF1[(1 + m + n)/(2*n), -p, 1
, (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^2*(1 + m +
n)*(1 + (c*x^(2*n))/a)^p)
```

Rubi [A] time = 0.223911, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1562, 511, 510}

$$\frac{x(fx)^m \left(a+cx^{2n}\right)^p \left(\frac{cx^{2n}}{a}+1\right)^{-p} F_1\left(\frac{m+1}{2n};-p,1;\frac{m+1}{2n}+1;-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d(m+1)} - \frac{ex^{n+1}(fx)^m \left(a+cx^{2n}\right)^p \left(\frac{cx^{2n}}{a}+1\right)^{-p} F_1\left(\frac{m+n+1}{2n};-\right.}{d^2(m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n),x]
```

```
[Out] (x*(f*x)^m*(a + c*x^(2*n))^p*AppellF1[(1 + m)/(2*n), -p, 1, 1 + (1 + m)/(2*
n), -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d*(1 + m)*(1 + (c*x^(2*n))/a)^p)
 - (e*x^(1 + n)*(f*x)^m*(a + c*x^(2*n))^p*AppellF1[(1 + m + n)/(2*n), -p, 1
, (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^2*(1 + m +
n)*(1 + (c*x^(2*n))/a)^p)
```

Rule 1562

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2
_))^(p_), x_Symbol] :> Dist[(f*x)^m/x^m, Int[ExpandIntegrand[x^m*(a + c*x^(
2*n))^p, (d/(d^2 - e^2*x^(2*n)) - (e*x^n)/(d^2 - e^2*x^(2*n)))^(-q), x], x]
, x] /; FreeQ[{a, c, d, e, f, m, n, p}, x] && EqQ[n2, 2*n] &&  !IntegerQ[p]
 && ILtQ[q, 0]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(fx)^m\left(a+cx^{2n}\right)^p}{d+ex^n}\,dx &= \left(x^{-m}(fx)^m\right)\int\left(\frac{dx^m\left(a+cx^{2n}\right)^p}{d^2-e^2x^{2n}}+\frac{ex^{m+n}\left(a+cx^{2n}\right)^p}{-d^2+e^2x^{2n}}\right)dx\\ &= \left(dx^{-m}(fx)^m\right)\int\frac{x^m\left(a+cx^{2n}\right)^p}{d^2-e^2x^{2n}}\,dx+\left(ex^{-m}(fx)^m\right)\int\frac{x^{m+n}\left(a+cx^{2n}\right)^p}{-d^2+e^2x^{2n}}\,dx\\ &= \left(dx^{-m}(fx)^m\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\right)\int\frac{x^m\left(1+\frac{cx^{2n}}{a}\right)^p}{d^2-e^2x^{2n}}\,dx+\left(ex^{-m}(fx)^m\left(a+cx^{2n}\right)^p\left(1+\frac{cx^2}{a}\right.\right.\\ &= \frac{x(fx)^m\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}F_1\left(\frac{1+m}{2n};-p,1;1+\frac{1+m}{2n};-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d(1+m)}-\frac{ex^{1+n}(fx)^m\left(a+cx^{2n}\right)^p}{}\end{aligned}$$

Mathematica [F] time = 0.116138, size = 0, normalized size = 0.

$$\int \frac{(fx)^m\left(a+cx^{2n}\right)^p}{d+ex^n}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n),x]
```

```
[Out] Integrate[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n), x]
```

Maple [F] time = 0.094, size = 0, normalized size = 0.

$$\int \frac{\left(fx\right)^{m} \left(a+c\,x^{2 n}\right)^{p}}{d+e\,x^{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n),x)
```

```
[Out] int((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c x^{2 n}+a\right)^{p} \left(fx\right)^{m}}{e x^{n}+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n),x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + a)^p*(f*x)^m/(e*x^n + d), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(c x^{2 n}+a\right)^{p} \left(fx\right)^{m}}{e x^{n}+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n),x, algorithm="fricas")
```

```
[Out] integral((c*x^(2*n) + a)^p*(f*x)^m/(e*x^n + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+c*x**(2*n))**p/(d+e*x**n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^{2n} + a\right)^{p} \left(fx\right)^{m}}{ex^{n} + d} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n),x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + a)^p*(f*x)^m/(e*x^n + d), x)
```

3.91 $\int \frac{(fx)^m\left(a+cx^{2n}\right)^p}{(d+ex^n)^2}\,dx$

Optimal. Leaf size=302

$$\frac{x(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{m+1}{2n};-p,2;\frac{m+1}{2n}+1;-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^2(m+1)}-\frac{2ex^{n+1}(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{m+n+1}{2n};\right.}{d^3(m+n+1)}$$

```
[Out] (x*(f*x)^m*(a + c*x^(2*n))^p*AppellF1[(1 + m)/(2*n), -p, 2, 1 + (1 + m)/(2*
n), -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^2*(1 + m)*(1 + (c*x^(2*n))/a)^
p) - (2*e*x^(1 + n)*(f*x)^m*(a + c*x^(2*n))^p*AppellF1[(1 + m + n)/(2*n), -
p, 2, (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^3*(1 +
m + n)*(1 + (c*x^(2*n))/a)^p) + (e^2*x^(1 + 2*n)*(f*x)^m*(a + c*x^(2*n))^p*
AppellF1[(1 + m + 2*n)/(2*n), -p, 2, (1 + m + 4*n)/(2*n), -((c*x^(2*n))/a),
 (e^2*x^(2*n))/d^2])/(d^4*(1 + m + 2*n)*(1 + (c*x^(2*n))/a)^p)
```

Rubi [A] time = 0.332553, antiderivative size = 302, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1562, 511, 510}

$$\frac{x(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{m+1}{2n};-p,2;\frac{m+1}{2n}+1;-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^2(m+1)}-\frac{2ex^{n+1}(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{m+n+1}{2n};\right.}{d^3(m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n)^2,x]
```

```
[Out] (x*(f*x)^m*(a + c*x^(2*n))^p*AppellF1[(1 + m)/(2*n), -p, 2, 1 + (1 + m)/(2*
n), -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^2*(1 + m)*(1 + (c*x^(2*n))/a)^
p) - (2*e*x^(1 + n)*(f*x)^m*(a + c*x^(2*n))^p*AppellF1[(1 + m + n)/(2*n), -
p, 2, (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^3*(1 +
m + n)*(1 + (c*x^(2*n))/a)^p) + (e^2*x^(1 + 2*n)*(f*x)^m*(a + c*x^(2*n))^p*
AppellF1[(1 + m + 2*n)/(2*n), -p, 2, (1 + m + 4*n)/(2*n), -((c*x^(2*n))/a),
 (e^2*x^(2*n))/d^2])/(d^4*(1 + m + 2*n)*(1 + (c*x^(2*n))/a)^p)
```

Rule 1562

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2
_))^(p_), x_Symbol] :> Dist[(f*x)^m/x^m, Int[ExpandIntegrand[x^m*(a + c*x^(
2*n))^p, (d/(d^2 - e^2*x^(2*n)) - (e*x^n)/(d^2 - e^2*x^(2*n)))^(-q), x], x]
```

```
, x] /; FreeQ[{a, c, d, e, f, m, n, p}, x] && EqQ[n2, 2*n] &&  !IntegerQ[p]
 && ILtQ[q, 0]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(fx)^m\left(a+cx^{2n}\right)^p}{(d+ex^n)^2}\,dx = \left(x^{-m}(fx)^m\right)\int \left(\frac{d^2x^m\left(a+cx^{2n}\right)^p}{\left(d^2-e^2x^{2n}\right)^2} - \frac{2dex^{m+n}\left(a+cx^{2n}\right)^p}{\left(-d^2+e^2x^{2n}\right)^2} + \frac{e^2x^{m+2n}\left(a+cx^{2n}\right)^p}{\left(-d^2+e^2x^{2n}\right)^2}\right)dx$$

$$= \left(d^2x^{-m}(fx)^m\right)\int \frac{x^m\left(a+cx^{2n}\right)^p}{\left(d^2-e^2x^{2n}\right)^2}\,dx - \left(2dex^{-m}(fx)^m\right)\int \frac{x^{m+n}\left(a+cx^{2n}\right)^p}{\left(-d^2+e^2x^{2n}\right)^2}\,dx + \left(e^2x^{-m}(fx)^m\right)$$

$$= \left(d^2x^{-m}(fx)^m\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\right)\int \frac{x^m\left(1+\frac{cx^{2n}}{a}\right)^p}{\left(d^2-e^2x^{2n}\right)^2}\,dx - \left(2dex^{-m}(fx)^m\left(a+cx^{2n}\right)^p\left(1+\right.\right.$$

$$= \frac{x(fx)^m\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}F_1\left(\frac{1+m}{2n};-p,2;1+\frac{1+m}{2n};-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^2(1+m)} - \frac{2ex^{1+n}(fx)^m\left(a+cx^{2n}\right)}{}$$

Mathematica [F] time = 0.16251, size = 0, normalized size = 0.

$$\int \frac{(fx)^m\left(a+cx^{2n}\right)^p}{(d+ex^n)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n)^2,x]

[Out] Integrate[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n)^2, x]
```

Maple [F] time = 0.094, size = 0, normalized size = 0.

$$\int \frac{(fx)^m (a+cx^{2n})^p}{(d+ex^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n)^2,x)

[Out] int((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^{2n}+a)^p (fx)^m}{(ex^n+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n)^2,x, algorithm="maxima")

[Out] integrate((c*x^(2*n) + a)^p*(f*x)^m/(e*x^n + d)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx^{2n}+a)^p (fx)^m}{e^2x^{2n}+2\,dex^n+d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral((c*x^(2*n) + a)^p*(f*x)^m/(e^2*x^(2*n) + 2*d*e*x^n + d^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+c*x**(2*n))**p/(d+e*x**n)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^{2n}+a\right)^{p}\left(fx\right)^{m}}{\left(ex^{n}+d\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + a)^p*(f*x)^m/(e*x^n + d)^2, x)
```

3.92 $\int \frac{(fx)^m\left(a+cx^{2n}\right)^p}{(d+ex^n)^3}\,dx$

Optimal. Leaf size=412

$$\frac{x(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{m+1}{2n};-p,3;\frac{m+1}{2n}+1;-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^3(m+1)}-\frac{3ex^{n+1}(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{m+n+1}{2n};\right.}{d^4(m+n+1)}$$

```
[Out] (x*(f*x)^m*(a + c*x^(2*n))^p*AppellF1[(1 + m)/(2*n), -p, 3, 1 + (1 + m)/(2*
n), -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^3*(1 + m)*(1 + (c*x^(2*n))/a)^
p) - (3*e*x^(1 + n)*(f*x)^m*(a + c*x^(2*n))^p*AppellF1[(1 + m + n)/(2*n), -
p, 3, (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^4*(1 +
m + n)*(1 + (c*x^(2*n))/a)^p) + (3*e^2*x^(1 + 2*n)*(f*x)^m*(a + c*x^(2*n))^
p*AppellF1[(1 + m + 2*n)/(2*n), -p, 3, (1 + m + 4*n)/(2*n), -((c*x^(2*n))/a
), (e^2*x^(2*n))/d^2])/(d^5*(1 + m + 2*n)*(1 + (c*x^(2*n))/a)^p) - (e^3*x^(
1 + 3*n)*(f*x)^m*(a + c*x^(2*n))^p*AppellF1[(1 + m + 3*n)/(2*n), -p, 3, (1
+ m + 5*n)/(2*n), -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^6*(1 + m + 3*n)*
(1 + (c*x^(2*n))/a)^p)
```

Rubi [A] time = 0.449459, antiderivative size = 412, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1562, 511, 510}

$$\frac{x(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{m+1}{2n};-p,3;\frac{m+1}{2n}+1;-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^3(m+1)}-\frac{3ex^{n+1}(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{m+n+1}{2n};\right.}{d^4(m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n)^3,x]
```


```
[Out] (x*(f*x)^m*(a + c*x^(2*n))^p*AppellF1[(1 + m)/(2*n), -p, 3, 1 + (1 + m)/(2*
n), -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^3*(1 + m)*(1 + (c*x^(2*n))/a)^
p) - (3*e*x^(1 + n)*(f*x)^m*(a + c*x^(2*n))^p*AppellF1[(1 + m + n)/(2*n), -
p, 3, (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^4*(1 +
m + n)*(1 + (c*x^(2*n))/a)^p) + (3*e^2*x^(1 + 2*n)*(f*x)^m*(a + c*x^(2*n))^
p*AppellF1[(1 + m + 2*n)/(2*n), -p, 3, (1 + m + 4*n)/(2*n), -((c*x^(2*n))/a
), (e^2*x^(2*n))/d^2])/(d^5*(1 + m + 2*n)*(1 + (c*x^(2*n))/a)^p) - (e^3*x^(
1 + 3*n)*(f*x)^m*(a + c*x^(2*n))^p*AppellF1[(1 + m + 3*n)/(2*n), -p, 3, (1
+ m + 5*n)/(2*n), -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^6*(1 + m + 3*n)*
(1 + (c*x^(2*n))/a)^p)
```

Rule 1562

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2
_))^(p_), x_Symbol] :> Dist[(f*x)^m/x^m, Int[ExpandIntegrand[x^m*(a + c*x^(
2*n))^p, (d/(d^2 - e^2*x^(2*n)) - (e*x^n)/(d^2 - e^2*x^(2*n)))^(-q), x], x]
, x] /; FreeQ[{a, c, d, e, f, m, n, p}, x] && EqQ[n2, 2*n] &&  !IntegerQ[p]
 && ILtQ[q, 0]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(fx)^m\left(a+cx^{2n}\right)^p}{(d+ex^n)^3}\,dx = \left(x^{-m}(fx)^m\right)\int\left(\frac{d^3x^m\left(a+cx^{2n}\right)^p}{\left(d^2-e^2x^{2n}\right)^3}+\frac{3d^2ex^{m+n}\left(a+cx^{2n}\right)^p}{\left(-d^2+e^2x^{2n}\right)^3}-\frac{3de^2x^{m+2n}\left(a+cx^{2n}\right)^p}{\left(-d^2+e^2x^{2n}\right)^3}+\frac{e^3x^{m+3n}}{\left(-d^2\right.}$$

$$= \left(d^3x^{-m}(fx)^m\right)\int\frac{x^m\left(a+cx^{2n}\right)^p}{\left(d^2-e^2x^{2n}\right)^3}\,dx+\left(3d^2ex^{-m}(fx)^m\right)\int\frac{x^{m+n}\left(a+cx^{2n}\right)^p}{\left(-d^2+e^2x^{2n}\right)^3}\,dx-\left(3de^2x^{-m}(fx)\right.$$

$$= \left(d^3x^{-m}(fx)^m\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\right)\int\frac{x^m\left(1+\frac{cx^{2n}}{a}\right)^p}{\left(d^2-e^2x^{2n}\right)^3}\,dx+\left(3d^2ex^{-m}(fx)^m\left(a+cx^{2n}\right)^p\left(1+\right.\right.$$

$$= \frac{x(fx)^m\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}F_1\left(\frac{1+m}{2n};-p,3;1+\frac{1+m}{2n};-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^3(1+m)}-\frac{3ex^{1+n}(fx)^m\left(a+cx^{2n}\right)}{}$$

Mathematica [F] time = 0.606542, size = 0, normalized size = 0.

$$\int \frac{(fx)^m \left(a+cx^{2n}\right)^p}{(d+ex^n)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n)^3,x]
```

```
[Out] Integrate[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n)^3, x]
```

Maple [F] time = 0.092, size = 0, normalized size = 0.

$$\int \frac{\left(fx\right)^m \left(a+cx^{2n}\right)^p}{(d+ex^n)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n)^3,x)
```

```
[Out] int((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^{2n}+a\right)^p \left(fx\right)^m}{(ex^n+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n)^3,x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + a)^p*(f*x)^m/(e*x^n + d)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(cx^{2n}+a\right)^p \left(fx\right)^m}{e^3x^{3n}+3\,de^2x^{2n}+3\,d^2ex^n+d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n)^3,x, algorithm="fricas")
```

```
[Out] integral((c*x^(2*n) + a)^p*(f*x)^m/(e^3*x^(3*n) + 3*d*e^2*x^(2*n) + 3*d^2*e
*x^n + d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+c*x**(2*n))**p/(d+e*x**n)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^{2n} + a\right)^{p} \left(fx\right)^{m}}{\left(ex^{n} + d\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + a)^p*(f*x)^m/(e*x^n + d)^3, x)
```

3.93 $\int (b+2cx)\left(a+bx+cx^2\right)^{13}\,dx$

Optimal. Leaf size=16

$$\frac{1}{14}\left(a+bx+cx^2\right)^{14}$$

```
[Out] (a + b*x + c*x^2)^14/14
```

Rubi [A] time = 0.0603052, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {629}

$$\frac{1}{14}\left(a+bx+cx^2\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)*(a + b*x + c*x^2)^13,x]
```

```
[Out] (a + b*x + c*x^2)^14/14
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (b+2cx)\left(a+bx+cx^2\right)^{13}\,dx=\frac{1}{14}\left(a+bx+cx^2\right)^{14}$$

Mathematica [B] time = 0.169213, size = 201, normalized size = 12.56

$$\frac{1}{14}x(b+cx)\left(364a^{11}x^2(b+cx)^2+1001a^{10}x^3(b+cx)^3+2002a^9x^4(b+cx)^4+3003a^8x^5(b+cx)^5+3432a^7x^6(b+cx)^6+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)*(a + b*x + c*x^2)^13,x]
```

```
[Out] (x*(b + c*x)*(14*a^13 + 91*a^12*x*(b + c*x) + 364*a^11*x^2*(b + c*x)^2 + 10
01*a^10*x^3*(b + c*x)^3 + 2002*a^9*x^4*(b + c*x)^4 + 3003*a^8*x^5*(b + c*x)
^5 + 3432*a^7*x^6*(b + c*x)^6 + 3003*a^6*x^7*(b + c*x)^7 + 2002*a^5*x^8*(b
+ c*x)^8 + 1001*a^4*x^9*(b + c*x)^9 + 364*a^3*x^10*(b + c*x)^10 + 91*a^2*x^
11*(b + c*x)^11 + 14*a*x^12*(b + c*x)^12 + x^13*(b + c*x)^13))/14
```

Maple [B] time = 0.004, size = 46548, normalized size = 2909.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)*(c*x^2+b*x+a)^13,x)
```

```
[Out] result too large to display
```

Maxima [A] time = 1.03581, size = 19, normalized size = 1.19

$$\frac{1}{14}\left(cx^2 + bx + a\right)^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x+a)^13,x, algorithm="maxima")
```

```
[Out] 1/14*(c*x^2 + b*x + a)^14
```

Fricas [B] time = 0.935923, size = 3474, normalized size = 217.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x+a)^13,x, algorithm="fricas")
```

```
[Out] 1/14*x^28*c^14 + x^27*c^13*b + 13/2*x^26*c^12*b^2 + x^26*c^13*a + 26*x^25*c
^11*b^3 + 13*x^25*c^12*b*a + 143/2*x^24*c^10*b^4 + 78*x^24*c^11*b^2*a + 13/
2*x^24*c^12*a^2 + 143*x^23*c^9*b^5 + 286*x^23*c^10*b^3*a + 78*x^23*c^11*b*a
^2 + 429/2*x^22*c^8*b^6 + 715*x^22*c^9*b^4*a + 429*x^22*c^10*b^2*a^2 + 26*x
^22*c^11*a^3 + 1716/7*x^21*c^7*b^7 + 1287*x^21*c^8*b^5*a + 1430*x^21*c^9*b^
3*a^2 + 286*x^21*c^10*b*a^3 + 429/2*x^20*c^6*b^8 + 1716*x^20*c^7*b^6*a + 64
35/2*x^20*c^8*b^4*a^2 + 1430*x^20*c^9*b^2*a^3 + 143/2*x^20*c^10*a^4 + 143*x
^19*c^5*b^9 + 1716*x^19*c^6*b^7*a + 5148*x^19*c^7*b^5*a^2 + 4290*x^19*c^8*b
^3*a^3 + 715*x^19*c^9*b*a^4 + 143/2*x^18*c^4*b^10 + 1287*x^18*c^5*b^8*a + 6
006*x^18*c^6*b^6*a^2 + 8580*x^18*c^7*b^4*a^3 + 6435/2*x^18*c^8*b^2*a^4 + 14
3*x^18*c^9*a^5 + 26*x^17*c^3*b^11 + 715*x^17*c^4*b^9*a + 5148*x^17*c^5*b^7*
a^2 + 12012*x^17*c^6*b^5*a^3 + 8580*x^17*c^7*b^3*a^4 + 1287*x^17*c^8*b*a^5
+ 13/2*x^16*c^2*b^12 + 286*x^16*c^3*b^10*a + 6435/2*x^16*c^4*b^8*a^2 + 1201
2*x^16*c^5*b^6*a^3 + 15015*x^16*c^6*b^4*a^4 + 5148*x^16*c^7*b^2*a^5 + 429/2
*x^16*c^8*a^6 + x^15*c*b^13 + 78*x^15*c^2*b^11*a + 1430*x^15*c^3*b^9*a^2 +
8580*x^15*c^4*b^7*a^3 + 18018*x^15*c^5*b^5*a^4 + 12012*x^15*c^6*b^3*a^5 + 1
716*x^15*c^7*b*a^6 + 1/14*x^14*b^14 + 13*x^14*c*b^12*a + 429*x^14*c^2*b^10*
a^2 + 4290*x^14*c^3*b^8*a^3 + 15015*x^14*c^4*b^6*a^4 + 18018*x^14*c^5*b^4*a
^5 + 6006*x^14*c^6*b^2*a^6 + 1716/7*x^14*c^7*a^7 + x^13*b^13*a + 78*x^13*c*
b^11*a^2 + 1430*x^13*c^2*b^9*a^3 + 8580*x^13*c^3*b^7*a^4 + 18018*x^13*c^4*b
^5*a^5 + 12012*x^13*c^5*b^3*a^6 + 1716*x^13*c^6*b*a^7 + 13/2*x^12*b^12*a^2
+ 286*x^12*c*b^10*a^3 + 6435/2*x^12*c^2*b^8*a^4 + 12012*x^12*c^3*b^6*a^5 +
15015*x^12*c^4*b^4*a^6 + 5148*x^12*c^5*b^2*a^7 + 429/2*x^12*c^6*a^8 + 26*x^
11*b^11*a^3 + 715*x^11*c*b^9*a^4 + 5148*x^11*c^2*b^7*a^5 + 12012*x^11*c^3*b
^5*a^6 + 8580*x^11*c^4*b^3*a^7 + 1287*x^11*c^5*b*a^8 + 143/2*x^10*b^10*a^4
+ 1287*x^10*c*b^8*a^5 + 6006*x^10*c^2*b^6*a^6 + 8580*x^10*c^3*b^4*a^7 + 643
5/2*x^10*c^4*b^2*a^8 + 143*x^10*c^5*a^9 + 143*x^9*b^9*a^5 + 1716*x^9*c*b^7*
a^6 + 5148*x^9*c^2*b^5*a^7 + 4290*x^9*c^3*b^3*a^8 + 715*x^9*c^4*b*a^9 + 429
/2*x^8*b^8*a^6 + 1716*x^8*c*b^6*a^7 + 6435/2*x^8*c^2*b^4*a^8 + 1430*x^8*c^3
*b^2*a^9 + 143/2*x^8*c^4*a^10 + 1716/7*x^7*b^7*a^7 + 1287*x^7*c*b^5*a^8 + 1
430*x^7*c^2*b^3*a^9 + 286*x^7*c^3*b*a^10 + 429/2*x^6*b^6*a^8 + 715*x^6*c*b^
4*a^9 + 429*x^6*c^2*b^2*a^10 + 26*x^6*c^3*a^11 + 143*x^5*b^5*a^9 + 286*x^5*
c*b^3*a^10 + 78*x^5*c^2*b*a^11 + 143/2*x^4*b^4*a^10 + 78*x^4*c*b^2*a^11 + 1
3/2*x^4*c^2*a^12 + 26*x^3*b^3*a^11 + 13*x^3*c*b*a^12 + 13/2*x^2*b^2*a^12 +
x^2*c*a^13 + x*b*a^13
```

Sympy [B] time = 0.305398, size = 1326, normalized size = 82.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x**2+b*x+a)**13,x)
```

```
[Out] a**13*b*x + b*c**13*x**27 + c**14*x**28/14 + x**26*(a*c**13 + 13*b**2*c**12
/2) + x**25*(13*a*b*c**12 + 26*b**3*c**11) + x**24*(13*a**2*c**12/2 + 78*a*
b**2*c**11 + 143*b**4*c**10/2) + x**23*(78*a**2*b*c**11 + 286*a*b**3*c**10
+ 143*b**5*c**9) + x**22*(26*a**3*c**11 + 429*a**2*b**2*c**10 + 715*a*b**4*
c**9 + 429*b**6*c**8/2) + x**21*(286*a**3*b*c**10 + 1430*a**2*b**3*c**9 + 1
287*a*b**5*c**8 + 1716*b**7*c**7/7) + x**20*(143*a**4*c**10/2 + 1430*a**3*b
**2*c**9 + 6435*a**2*b**4*c**8/2 + 1716*a*b**6*c**7 + 429*b**8*c**6/2) + x*
*19*(715*a**4*b*c**9 + 4290*a**3*b**3*c**8 + 5148*a**2*b**5*c**7 + 1716*a*b
**7*c**6 + 143*b**9*c**5) + x**18*(143*a**5*c**9 + 6435*a**4*b**2*c**8/2 +
8580*a**3*b**4*c**7 + 6006*a**2*b**6*c**6 + 1287*a*b**8*c**5 + 143*b**10*c*
*4/2) + x**17*(1287*a**5*b*c**8 + 8580*a**4*b**3*c**7 + 12012*a**3*b**5*c**
6 + 5148*a**2*b**7*c**5 + 715*a*b**9*c**4 + 26*b**11*c**3) + x**16*(429*a**
6*c**8/2 + 5148*a**5*b**2*c**7 + 15015*a**4*b**4*c**6 + 12012*a**3*b**6*c**
5 + 6435*a**2*b**8*c**4/2 + 286*a*b**10*c**3 + 13*b**12*c**2/2) + x**15*(17
16*a**6*b*c**7 + 12012*a**5*b**3*c**6 + 18018*a**4*b**5*c**5 + 8580*a**3*b*
*7*c**4 + 1430*a**2*b**9*c**3 + 78*a*b**11*c**2 + b**13*c) + x**14*(1716*a*
*7*c**7/7 + 6006*a**6*b**2*c**6 + 18018*a**5*b**4*c**5 + 15015*a**4*b**6*c*
*4 + 4290*a**3*b**8*c**3 + 429*a**2*b**10*c**2 + 13*a*b**12*c + b**14/14) +
 x**13*(1716*a**7*b*c**6 + 12012*a**6*b**3*c**5 + 18018*a**5*b**5*c**4 + 85
80*a**4*b**7*c**3 + 1430*a**3*b**9*c**2 + 78*a**2*b**11*c + a*b**13) + x**1
2*(429*a**8*c**6/2 + 5148*a**7*b**2*c**5 + 15015*a**6*b**4*c**4 + 12012*a**
5*b**6*c**3 + 6435*a**4*b**8*c**2/2 + 286*a**3*b**10*c + 13*a**2*b**12/2) +
 x**11*(1287*a**8*b*c**5 + 8580*a**7*b**3*c**4 + 12012*a**6*b**5*c**3 + 514
8*a**5*b**7*c**2 + 715*a**4*b**9*c + 26*a**3*b**11) + x**10*(143*a**9*c**5
+ 6435*a**8*b**2*c**4/2 + 8580*a**7*b**4*c**3 + 6006*a**6*b**6*c**2 + 1287*
a**5*b**8*c + 143*a**4*b**10/2) + x**9*(715*a**9*b*c**4 + 4290*a**8*b**3*c*
*3 + 5148*a**7*b**5*c**2 + 1716*a**6*b**7*c + 143*a**5*b**9) + x**8*(143*a*
*10*c**4/2 + 1430*a**9*b**2*c**3 + 6435*a**8*b**4*c**2/2 + 1716*a**7*b**6*c
 + 429*a**6*b**8/2) + x**7*(286*a**10*b*c**3 + 1430*a**9*b**3*c**2 + 1287*a
**8*b**5*c + 1716*a**7*b**7/7) + x**6*(26*a**11*c**3 + 429*a**10*b**2*c**2
+ 715*a**9*b**4*c + 429*a**8*b**6/2) + x**5*(78*a**11*b*c**2 + 286*a**10*b*
*3*c + 143*a**9*b**5) + x**4*(13*a**12*c**2/2 + 78*a**11*b**2*c + 143*a**10
*b**4/2) + x**3*(13*a**12*b*c + 26*a**11*b**3) + x**2*(a**13*c + 13*a**12*b
**2/2)
```

Giac [B] time = 1.11276, size = 1952, normalized size = 122.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x+a)^13,x, algorithm="giac")
```

```
[Out] 1/14*c^14*x^28 + b*c^13*x^27 + 13/2*b^2*c^12*x^26 + a*c^13*x^26 + 26*b^3*c^
11*x^25 + 13*a*b*c^12*x^25 + 143/2*b^4*c^10*x^24 + 78*a*b^2*c^11*x^24 + 13/
2*a^2*c^12*x^24 + 143*b^5*c^9*x^23 + 286*a*b^3*c^10*x^23 + 78*a^2*b*c^11*x^
23 + 429/2*b^6*c^8*x^22 + 715*a*b^4*c^9*x^22 + 429*a^2*b^2*c^10*x^22 + 26*a
^3*c^11*x^22 + 1716/7*b^7*c^7*x^21 + 1287*a*b^5*c^8*x^21 + 1430*a^2*b^3*c^9
*x^21 + 286*a^3*b*c^10*x^21 + 429/2*b^8*c^6*x^20 + 1716*a*b^6*c^7*x^20 + 64
35/2*a^2*b^4*c^8*x^20 + 1430*a^3*b^2*c^9*x^20 + 143/2*a^4*c^10*x^20 + 143*b
^9*c^5*x^19 + 1716*a*b^7*c^6*x^19 + 5148*a^2*b^5*c^7*x^19 + 4290*a^3*b^3*c^
8*x^19 + 715*a^4*b*c^9*x^19 + 143/2*b^10*c^4*x^18 + 1287*a*b^8*c^5*x^18 + 6
006*a^2*b^6*c^6*x^18 + 8580*a^3*b^4*c^7*x^18 + 6435/2*a^4*b^2*c^8*x^18 + 14
3*a^5*c^9*x^18 + 26*b^11*c^3*x^17 + 715*a*b^9*c^4*x^17 + 5148*a^2*b^7*c^5*x
^17 + 12012*a^3*b^5*c^6*x^17 + 8580*a^4*b^3*c^7*x^17 + 1287*a^5*b*c^8*x^17
+ 13/2*b^12*c^2*x^16 + 286*a*b^10*c^3*x^16 + 6435/2*a^2*b^8*c^4*x^16 + 1201
2*a^3*b^6*c^5*x^16 + 15015*a^4*b^4*c^6*x^16 + 5148*a^5*b^2*c^7*x^16 + 429/2
*a^6*c^8*x^16 + b^13*c*x^15 + 78*a*b^11*c^2*x^15 + 1430*a^2*b^9*c^3*x^15 +
8580*a^3*b^7*c^4*x^15 + 18018*a^4*b^5*c^5*x^15 + 12012*a^5*b^3*c^6*x^15 + 1
716*a^6*b*c^7*x^15 + 1/14*b^14*x^14 + 13*a*b^12*c*x^14 + 429*a^2*b^10*c^2*x
^14 + 4290*a^3*b^8*c^3*x^14 + 15015*a^4*b^6*c^4*x^14 + 18018*a^5*b^4*c^5*x^
14 + 6006*a^6*b^2*c^6*x^14 + 1716/7*a^7*c^7*x^14 + a*b^13*x^13 + 78*a^2*b^1
1*c*x^13 + 1430*a^3*b^9*c^2*x^13 + 8580*a^4*b^7*c^3*x^13 + 18018*a^5*b^5*c^
4*x^13 + 12012*a^6*b^3*c^5*x^13 + 1716*a^7*b*c^6*x^13 + 13/2*a^2*b^12*x^12
+ 286*a^3*b^10*c*x^12 + 6435/2*a^4*b^8*c^2*x^12 + 12012*a^5*b^6*c^3*x^12 +
15015*a^6*b^4*c^4*x^12 + 5148*a^7*b^2*c^5*x^12 + 429/2*a^8*c^6*x^12 + 26*a^
3*b^11*x^11 + 715*a^4*b^9*c*x^11 + 5148*a^5*b^7*c^2*x^11 + 12012*a^6*b^5*c^
3*x^11 + 8580*a^7*b^3*c^4*x^11 + 1287*a^8*b*c^5*x^11 + 143/2*a^4*b^10*x^10
+ 1287*a^5*b^8*c*x^10 + 6006*a^6*b^6*c^2*x^10 + 8580*a^7*b^4*c^3*x^10 + 643
5/2*a^8*b^2*c^4*x^10 + 143*a^9*c^5*x^10 + 143*a^5*b^9*x^9 + 1716*a^6*b^7*c*
x^9 + 5148*a^7*b^5*c^2*x^9 + 4290*a^8*b^3*c^3*x^9 + 715*a^9*b*c^4*x^9 + 429
/2*a^6*b^8*x^8 + 1716*a^7*b^6*c*x^8 + 6435/2*a^8*b^4*c^2*x^8 + 1430*a^9*b^2
*c^3*x^8 + 143/2*a^10*c^4*x^8 + 1716/7*a^7*b^7*x^7 + 1287*a^8*b^5*c*x^7 + 1
430*a^9*b^3*c^2*x^7 + 286*a^10*b*c^3*x^7 + 429/2*a^8*b^6*x^6 + 715*a^9*b^4*
c*x^6 + 429*a^10*b^2*c^2*x^6 + 26*a^11*c^3*x^6 + 143*a^9*b^5*x^5 + 286*a^10
*b^3*c*x^5 + 78*a^11*b*c^2*x^5 + 143/2*a^10*b^4*x^4 + 78*a^11*b^2*c*x^4 + 1
3/2*a^12*c^2*x^4 + 26*a^11*b^3*x^3 + 13*a^12*b*c*x^3 + 13/2*a^12*b^2*x^2 +
a^13*c*x^2 + a^13*b*x
```

3.94 $\int x\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{13}\,dx$

Optimal. Leaf size=18

$$\frac{1}{28}\left(a+bx^2+cx^4\right)^{14}$$

```
[Out] (a + b*x^2 + c*x^4)^14/28
```

Rubi [A] time = 0.329237, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1247, 629}

$$\frac{1}{28}\left(a+bx^2+cx^4\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[x*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^13,x]
```

```
[Out] (a + b*x^2 + c*x^4)^14/28
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int x\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{13}\,dx &= \frac{1}{2}\text{Subst}\left(\int (b+2cx)\left(a+bx+cx^2\right)^{13}\,dx,x,x^2\right)\\ &= \frac{1}{28}\left(a+bx^2+cx^4\right)^{14}\end{aligned}$$

Mathematica [B] time = 0.174441, size = 233, normalized size = 12.94

$$\frac{1}{28}x^2\left(b+cx^2\right)\left(91a^2x^{22}\left(b+cx^2\right)^{11}+364a^3x^{20}\left(b+cx^2\right)^{10}+1001a^4x^{18}\left(b+cx^2\right)^9+2002a^5x^{16}\left(b+cx^2\right)^8+3003a^6x^{14}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^13,x]
```

```
[Out] (x^2*(b + c*x^2)*(14*a^13 + 91*a^12*x^2*(b + c*x^2) + 364*a^11*x^4*(b + c*x
^2)^2 + 1001*a^10*x^6*(b + c*x^2)^3 + 2002*a^9*x^8*(b + c*x^2)^4 + 3003*a^8
*x^10*(b + c*x^2)^5 + 3432*a^7*x^12*(b + c*x^2)^6 + 3003*a^6*x^14*(b + c*x^
2)^7 + 2002*a^5*x^16*(b + c*x^2)^8 + 1001*a^4*x^18*(b + c*x^2)^9 + 364*a^3*
x^20*(b + c*x^2)^10 + 91*a^2*x^22*(b + c*x^2)^11 + 14*a*x^24*(b + c*x^2)^12
 + x^26*(b + c*x^2)^13))/28
```

Maple [B] time = 0.003, size = 46552, normalized size = 2586.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^13,x)
```

```
[Out] result too large to display
```

Maxima [B] time = 1.037, size = 1674, normalized size = 93.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^13,x, algorithm="maxima")
```

```
[Out] 1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 1/4*(13*b^2*c^12 + 2*a*c^13)*x^52 + 13/2
*(2*b^3*c^11 + a*b*c^12)*x^50 + 13/4*(11*b^4*c^10 + 12*a*b^2*c^11 + a^2*c^1
2)*x^48 + 13/2*(11*b^5*c^9 + 22*a*b^3*c^10 + 6*a^2*b*c^11)*x^46 + 13/4*(33*
b^6*c^8 + 110*a*b^4*c^9 + 66*a^2*b^2*c^10 + 4*a^3*c^11)*x^44 + 143/14*(12*b
^7*c^7 + 63*a*b^5*c^8 + 70*a^2*b^3*c^9 + 14*a^3*b*c^10)*x^42 + 143/4*(3*b^8
```

```
*c^6 + 24*a*b^6*c^7 + 45*a^2*b^4*c^8 + 20*a^3*b^2*c^9 + a^4*c^10)*x^40 + 14
3/2*(b^9*c^5 + 12*a*b^7*c^6 + 36*a^2*b^5*c^7 + 30*a^3*b^3*c^8 + 5*a^4*b*c^9
)*x^38 + 143/4*(b^10*c^4 + 18*a*b^8*c^5 + 84*a^2*b^6*c^6 + 120*a^3*b^4*c^7
+ 45*a^4*b^2*c^8 + 2*a^5*c^9)*x^36 + 13/2*(2*b^11*c^3 + 55*a*b^9*c^4 + 396*
a^2*b^7*c^5 + 924*a^3*b^5*c^6 + 660*a^4*b^3*c^7 + 99*a^5*b*c^8)*x^34 + 13/4
*(b^12*c^2 + 44*a*b^10*c^3 + 495*a^2*b^8*c^4 + 1848*a^3*b^6*c^5 + 2310*a^4*
b^4*c^6 + 792*a^5*b^2*c^7 + 33*a^6*c^8)*x^32 + 1/2*(b^13*c + 78*a*b^11*c^2
+ 1430*a^2*b^9*c^3 + 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 + 12012*a^5*b^3*c
^6 + 1716*a^6*b*c^7)*x^30 + 1/28*(b^14 + 182*a*b^12*c + 6006*a^2*b^10*c^2 +
 60060*a^3*b^8*c^3 + 210210*a^4*b^6*c^4 + 252252*a^5*b^4*c^5 + 84084*a^6*b^
2*c^6 + 3432*a^7*c^7)*x^28 + 1/2*(a*b^13 + 78*a^2*b^11*c + 1430*a^3*b^9*c^2
 + 8580*a^4*b^7*c^3 + 18018*a^5*b^5*c^4 + 12012*a^6*b^3*c^5 + 1716*a^7*b*c^
6)*x^26 + 13/4*(a^2*b^12 + 44*a^3*b^10*c + 495*a^4*b^8*c^2 + 1848*a^5*b^6*c
^3 + 2310*a^6*b^4*c^4 + 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^24 + 13/2*(2*a^3*b^
11 + 55*a^4*b^9*c + 396*a^5*b^7*c^2 + 924*a^6*b^5*c^3 + 660*a^7*b^3*c^4 + 9
9*a^8*b*c^5)*x^22 + 143/4*(a^4*b^10 + 18*a^5*b^8*c + 84*a^6*b^6*c^2 + 120*a
^7*b^4*c^3 + 45*a^8*b^2*c^4 + 2*a^9*c^5)*x^20 + 143/2*(a^5*b^9 + 12*a^6*b^7
*c + 36*a^7*b^5*c^2 + 30*a^8*b^3*c^3 + 5*a^9*b*c^4)*x^18 + 143/4*(3*a^6*b^8
 + 24*a^7*b^6*c + 45*a^8*b^4*c^2 + 20*a^9*b^2*c^3 + a^10*c^4)*x^16 + 1/2*a^
13*b*x^2 + 143/14*(12*a^7*b^7 + 63*a^8*b^5*c + 70*a^9*b^3*c^2 + 14*a^10*b*c
^3)*x^14 + 13/4*(33*a^8*b^6 + 110*a^9*b^4*c + 66*a^10*b^2*c^2 + 4*a^11*c^3)
*x^12 + 13/2*(11*a^9*b^5 + 22*a^10*b^3*c + 6*a^11*b*c^2)*x^10 + 13/4*(11*a^
10*b^4 + 12*a^11*b^2*c + a^12*c^2)*x^8 + 13/2*(2*a^11*b^3 + a^12*b*c)*x^6 +
 1/4*(13*a^12*b^2 + 2*a^13*c)*x^4
```

Fricas [B] time = 0.892736, size = 3578, normalized size = 198.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^13,x, algorithm="fricas")
```

```
[Out] 1/28*x^56*c^14 + 1/2*x^54*c^13*b + 13/4*x^52*c^12*b^2 + 1/2*x^52*c^13*a + 1
3*x^50*c^11*b^3 + 13/2*x^50*c^12*b*a + 143/4*x^48*c^10*b^4 + 39*x^48*c^11*b
^2*a + 13/4*x^48*c^12*a^2 + 143/2*x^46*c^9*b^5 + 143*x^46*c^10*b^3*a + 39*x
^46*c^11*b*a^2 + 429/4*x^44*c^8*b^6 + 715/2*x^44*c^9*b^4*a + 429/2*x^44*c^1
0*b^2*a^2 + 13*x^44*c^11*a^3 + 858/7*x^42*c^7*b^7 + 1287/2*x^42*c^8*b^5*a +
 715*x^42*c^9*b^3*a^2 + 143*x^42*c^10*b*a^3 + 429/4*x^40*c^6*b^8 + 858*x^40
*c^7*b^6*a + 6435/4*x^40*c^8*b^4*a^2 + 715*x^40*c^9*b^2*a^3 + 143/4*x^40*c^
10*a^4 + 143/2*x^38*c^5*b^9 + 858*x^38*c^6*b^7*a + 2574*x^38*c^7*b^5*a^2 +
2145*x^38*c^8*b^3*a^3 + 715/2*x^38*c^9*b*a^4 + 143/4*x^36*c^4*b^10 + 1287/2
```

```
*x^36*c^5*b^8*a + 3003*x^36*c^6*b^6*a^2 + 4290*x^36*c^7*b^4*a^3 + 6435/4*x^
36*c^8*b^2*a^4 + 143/2*x^36*c^9*a^5 + 13*x^34*c^3*b^11 + 715/2*x^34*c^4*b^9
*a + 2574*x^34*c^5*b^7*a^2 + 6006*x^34*c^6*b^5*a^3 + 4290*x^34*c^7*b^3*a^4
+ 1287/2*x^34*c^8*b*a^5 + 13/4*x^32*c^2*b^12 + 143*x^32*c^3*b^10*a + 6435/4
*x^32*c^4*b^8*a^2 + 6006*x^32*c^5*b^6*a^3 + 15015/2*x^32*c^6*b^4*a^4 + 2574
*x^32*c^7*b^2*a^5 + 429/4*x^32*c^8*a^6 + 1/2*x^30*c*b^13 + 39*x^30*c^2*b^11
*a + 715*x^30*c^3*b^9*a^2 + 4290*x^30*c^4*b^7*a^3 + 9009*x^30*c^5*b^5*a^4 +
 6006*x^30*c^6*b^3*a^5 + 858*x^30*c^7*b*a^6 + 1/28*x^28*b^14 + 13/2*x^28*c*
b^12*a + 429/2*x^28*c^2*b^10*a^2 + 2145*x^28*c^3*b^8*a^3 + 15015/2*x^28*c^4
*b^6*a^4 + 9009*x^28*c^5*b^4*a^5 + 3003*x^28*c^6*b^2*a^6 + 858/7*x^28*c^7*a
^7 + 1/2*x^26*b^13*a + 39*x^26*c*b^11*a^2 + 715*x^26*c^2*b^9*a^3 + 4290*x^2
6*c^3*b^7*a^4 + 9009*x^26*c^4*b^5*a^5 + 6006*x^26*c^5*b^3*a^6 + 858*x^26*c^
6*b*a^7 + 13/4*x^24*b^12*a^2 + 143*x^24*c*b^10*a^3 + 6435/4*x^24*c^2*b^8*a^
4 + 6006*x^24*c^3*b^6*a^5 + 15015/2*x^24*c^4*b^4*a^6 + 2574*x^24*c^5*b^2*a^
7 + 429/4*x^24*c^6*a^8 + 13*x^22*b^11*a^3 + 715/2*x^22*c*b^9*a^4 + 2574*x^2
2*c^2*b^7*a^5 + 6006*x^22*c^3*b^5*a^6 + 4290*x^22*c^4*b^3*a^7 + 1287/2*x^22
*c^5*b*a^8 + 143/4*x^20*b^10*a^4 + 1287/2*x^20*c*b^8*a^5 + 3003*x^20*c^2*b^
6*a^6 + 4290*x^20*c^3*b^4*a^7 + 6435/4*x^20*c^4*b^2*a^8 + 143/2*x^20*c^5*a^
9 + 143/2*x^18*b^9*a^5 + 858*x^18*c*b^7*a^6 + 2574*x^18*c^2*b^5*a^7 + 2145*
x^18*c^3*b^3*a^8 + 715/2*x^18*c^4*b*a^9 + 429/4*x^16*b^8*a^6 + 858*x^16*c*b
^6*a^7 + 6435/4*x^16*c^2*b^4*a^8 + 715*x^16*c^3*b^2*a^9 + 143/4*x^16*c^4*a^
10 + 858/7*x^14*b^7*a^7 + 1287/2*x^14*c*b^5*a^8 + 715*x^14*c^2*b^3*a^9 + 14
3*x^14*c^3*b*a^10 + 429/4*x^12*b^6*a^8 + 715/2*x^12*c*b^4*a^9 + 429/2*x^12*
c^2*b^2*a^10 + 13*x^12*c^3*a^11 + 143/2*x^10*b^5*a^9 + 143*x^10*c*b^3*a^10
+ 39*x^10*c^2*b*a^11 + 143/4*x^8*b^4*a^10 + 39*x^8*c*b^2*a^11 + 13/4*x^8*c^
2*a^12 + 13*x^6*b^3*a^11 + 13/2*x^6*c*b*a^12 + 13/4*x^4*b^2*a^12 + 1/2*x^4*
c*a^13 + 1/2*x^2*b*a^13
```

Sympy [B] time = 0.306524, size = 1384, normalized size = 76.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x**2+b)*(c*x**4+b*x**2+a)**13,x)
```

```
[Out] a**13*b*x**2/2 + b*c**13*x**54/2 + c**14*x**56/28 + x**52*(a*c**13/2 + 13*b
**2*c**12/4) + x**50*(13*a*b*c**12/2 + 13*b**3*c**11) + x**48*(13*a**2*c**1
2/4 + 39*a*b**2*c**11 + 143*b**4*c**10/4) + x**46*(39*a**2*b*c**11 + 143*a*
b**3*c**10 + 143*b**5*c**9/2) + x**44*(13*a**3*c**11 + 429*a**2*b**2*c**10/
2 + 715*a*b**4*c**9/2 + 429*b**6*c**8/4) + x**42*(143*a**3*b*c**10 + 715*a*
*2*b**3*c**9 + 1287*a*b**5*c**8/2 + 858*b**7*c**7/7) + x**40*(143*a**4*c**1
```

```
0/4 + 715*a**3*b**2*c**9 + 6435*a**2*b**4*c**8/4 + 858*a*b**6*c**7 + 429*b*
*8*c**6/4) + x**38*(715*a**4*b*c**9/2 + 2145*a**3*b**3*c**8 + 2574*a**2*b**
5*c**7 + 858*a*b**7*c**6 + 143*b**9*c**5/2) + x**36*(143*a**5*c**9/2 + 6435
*a**4*b**2*c**8/4 + 4290*a**3*b**4*c**7 + 3003*a**2*b**6*c**6 + 1287*a*b**8
*c**5/2 + 143*b**10*c**4/4) + x**34*(1287*a**5*b*c**8/2 + 4290*a**4*b**3*c*
*7 + 6006*a**3*b**5*c**6 + 2574*a**2*b**7*c**5 + 715*a*b**9*c**4/2 + 13*b**
11*c**3) + x**32*(429*a**6*c**8/4 + 2574*a**5*b**2*c**7 + 15015*a**4*b**4*c
**6/2 + 6006*a**3*b**6*c**5 + 6435*a**2*b**8*c**4/4 + 143*a*b**10*c**3 + 13
*b**12*c**2/4) + x**30*(858*a**6*b*c**7 + 6006*a**5*b**3*c**6 + 9009*a**4*b
**5*c**5 + 4290*a**3*b**7*c**4 + 715*a**2*b**9*c**3 + 39*a*b**11*c**2 + b**
13*c/2) + x**28*(858*a**7*c**7/7 + 3003*a**6*b**2*c**6 + 9009*a**5*b**4*c**
5 + 15015*a**4*b**6*c**4/2 + 2145*a**3*b**8*c**3 + 429*a**2*b**10*c**2/2 +
13*a*b**12*c/2 + b**14/28) + x**26*(858*a**7*b*c**6 + 6006*a**6*b**3*c**5 +
 9009*a**5*b**5*c**4 + 4290*a**4*b**7*c**3 + 715*a**3*b**9*c**2 + 39*a**2*b
**11*c + a*b**13/2) + x**24*(429*a**8*c**6/4 + 2574*a**7*b**2*c**5 + 15015*
a**6*b**4*c**4/2 + 6006*a**5*b**6*c**3 + 6435*a**4*b**8*c**2/4 + 143*a**3*b
**10*c + 13*a**2*b**12/4) + x**22*(1287*a**8*b*c**5/2 + 4290*a**7*b**3*c**4
 + 6006*a**6*b**5*c**3 + 2574*a**5*b**7*c**2 + 715*a**4*b**9*c/2 + 13*a**3*
b**11) + x**20*(143*a**9*c**5/2 + 6435*a**8*b**2*c**4/4 + 4290*a**7*b**4*c*
*3 + 3003*a**6*b**6*c**2 + 1287*a**5*b**8*c/2 + 143*a**4*b**10/4) + x**18*(
715*a**9*b*c**4/2 + 2145*a**8*b**3*c**3 + 2574*a**7*b**5*c**2 + 858*a**6*b*
*7*c + 143*a**5*b**9/2) + x**16*(143*a**10*c**4/4 + 715*a**9*b**2*c**3 + 64
35*a**8*b**4*c**2/4 + 858*a**7*b**6*c + 429*a**6*b**8/4) + x**14*(143*a**10
*b*c**3 + 715*a**9*b**3*c**2 + 1287*a**8*b**5*c/2 + 858*a**7*b**7/7) + x**1
2*(13*a**11*c**3 + 429*a**10*b**2*c**2/2 + 715*a**9*b**4*c/2 + 429*a**8*b**
6/4) + x**10*(39*a**11*b*c**2 + 143*a**10*b**3*c + 143*a**9*b**5/2) + x**8*
(13*a**12*c**2/4 + 39*a**11*b**2*c + 143*a**10*b**4/4) + x**6*(13*a**12*b*c
/2 + 13*a**11*b**3) + x**4*(a**13*c/2 + 13*a**12*b**2/4)
```

Giac [B] time = 1.14995, size = 1963, normalized size = 109.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^13,x, algorithm="giac")
```

```
[Out] 1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 13/4*b^2*c^12*x^52 + 1/2*a*c^13*x^52 + 1
3*b^3*c^11*x^50 + 13/2*a*b*c^12*x^50 + 143/4*b^4*c^10*x^48 + 39*a*b^2*c^11*
x^48 + 13/4*a^2*c^12*x^48 + 143/2*b^5*c^9*x^46 + 143*a*b^3*c^10*x^46 + 39*a
^2*b*c^11*x^46 + 429/4*b^6*c^8*x^44 + 715/2*a*b^4*c^9*x^44 + 429/2*a^2*b^2*
c^10*x^44 + 13*a^3*c^11*x^44 + 858/7*b^7*c^7*x^42 + 1287/2*a*b^5*c^8*x^42 +
```

```
 715*a^2*b^3*c^9*x^42 + 143*a^3*b*c^10*x^42 + 429/4*b^8*c^6*x^40 + 858*a*b^
6*c^7*x^40 + 6435/4*a^2*b^4*c^8*x^40 + 715*a^3*b^2*c^9*x^40 + 143/4*a^4*c^1
0*x^40 + 143/2*b^9*c^5*x^38 + 858*a*b^7*c^6*x^38 + 2574*a^2*b^5*c^7*x^38 +
2145*a^3*b^3*c^8*x^38 + 715/2*a^4*b*c^9*x^38 + 143/4*b^10*c^4*x^36 + 1287/2
*a*b^8*c^5*x^36 + 3003*a^2*b^6*c^6*x^36 + 4290*a^3*b^4*c^7*x^36 + 6435/4*a^
4*b^2*c^8*x^36 + 143/2*a^5*c^9*x^36 + 13*b^11*c^3*x^34 + 715/2*a*b^9*c^4*x^
34 + 2574*a^2*b^7*c^5*x^34 + 6006*a^3*b^5*c^6*x^34 + 4290*a^4*b^3*c^7*x^34
+ 1287/2*a^5*b*c^8*x^34 + 13/4*b^12*c^2*x^32 + 143*a*b^10*c^3*x^32 + 6435/4
*a^2*b^8*c^4*x^32 + 6006*a^3*b^6*c^5*x^32 + 15015/2*a^4*b^4*c^6*x^32 + 2574
*a^5*b^2*c^7*x^32 + 429/4*a^6*c^8*x^32 + 1/2*b^13*c*x^30 + 39*a*b^11*c^2*x^
30 + 715*a^2*b^9*c^3*x^30 + 4290*a^3*b^7*c^4*x^30 + 9009*a^4*b^5*c^5*x^30 +
 6006*a^5*b^3*c^6*x^30 + 858*a^6*b*c^7*x^30 + 1/28*b^14*x^28 + 13/2*a*b^12*
c*x^28 + 429/2*a^2*b^10*c^2*x^28 + 2145*a^3*b^8*c^3*x^28 + 15015/2*a^4*b^6*
c^4*x^28 + 9009*a^5*b^4*c^5*x^28 + 3003*a^6*b^2*c^6*x^28 + 858/7*a^7*c^7*x^
28 + 1/2*a*b^13*x^26 + 39*a^2*b^11*c*x^26 + 715*a^3*b^9*c^2*x^26 + 4290*a^4
*b^7*c^3*x^26 + 9009*a^5*b^5*c^4*x^26 + 6006*a^6*b^3*c^5*x^26 + 858*a^7*b*c
^6*x^26 + 13/4*a^2*b^12*x^24 + 143*a^3*b^10*c*x^24 + 6435/4*a^4*b^8*c^2*x^2
4 + 6006*a^5*b^6*c^3*x^24 + 15015/2*a^6*b^4*c^4*x^24 + 2574*a^7*b^2*c^5*x^2
4 + 429/4*a^8*c^6*x^24 + 13*a^3*b^11*x^22 + 715/2*a^4*b^9*c*x^22 + 2574*a^5
*b^7*c^2*x^22 + 6006*a^6*b^5*c^3*x^22 + 4290*a^7*b^3*c^4*x^22 + 1287/2*a^8*
b*c^5*x^22 + 143/4*a^4*b^10*x^20 + 1287/2*a^5*b^8*c*x^20 + 3003*a^6*b^6*c^2
*x^20 + 4290*a^7*b^4*c^3*x^20 + 6435/4*a^8*b^2*c^4*x^20 + 143/2*a^9*c^5*x^2
0 + 143/2*a^5*b^9*x^18 + 858*a^6*b^7*c*x^18 + 2574*a^7*b^5*c^2*x^18 + 2145*
a^8*b^3*c^3*x^18 + 715/2*a^9*b*c^4*x^18 + 429/4*a^6*b^8*x^16 + 858*a^7*b^6*
c*x^16 + 6435/4*a^8*b^4*c^2*x^16 + 715*a^9*b^2*c^3*x^16 + 143/4*a^10*c^4*x^
16 + 858/7*a^7*b^7*x^14 + 1287/2*a^8*b^5*c*x^14 + 715*a^9*b^3*c^2*x^14 + 14
3*a^10*b*c^3*x^14 + 429/4*a^8*b^6*x^12 + 715/2*a^9*b^4*c*x^12 + 429/2*a^10*
b^2*c^2*x^12 + 13*a^11*c^3*x^12 + 143/2*a^9*b^5*x^10 + 143*a^10*b^3*c*x^10
+ 39*a^11*b*c^2*x^10 + 143/4*a^10*b^4*x^8 + 39*a^11*b^2*c*x^8 + 13/4*a^12*c
^2*x^8 + 13*a^11*b^3*x^6 + 13/2*a^12*b*c*x^6 + 13/4*a^12*b^2*x^4 + 1/2*a^13
*c*x^4 + 1/2*a^13*b*x^2
```

3.95 $\int x^2 (b+2cx^3) (a+bx^3+cx^6)^{13} dx$

Optimal. Leaf size=18

$$\frac{1}{42}\left(a+bx^3+cx^6\right)^{14}$$

```
[Out] (a + b*x^3 + c*x^6)^14/42
```

Rubi [A] time = 0.302002, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1468, 629}

$$\frac{1}{42}\left(a+bx^3+cx^6\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^13,x]
```

```
[Out] (a + b*x^3 + c*x^6)^14/42
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int x^2 \left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{13} dx &= \frac{1}{3}\text{Subst}\left(\int (b+2cx)\left(a+bx+cx^2\right)^{13} dx, x, x^3\right)\\ &= \frac{1}{42}\left(a+bx^3+cx^6\right)^{14}\end{aligned}$$

Mathematica [B] time = 0.178563, size = 233, normalized size = 12.94

$$\frac{1}{42}x^3\left(b+cx^3\right)\left(91a^2x^{33}\left(b+cx^3\right)^{11}+364a^3x^{30}\left(b+cx^3\right)^{10}+1001a^4x^{27}\left(b+cx^3\right)^9+2002a^5x^{24}\left(b+cx^3\right)^8+3003a^6x^{21}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^13,x]
```

```
[Out] (x^3*(b + c*x^3)*(14*a^13 + 91*a^12*x^3*(b + c*x^3) + 364*a^11*x^6*(b + c*x
^3)^2 + 1001*a^10*x^9*(b + c*x^3)^3 + 2002*a^9*x^12*(b + c*x^3)^4 + 3003*a^
8*x^15*(b + c*x^3)^5 + 3432*a^7*x^18*(b + c*x^3)^6 + 3003*a^6*x^21*(b + c*x
^3)^7 + 2002*a^5*x^24*(b + c*x^3)^8 + 1001*a^4*x^27*(b + c*x^3)^9 + 364*a^3
*x^30*(b + c*x^3)^10 + 91*a^2*x^33*(b + c*x^3)^11 + 14*a*x^36*(b + c*x^3)^1
2 + x^39*(b + c*x^3)^13))/42
```

Maple [B] time = 0.003, size = 46552, normalized size = 2586.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^13,x)
```

```
[Out] result too large to display
```

Maxima [B] time = 1.07024, size = 1674, normalized size = 93.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^13,x, algorithm="maxima")
```

```
[Out] 1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 1/6*(13*b^2*c^12 + 2*a*c^13)*x^78 + 13/3
*(2*b^3*c^11 + a*b*c^12)*x^75 + 13/6*(11*b^4*c^10 + 12*a*b^2*c^11 + a^2*c^1
2)*x^72 + 13/3*(11*b^5*c^9 + 22*a*b^3*c^10 + 6*a^2*b*c^11)*x^69 + 13/6*(33*
b^6*c^8 + 110*a*b^4*c^9 + 66*a^2*b^2*c^10 + 4*a^3*c^11)*x^66 + 143/21*(12*b
^7*c^7 + 63*a*b^5*c^8 + 70*a^2*b^3*c^9 + 14*a^3*b*c^10)*x^63 + 143/6*(3*b^8
```

```
*c^6 + 24*a*b^6*c^7 + 45*a^2*b^4*c^8 + 20*a^3*b^2*c^9 + a^4*c^10)*x^60 + 14
3/3*(b^9*c^5 + 12*a*b^7*c^6 + 36*a^2*b^5*c^7 + 30*a^3*b^3*c^8 + 5*a^4*b*c^9
)*x^57 + 143/6*(b^10*c^4 + 18*a*b^8*c^5 + 84*a^2*b^6*c^6 + 120*a^3*b^4*c^7
+ 45*a^4*b^2*c^8 + 2*a^5*c^9)*x^54 + 13/3*(2*b^11*c^3 + 55*a*b^9*c^4 + 396*
a^2*b^7*c^5 + 924*a^3*b^5*c^6 + 660*a^4*b^3*c^7 + 99*a^5*b*c^8)*x^51 + 13/6
*(b^12*c^2 + 44*a*b^10*c^3 + 495*a^2*b^8*c^4 + 1848*a^3*b^6*c^5 + 2310*a^4*
b^4*c^6 + 792*a^5*b^2*c^7 + 33*a^6*c^8)*x^48 + 1/3*(b^13*c + 78*a*b^11*c^2
+ 1430*a^2*b^9*c^3 + 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 + 12012*a^5*b^3*c
^6 + 1716*a^6*b*c^7)*x^45 + 1/42*(b^14 + 182*a*b^12*c + 6006*a^2*b^10*c^2 +
 60060*a^3*b^8*c^3 + 210210*a^4*b^6*c^4 + 252252*a^5*b^4*c^5 + 84084*a^6*b^
2*c^6 + 3432*a^7*c^7)*x^42 + 1/3*(a*b^13 + 78*a^2*b^11*c + 1430*a^3*b^9*c^2
 + 8580*a^4*b^7*c^3 + 18018*a^5*b^5*c^4 + 12012*a^6*b^3*c^5 + 1716*a^7*b*c^
6)*x^39 + 13/6*(a^2*b^12 + 44*a^3*b^10*c + 495*a^4*b^8*c^2 + 1848*a^5*b^6*c
^3 + 2310*a^6*b^4*c^4 + 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^36 + 13/3*(2*a^3*b^
11 + 55*a^4*b^9*c + 396*a^5*b^7*c^2 + 924*a^6*b^5*c^3 + 660*a^7*b^3*c^4 + 9
9*a^8*b*c^5)*x^33 + 143/6*(a^4*b^10 + 18*a^5*b^8*c + 84*a^6*b^6*c^2 + 120*a
^7*b^4*c^3 + 45*a^8*b^2*c^4 + 2*a^9*c^5)*x^30 + 143/3*(a^5*b^9 + 12*a^6*b^7
*c + 36*a^7*b^5*c^2 + 30*a^8*b^3*c^3 + 5*a^9*b*c^4)*x^27 + 143/6*(3*a^6*b^8
 + 24*a^7*b^6*c + 45*a^8*b^4*c^2 + 20*a^9*b^2*c^3 + a^10*c^4)*x^24 + 143/21
*(12*a^7*b^7 + 63*a^8*b^5*c + 70*a^9*b^3*c^2 + 14*a^10*b*c^3)*x^21 + 13/6*(
33*a^8*b^6 + 110*a^9*b^4*c + 66*a^10*b^2*c^2 + 4*a^11*c^3)*x^18 + 1/3*a^13*
b*x^3 + 13/3*(11*a^9*b^5 + 22*a^10*b^3*c + 6*a^11*b*c^2)*x^15 + 13/6*(11*a^
10*b^4 + 12*a^11*b^2*c + a^12*c^2)*x^12 + 13/3*(2*a^11*b^3 + a^12*b*c)*x^9
+ 1/6*(13*a^12*b^2 + 2*a^13*c)*x^6
```

Fricas [B] time = 0.927339, size = 3594, normalized size = 199.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^13,x, algorithm="fricas")
```

```
[Out] 1/42*x^84*c^14 + 1/3*x^81*c^13*b + 13/6*x^78*c^12*b^2 + 1/3*x^78*c^13*a + 2
6/3*x^75*c^11*b^3 + 13/3*x^75*c^12*b*a + 143/6*x^72*c^10*b^4 + 26*x^72*c^11
*b^2*a + 13/6*x^72*c^12*a^2 + 143/3*x^69*c^9*b^5 + 286/3*x^69*c^10*b^3*a +
26*x^69*c^11*b*a^2 + 143/2*x^66*c^8*b^6 + 715/3*x^66*c^9*b^4*a + 143*x^66*c
^10*b^2*a^2 + 26/3*x^66*c^11*a^3 + 572/7*x^63*c^7*b^7 + 429*x^63*c^8*b^5*a
+ 1430/3*x^63*c^9*b^3*a^2 + 286/3*x^63*c^10*b*a^3 + 143/2*x^60*c^6*b^8 + 57
2*x^60*c^7*b^6*a + 2145/2*x^60*c^8*b^4*a^2 + 1430/3*x^60*c^9*b^2*a^3 + 143/
6*x^60*c^10*a^4 + 143/3*x^57*c^5*b^9 + 572*x^57*c^6*b^7*a + 1716*x^57*c^7*b
^5*a^2 + 1430*x^57*c^8*b^3*a^3 + 715/3*x^57*c^9*b*a^4 + 143/6*x^54*c^4*b^10
```

```
 + 429*x^54*c^5*b^8*a + 2002*x^54*c^6*b^6*a^2 + 2860*x^54*c^7*b^4*a^3 + 214
5/2*x^54*c^8*b^2*a^4 + 143/3*x^54*c^9*a^5 + 26/3*x^51*c^3*b^11 + 715/3*x^51
*c^4*b^9*a + 1716*x^51*c^5*b^7*a^2 + 4004*x^51*c^6*b^5*a^3 + 2860*x^51*c^7*
b^3*a^4 + 429*x^51*c^8*b*a^5 + 13/6*x^48*c^2*b^12 + 286/3*x^48*c^3*b^10*a +
 2145/2*x^48*c^4*b^8*a^2 + 4004*x^48*c^5*b^6*a^3 + 5005*x^48*c^6*b^4*a^4 +
1716*x^48*c^7*b^2*a^5 + 143/2*x^48*c^8*a^6 + 1/3*x^45*c*b^13 + 26*x^45*c^2*
b^11*a + 1430/3*x^45*c^3*b^9*a^2 + 2860*x^45*c^4*b^7*a^3 + 6006*x^45*c^5*b^
5*a^4 + 4004*x^45*c^6*b^3*a^5 + 572*x^45*c^7*b*a^6 + 1/42*x^42*b^14 + 13/3*
x^42*c*b^12*a + 143*x^42*c^2*b^10*a^2 + 1430*x^42*c^3*b^8*a^3 + 5005*x^42*c
^4*b^6*a^4 + 6006*x^42*c^5*b^4*a^5 + 2002*x^42*c^6*b^2*a^6 + 572/7*x^42*c^7
*a^7 + 1/3*x^39*b^13*a + 26*x^39*c*b^11*a^2 + 1430/3*x^39*c^2*b^9*a^3 + 286
0*x^39*c^3*b^7*a^4 + 6006*x^39*c^4*b^5*a^5 + 4004*x^39*c^5*b^3*a^6 + 572*x^
39*c^6*b*a^7 + 13/6*x^36*b^12*a^2 + 286/3*x^36*c*b^10*a^3 + 2145/2*x^36*c^2
*b^8*a^4 + 4004*x^36*c^3*b^6*a^5 + 5005*x^36*c^4*b^4*a^6 + 1716*x^36*c^5*b^
2*a^7 + 143/2*x^36*c^6*a^8 + 26/3*x^33*b^11*a^3 + 715/3*x^33*c*b^9*a^4 + 17
16*x^33*c^2*b^7*a^5 + 4004*x^33*c^3*b^5*a^6 + 2860*x^33*c^4*b^3*a^7 + 429*x
^33*c^5*b*a^8 + 143/6*x^30*b^10*a^4 + 429*x^30*c*b^8*a^5 + 2002*x^30*c^2*b^
6*a^6 + 2860*x^30*c^3*b^4*a^7 + 2145/2*x^30*c^4*b^2*a^8 + 143/3*x^30*c^5*a^
9 + 143/3*x^27*b^9*a^5 + 572*x^27*c*b^7*a^6 + 1716*x^27*c^2*b^5*a^7 + 1430*
x^27*c^3*b^3*a^8 + 715/3*x^27*c^4*b*a^9 + 143/2*x^24*b^8*a^6 + 572*x^24*c*b
^6*a^7 + 2145/2*x^24*c^2*b^4*a^8 + 1430/3*x^24*c^3*b^2*a^9 + 143/6*x^24*c^4
*a^10 + 572/7*x^21*b^7*a^7 + 429*x^21*c*b^5*a^8 + 1430/3*x^21*c^2*b^3*a^9 +
 286/3*x^21*c^3*b*a^10 + 143/2*x^18*b^6*a^8 + 715/3*x^18*c*b^4*a^9 + 143*x^
18*c^2*b^2*a^10 + 26/3*x^18*c^3*a^11 + 143/3*x^15*b^5*a^9 + 286/3*x^15*c*b^
3*a^10 + 26*x^15*c^2*b*a^11 + 143/6*x^12*b^4*a^10 + 26*x^12*c*b^2*a^11 + 13
/6*x^12*c^2*a^12 + 26/3*x^9*b^3*a^11 + 13/3*x^9*c*b*a^12 + 13/6*x^6*b^2*a^1
2 + 1/3*x^6*c*a^13 + 1/3*x^3*b*a^13
```

Sympy [B] time = 0.305405, size = 1394, normalized size = 77.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*c*x**3+b)*(c*x**6+b*x**3+a)**13,x)
```

```
[Out] a**13*b*x**3/3 + b*c**13*x**81/3 + c**14*x**84/42 + x**78*(a*c**13/3 + 13*b
**2*c**12/6) + x**75*(13*a*b*c**12/3 + 26*b**3*c**11/3) + x**72*(13*a**2*c*
*12/6 + 26*a*b**2*c**11 + 143*b**4*c**10/6) + x**69*(26*a**2*b*c**11 + 286*
a*b**3*c**10/3 + 143*b**5*c**9/3) + x**66*(26*a**3*c**11/3 + 143*a**2*b**2*
c**10 + 715*a*b**4*c**9/3 + 143*b**6*c**8/2) + x**63*(286*a**3*b*c**10/3 +
1430*a**2*b**3*c**9/3 + 429*a*b**5*c**8 + 572*b**7*c**7/7) + x**60*(143*a**
```

```
4*c**10/6 + 1430*a**3*b**2*c**9/3 + 2145*a**2*b**4*c**8/2 + 572*a*b**6*c**7
 + 143*b**8*c**6/2) + x**57*(715*a**4*b*c**9/3 + 1430*a**3*b**3*c**8 + 1716
*a**2*b**5*c**7 + 572*a*b**7*c**6 + 143*b**9*c**5/3) + x**54*(143*a**5*c**9
/3 + 2145*a**4*b**2*c**8/2 + 2860*a**3*b**4*c**7 + 2002*a**2*b**6*c**6 + 42
9*a*b**8*c**5 + 143*b**10*c**4/6) + x**51*(429*a**5*b*c**8 + 2860*a**4*b**3
*c**7 + 4004*a**3*b**5*c**6 + 1716*a**2*b**7*c**5 + 715*a*b**9*c**4/3 + 26*
b**11*c**3/3) + x**48*(143*a**6*c**8/2 + 1716*a**5*b**2*c**7 + 5005*a**4*b*
*4*c**6 + 4004*a**3*b**6*c**5 + 2145*a**2*b**8*c**4/2 + 286*a*b**10*c**3/3
+ 13*b**12*c**2/6) + x**45*(572*a**6*b*c**7 + 4004*a**5*b**3*c**6 + 6006*a*
*4*b**5*c**5 + 2860*a**3*b**7*c**4 + 1430*a**2*b**9*c**3/3 + 26*a*b**11*c**
2 + b**13*c/3) + x**42*(572*a**7*c**7/7 + 2002*a**6*b**2*c**6 + 6006*a**5*b
**4*c**5 + 5005*a**4*b**6*c**4 + 1430*a**3*b**8*c**3 + 143*a**2*b**10*c**2
+ 13*a*b**12*c/3 + b**14/42) + x**39*(572*a**7*b*c**6 + 4004*a**6*b**3*c**5
 + 6006*a**5*b**5*c**4 + 2860*a**4*b**7*c**3 + 1430*a**3*b**9*c**2/3 + 26*a
**2*b**11*c + a*b**13/3) + x**36*(143*a**8*c**6/2 + 1716*a**7*b**2*c**5 + 5
005*a**6*b**4*c**4 + 4004*a**5*b**6*c**3 + 2145*a**4*b**8*c**2/2 + 286*a**3
*b**10*c/3 + 13*a**2*b**12/6) + x**33*(429*a**8*b*c**5 + 2860*a**7*b**3*c**
4 + 4004*a**6*b**5*c**3 + 1716*a**5*b**7*c**2 + 715*a**4*b**9*c/3 + 26*a**3
*b**11/3) + x**30*(143*a**9*c**5/3 + 2145*a**8*b**2*c**4/2 + 2860*a**7*b**4
*c**3 + 2002*a**6*b**6*c**2 + 429*a**5*b**8*c + 143*a**4*b**10/6) + x**27*(
715*a**9*b*c**4/3 + 1430*a**8*b**3*c**3 + 1716*a**7*b**5*c**2 + 572*a**6*b*
*7*c + 143*a**5*b**9/3) + x**24*(143*a**10*c**4/6 + 1430*a**9*b**2*c**3/3 +
 2145*a**8*b**4*c**2/2 + 572*a**7*b**6*c + 143*a**6*b**8/2) + x**21*(286*a*
*10*b*c**3/3 + 1430*a**9*b**3*c**2/3 + 429*a**8*b**5*c + 572*a**7*b**7/7) +
 x**18*(26*a**11*c**3/3 + 143*a**10*b**2*c**2 + 715*a**9*b**4*c/3 + 143*a**
8*b**6/2) + x**15*(26*a**11*b*c**2 + 286*a**10*b**3*c/3 + 143*a**9*b**5/3)
+ x**12*(13*a**12*c**2/6 + 26*a**11*b**2*c + 143*a**10*b**4/6) + x**9*(13*a
**12*b*c/3 + 26*a**11*b**3/3) + x**6*(a**13*c/3 + 13*a**12*b**2/6)
```

Giac [B] time = 1.14702, size = 1963, normalized size = 109.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^13,x, algorithm="giac")
```

```
[Out] 1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 13/6*b^2*c^12*x^78 + 1/3*a*c^13*x^78 + 2
6/3*b^3*c^11*x^75 + 13/3*a*b*c^12*x^75 + 143/6*b^4*c^10*x^72 + 26*a*b^2*c^1
1*x^72 + 13/6*a^2*c^12*x^72 + 143/3*b^5*c^9*x^69 + 286/3*a*b^3*c^10*x^69 +
26*a^2*b*c^11*x^69 + 143/2*b^6*c^8*x^66 + 715/3*a*b^4*c^9*x^66 + 143*a^2*b^
2*c^10*x^66 + 26/3*a^3*c^11*x^66 + 572/7*b^7*c^7*x^63 + 429*a*b^5*c^8*x^63
```

```
+ 1430/3*a^2*b^3*c^9*x^63 + 286/3*a^3*b*c^10*x^63 + 143/2*b^8*c^6*x^60 + 57
2*a*b^6*c^7*x^60 + 2145/2*a^2*b^4*c^8*x^60 + 1430/3*a^3*b^2*c^9*x^60 + 143/
6*a^4*c^10*x^60 + 143/3*b^9*c^5*x^57 + 572*a*b^7*c^6*x^57 + 1716*a^2*b^5*c^
7*x^57 + 1430*a^3*b^3*c^8*x^57 + 715/3*a^4*b*c^9*x^57 + 143/6*b^10*c^4*x^54
 + 429*a*b^8*c^5*x^54 + 2002*a^2*b^6*c^6*x^54 + 2860*a^3*b^4*c^7*x^54 + 214
5/2*a^4*b^2*c^8*x^54 + 143/3*a^5*c^9*x^54 + 26/3*b^11*c^3*x^51 + 715/3*a*b^
9*c^4*x^51 + 1716*a^2*b^7*c^5*x^51 + 4004*a^3*b^5*c^6*x^51 + 2860*a^4*b^3*c
^7*x^51 + 429*a^5*b*c^8*x^51 + 13/6*b^12*c^2*x^48 + 286/3*a*b^10*c^3*x^48 +
 2145/2*a^2*b^8*c^4*x^48 + 4004*a^3*b^6*c^5*x^48 + 5005*a^4*b^4*c^6*x^48 +
1716*a^5*b^2*c^7*x^48 + 143/2*a^6*c^8*x^48 + 1/3*b^13*c*x^45 + 26*a*b^11*c^
2*x^45 + 1430/3*a^2*b^9*c^3*x^45 + 2860*a^3*b^7*c^4*x^45 + 6006*a^4*b^5*c^5
*x^45 + 4004*a^5*b^3*c^6*x^45 + 572*a^6*b*c^7*x^45 + 1/42*b^14*x^42 + 13/3*
a*b^12*c*x^42 + 143*a^2*b^10*c^2*x^42 + 1430*a^3*b^8*c^3*x^42 + 5005*a^4*b^
6*c^4*x^42 + 6006*a^5*b^4*c^5*x^42 + 2002*a^6*b^2*c^6*x^42 + 572/7*a^7*c^7*
x^42 + 1/3*a*b^13*x^39 + 26*a^2*b^11*c*x^39 + 1430/3*a^3*b^9*c^2*x^39 + 286
0*a^4*b^7*c^3*x^39 + 6006*a^5*b^5*c^4*x^39 + 4004*a^6*b^3*c^5*x^39 + 572*a^
7*b*c^6*x^39 + 13/6*a^2*b^12*x^36 + 286/3*a^3*b^10*c*x^36 + 2145/2*a^4*b^8*
c^2*x^36 + 4004*a^5*b^6*c^3*x^36 + 5005*a^6*b^4*c^4*x^36 + 1716*a^7*b^2*c^5
*x^36 + 143/2*a^8*c^6*x^36 + 26/3*a^3*b^11*x^33 + 715/3*a^4*b^9*c*x^33 + 17
16*a^5*b^7*c^2*x^33 + 4004*a^6*b^5*c^3*x^33 + 2860*a^7*b^3*c^4*x^33 + 429*a
^8*b*c^5*x^33 + 143/6*a^4*b^10*x^30 + 429*a^5*b^8*c*x^30 + 2002*a^6*b^6*c^2
*x^30 + 2860*a^7*b^4*c^3*x^30 + 2145/2*a^8*b^2*c^4*x^30 + 143/3*a^9*c^5*x^3
0 + 143/3*a^5*b^9*x^27 + 572*a^6*b^7*c*x^27 + 1716*a^7*b^5*c^2*x^27 + 1430*
a^8*b^3*c^3*x^27 + 715/3*a^9*b*c^4*x^27 + 143/2*a^6*b^8*x^24 + 572*a^7*b^6*
c*x^24 + 2145/2*a^8*b^4*c^2*x^24 + 1430/3*a^9*b^2*c^3*x^24 + 143/6*a^10*c^4
*x^24 + 572/7*a^7*b^7*x^21 + 429*a^8*b^5*c*x^21 + 1430/3*a^9*b^3*c^2*x^21 +
 286/3*a^10*b*c^3*x^21 + 143/2*a^8*b^6*x^18 + 715/3*a^9*b^4*c*x^18 + 143*a^
10*b^2*c^2*x^18 + 26/3*a^11*c^3*x^18 + 143/3*a^9*b^5*x^15 + 286/3*a^10*b^3*
c*x^15 + 26*a^11*b*c^2*x^15 + 143/6*a^10*b^4*x^12 + 26*a^11*b^2*c*x^12 + 13
/6*a^12*c^2*x^12 + 26/3*a^11*b^3*x^9 + 13/3*a^12*b*c*x^9 + 13/6*a^12*b^2*x^
6 + 1/3*a^13*c*x^6 + 1/3*a^13*b*x^3
```

3.96 $\int x^{-1+n}\left(b+2cx^n\right)\left(a+bx^n+cx^{2n}\right)^{13}\,dx$

Optimal. Leaf size=23

$$\frac{\left(a+bx^n+cx^{2n}\right)^{14}}{14n}$$

```
[Out] (a + b*x^n + c*x^(2*n))^14/(14*n)
```

Rubi [A] time = 0.0559981, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1468, 629}

$$\frac{\left(a+bx^n+cx^{2n}\right)^{14}}{14n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*(b + 2*c*x^n)*(a + b*x^n + c*x^(2*n))^13,x]
```

```
[Out] (a + b*x^n + c*x^(2*n))^14/(14*n)
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int x^{-1+n}\left(b+2cx^n\right)\left(a+bx^n+cx^{2n}\right)^{13}\,dx=\frac{\text{Subst}\left(\int(b+2cx)\left(a+bx+cx^2\right)^{13}\,dx,x,x^n\right)}{n}$$
$$=\frac{\left(a+bx^n+cx^{2n}\right)^{14}}{14n}$$

Mathematica [A] time = 0.0678214, size = 22, normalized size = 0.96

$$\frac{\left(a+x^n\left(b+cx^n\right)\right)^{14}}{14n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*(b + 2*c*x^n)*(a + b*x^n + c*x^(2*n))^13,x]
```

```
[Out] (a + x^n*(b + c*x^n))^14/(14*n)
```

Maple [B] time = 0.062, size = 2042, normalized size = 88.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n)*(b+2*c*x^n)*(a+b*x^n+c*x^(2*n))^13,x)
```

```
[Out] 1716/7*b^7*c^7/n*(x^n)^21+143*b^5*c^9/n*(x^n)^23+26*b^3*c^11/n*(x^n)^25+26*
a^11*b^3/n*(x^n)^3+143*b^5*a^9/n*(x^n)^5+1716/7*b^7*a^7/n*(x^n)^7+143*a^5*b
^9/n*(x^n)^9+26*b^11*a^3/n*(x^n)^11+a*b^13/n*(x^n)^13+b^13*c/n*(x^n)^15+26*
b^11*c^3/n*(x^n)^17+143*b^9*c^5/n*(x^n)^19+a^13/n*(x^n)^2*c+13/2*a^12/n*(x^
n)^2*b^2+1716/7/n*(x^n)^14*a^7*c^7+c^13/n*(x^n)^26*a+13/2*c^12/n*(x^n)^26*b
^2+143/2*c^10/n*(x^n)^24*b^4+429/2*c^6/n*(x^n)^20*b^8+26*c^11/n*(x^n)^22*a^
3+429/2*c^8/n*(x^n)^22*b^6+13/2*c^12/n*(x^n)^24*a^2+143*a^9/n*(x^n)^10*c^5+
143/2*a^4/n*(x^n)^10*b^10+429/2*a^8/n*(x^n)^12*c^6+13/2*a^2/n*(x^n)^12*b^12
+143*c^9/n*(x^n)^18*a^5+143/2*c^4/n*(x^n)^18*b^10+429/2*c^8/n*(x^n)^16*a^6+
13/2*c^2/n*(x^n)^16*b^12+143/2*c^10/n*(x^n)^20*a^4+13/2*a^12/n*(x^n)^4*c^2+
143/2*a^10/n*(x^n)^4*b^4+143/2*a^10/n*(x^n)^8*c^4+429/2*a^6/n*(x^n)^8*b^8+2
6*a^11/n*(x^n)^6*c^3+429/2*a^8/n*(x^n)^6*b^6+b*c^13/n*(x^n)^27+b*a^13/n*x^n
+1/14/n*(x^n)^14*b^14+8580*a^7/n*(x^n)^10*b^4*c^3+6006*a^6/n*(x^n)^10*b^6*c
^2+1287*a^5/n*(x^n)^10*b^8*c+5148*a^7/n*(x^n)^12*b^2*c^5+15015*a^6/n*(x^n)^
```

```
12*b^4*c^4+12012*a^5/n*(x^n)^12*b^6*c^3+6435/2*a^4/n*(x^n)^12*b^8*c^2+6435/
2*a^8/n*(x^n)^8*b^4*c^2+1716*a^7/n*(x^n)^8*b^6*c+429*a^10/n*(x^n)^6*b^2*c^2
+715*a^9/n*(x^n)^6*b^4*c+6435/2*a^8/n*(x^n)^10*b^2*c^4+78*b*c^11/n*(x^n)^23
*a^2+286*b^3*c^10/n*(x^n)^23*a+13*b*c^12/n*(x^n)^25*a+715*b*c^9/n*(x^n)^19*
a^4+4290*b^3*c^8/n*(x^n)^19*a^3+5148*b^5*c^7/n*(x^n)^19*a^2+1716*b^7*c^6/n*
(x^n)^19*a+286*b*c^10/n*(x^n)^21*a^3+1430*b^3*c^9/n*(x^n)^21*a^2+1287*b^5*c
^8/n*(x^n)^21*a+1287*b*c^8/n*(x^n)^17*a^5+8580*b^3*c^7/n*(x^n)^17*a^4+12012
*b^5*c^6/n*(x^n)^17*a^3+5148*b^7*c^5/n*(x^n)^17*a^2+715*b^9*c^4/n*(x^n)^17*
a+8580*a^4*b^7/n*(x^n)^13*c^3+1430*a^3*b^9/n*(x^n)^13*c^2+78*a^2*b^11/n*(x^
n)^13*c+1716*b*c^7/n*(x^n)^15*a^6+12012*b^3*c^6/n*(x^n)^15*a^5+18018*b^5*c^
5/n*(x^n)^15*a^4+8580*b^7*c^4/n*(x^n)^15*a^3+1430*b^9*c^3/n*(x^n)^15*a^2+78
*b^11*c^2/n*(x^n)^15*a+715*a^9*b/n*(x^n)^9*c^4+4290*a^8*b^3/n*(x^n)^9*c^3+5
148*a^7*b^5/n*(x^n)^9*c^2+1716*a^6*b^7/n*(x^n)^9*c+1287*b*a^8/n*(x^n)^11*c^
5+8580*b^3*a^7/n*(x^n)^11*c^4+12012*b^5*a^6/n*(x^n)^11*c^3+5148*b^7*a^5/n*(
x^n)^11*c^2+715*b^9*a^4/n*(x^n)^11*c+1716*a^7*b/n*(x^n)^13*c^6+12012*a^6*b^
3/n*(x^n)^13*c^5+18018*a^5*b^5/n*(x^n)^13*c^4+78*b*a^11/n*(x^n)^5*c^2+286*b
^3*a^10/n*(x^n)^5*c+286*b*a^10/n*(x^n)^7*c^3+1430*b^3*a^9/n*(x^n)^7*c^2+128
7*b^5*a^8/n*(x^n)^7*c+6006/n*(x^n)^14*a^6*b^2*c^6+18018/n*(x^n)^14*a^5*b^4*
c^5+15015/n*(x^n)^14*a^4*b^6*c^4+4290/n*(x^n)^14*a^3*b^8*c^3+429/n*(x^n)^14
*a^2*b^10*c^2+13/n*(x^n)^14*a*b^12*c+13*a^12*b/n*(x^n)^3*c+1430*c^9/n*(x^n)
^20*a^3*b^2+6435/2*c^8/n*(x^n)^20*a^2*b^4+1716*c^7/n*(x^n)^20*a*b^6+429*c^1
0/n*(x^n)^22*a^2*b^2+715*c^9/n*(x^n)^22*a*b^4+78*c^11/n*(x^n)^24*a*b^2+286*
a^3/n*(x^n)^12*b^10*c+6435/2*c^8/n*(x^n)^18*a^4*b^2+8580*c^7/n*(x^n)^18*a^3
*b^4+6006*c^6/n*(x^n)^18*a^2*b^6+1287*c^5/n*(x^n)^18*a*b^8+5148*c^7/n*(x^n)
^16*a^5*b^2+15015*c^6/n*(x^n)^16*a^4*b^4+12012*c^5/n*(x^n)^16*a^3*b^6+6435/
2*c^4/n*(x^n)^16*a^2*b^8+286*c^3/n*(x^n)^16*a*b^10+1/14*c^14/n*(x^n)^28+78*
a^11/n*(x^n)^4*b^2*c+1430*a^9/n*(x^n)^8*b^2*c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(a+b*x^n+c*x^(2*n))^13,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.29833, size = 2969, normalized size = 129.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(a+b*x^n+c*x^(2*n))^13,x, algorithm="fricas"
)
```

```
[Out] 1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 14*a^13*b*x^n + 7*(13*b^2*c^12 +
 2*a*c^13)*x^(26*n) + 182*(2*b^3*c^11 + a*b*c^12)*x^(25*n) + 91*(11*b^4*c^1
0 + 12*a*b^2*c^11 + a^2*c^12)*x^(24*n) + 182*(11*b^5*c^9 + 22*a*b^3*c^10 +
6*a^2*b*c^11)*x^(23*n) + 91*(33*b^6*c^8 + 110*a*b^4*c^9 + 66*a^2*b^2*c^10 +
 4*a^3*c^11)*x^(22*n) + 286*(12*b^7*c^7 + 63*a*b^5*c^8 + 70*a^2*b^3*c^9 + 1
4*a^3*b*c^10)*x^(21*n) + 1001*(3*b^8*c^6 + 24*a*b^6*c^7 + 45*a^2*b^4*c^8 +
20*a^3*b^2*c^9 + a^4*c^10)*x^(20*n) + 2002*(b^9*c^5 + 12*a*b^7*c^6 + 36*a^2
*b^5*c^7 + 30*a^3*b^3*c^8 + 5*a^4*b*c^9)*x^(19*n) + 1001*(b^10*c^4 + 18*a*b
^8*c^5 + 84*a^2*b^6*c^6 + 120*a^3*b^4*c^7 + 45*a^4*b^2*c^8 + 2*a^5*c^9)*x^(
18*n) + 182*(2*b^11*c^3 + 55*a*b^9*c^4 + 396*a^2*b^7*c^5 + 924*a^3*b^5*c^6
+ 660*a^4*b^3*c^7 + 99*a^5*b*c^8)*x^(17*n) + 91*(b^12*c^2 + 44*a*b^10*c^3 +
 495*a^2*b^8*c^4 + 1848*a^3*b^6*c^5 + 2310*a^4*b^4*c^6 + 792*a^5*b^2*c^7 +
33*a^6*c^8)*x^(16*n) + 14*(b^13*c + 78*a*b^11*c^2 + 1430*a^2*b^9*c^3 + 8580
*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 + 12012*a^5*b^3*c^6 + 1716*a^6*b*c^7)*x^(1
5*n) + (b^14 + 182*a*b^12*c + 6006*a^2*b^10*c^2 + 60060*a^3*b^8*c^3 + 21021
0*a^4*b^6*c^4 + 252252*a^5*b^4*c^5 + 84084*a^6*b^2*c^6 + 3432*a^7*c^7)*x^(1
4*n) + 14*(a*b^13 + 78*a^2*b^11*c + 1430*a^3*b^9*c^2 + 8580*a^4*b^7*c^3 + 1
8018*a^5*b^5*c^4 + 12012*a^6*b^3*c^5 + 1716*a^7*b*c^6)*x^(13*n) + 91*(a^2*b
^12 + 44*a^3*b^10*c + 495*a^4*b^8*c^2 + 1848*a^5*b^6*c^3 + 2310*a^6*b^4*c^4
 + 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^(12*n) + 182*(2*a^3*b^11 + 55*a^4*b^9*c
+ 396*a^5*b^7*c^2 + 924*a^6*b^5*c^3 + 660*a^7*b^3*c^4 + 99*a^8*b*c^5)*x^(11
*n) + 1001*(a^4*b^10 + 18*a^5*b^8*c + 84*a^6*b^6*c^2 + 120*a^7*b^4*c^3 + 45
*a^8*b^2*c^4 + 2*a^9*c^5)*x^(10*n) + 2002*(a^5*b^9 + 12*a^6*b^7*c + 36*a^7*
b^5*c^2 + 30*a^8*b^3*c^3 + 5*a^9*b*c^4)*x^(9*n) + 1001*(3*a^6*b^8 + 24*a^7*
b^6*c + 45*a^8*b^4*c^2 + 20*a^9*b^2*c^3 + a^10*c^4)*x^(8*n) + 286*(12*a^7*b
^7 + 63*a^8*b^5*c + 70*a^9*b^3*c^2 + 14*a^10*b*c^3)*x^(7*n) + 91*(33*a^8*b^
6 + 110*a^9*b^4*c + 66*a^10*b^2*c^2 + 4*a^11*c^3)*x^(6*n) + 182*(11*a^9*b^5
 + 22*a^10*b^3*c + 6*a^11*b*c^2)*x^(5*n) + 91*(11*a^10*b^4 + 12*a^11*b^2*c
+ a^12*c^2)*x^(4*n) + 182*(2*a^11*b^3 + a^12*b*c)*x^(3*n) + 7*(13*a^12*b^2
+ 2*a^13*c)*x^(2*n))/n
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)*(a+b*x**n+c*x**(2*n))**13,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.29113, size = 2286, normalized size = 99.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(a+b*x^n+c*x^(2*n))^13,x, algorithm="giac")
```

```
[Out] 1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 91*b^2*c^12*x^(26*n) + 14*a*c^13
*x^(26*n) + 364*b^3*c^11*x^(25*n) + 182*a*b*c^12*x^(25*n) + 1001*b^4*c^10*x
^(24*n) + 1092*a*b^2*c^11*x^(24*n) + 91*a^2*c^12*x^(24*n) + 2002*b^5*c^9*x^
(23*n) + 4004*a*b^3*c^10*x^(23*n) + 1092*a^2*b*c^11*x^(23*n) + 3003*b^6*c^8
*x^(22*n) + 10010*a*b^4*c^9*x^(22*n) + 6006*a^2*b^2*c^10*x^(22*n) + 364*a^3
*c^11*x^(22*n) + 3432*b^7*c^7*x^(21*n) + 18018*a*b^5*c^8*x^(21*n) + 20020*a
^2*b^3*c^9*x^(21*n) + 4004*a^3*b*c^10*x^(21*n) + 3003*b^8*c^6*x^(20*n) + 24
024*a*b^6*c^7*x^(20*n) + 45045*a^2*b^4*c^8*x^(20*n) + 20020*a^3*b^2*c^9*x^(
20*n) + 1001*a^4*c^10*x^(20*n) + 2002*b^9*c^5*x^(19*n) + 24024*a*b^7*c^6*x^
(19*n) + 72072*a^2*b^5*c^7*x^(19*n) + 60060*a^3*b^3*c^8*x^(19*n) + 10010*a^
4*b*c^9*x^(19*n) + 1001*b^10*c^4*x^(18*n) + 18018*a*b^8*c^5*x^(18*n) + 8408
4*a^2*b^6*c^6*x^(18*n) + 120120*a^3*b^4*c^7*x^(18*n) + 45045*a^4*b^2*c^8*x^
(18*n) + 2002*a^5*c^9*x^(18*n) + 364*b^11*c^3*x^(17*n) + 10010*a*b^9*c^4*x^
(17*n) + 72072*a^2*b^7*c^5*x^(17*n) + 168168*a^3*b^5*c^6*x^(17*n) + 120120*
a^4*b^3*c^7*x^(17*n) + 18018*a^5*b*c^8*x^(17*n) + 91*b^12*c^2*x^(16*n) + 40
04*a*b^10*c^3*x^(16*n) + 45045*a^2*b^8*c^4*x^(16*n) + 168168*a^3*b^6*c^5*x^
(16*n) + 210210*a^4*b^4*c^6*x^(16*n) + 72072*a^5*b^2*c^7*x^(16*n) + 3003*a^
6*c^8*x^(16*n) + 14*b^13*c*x^(15*n) + 1092*a*b^11*c^2*x^(15*n) + 20020*a^2*
b^9*c^3*x^(15*n) + 120120*a^3*b^7*c^4*x^(15*n) + 252252*a^4*b^5*c^5*x^(15*n
) + 168168*a^5*b^3*c^6*x^(15*n) + 24024*a^6*b*c^7*x^(15*n) + b^14*x^(14*n)
+ 182*a*b^12*c*x^(14*n) + 6006*a^2*b^10*c^2*x^(14*n) + 60060*a^3*b^8*c^3*x^
(14*n) + 210210*a^4*b^6*c^4*x^(14*n) + 252252*a^5*b^4*c^5*x^(14*n) + 84084*
a^6*b^2*c^6*x^(14*n) + 3432*a^7*c^7*x^(14*n) + 14*a*b^13*x^(13*n) + 1092*a^
2*b^11*c*x^(13*n) + 20020*a^3*b^9*c^2*x^(13*n) + 120120*a^4*b^7*c^3*x^(13*n
) + 252252*a^5*b^5*c^4*x^(13*n) + 168168*a^6*b^3*c^5*x^(13*n) + 24024*a^7*b
*c^6*x^(13*n) + 91*a^2*b^12*x^(12*n) + 4004*a^3*b^10*c*x^(12*n) + 45045*a^4
*b^8*c^2*x^(12*n) + 168168*a^5*b^6*c^3*x^(12*n) + 210210*a^6*b^4*c^4*x^(12*
n) + 72072*a^7*b^2*c^5*x^(12*n) + 3003*a^8*c^6*x^(12*n) + 364*a^3*b^11*x^(1
```

```
1*n) + 10010*a^4*b^9*c*x^(11*n) + 72072*a^5*b^7*c^2*x^(11*n) + 168168*a^6*b
^5*c^3*x^(11*n) + 120120*a^7*b^3*c^4*x^(11*n) + 18018*a^8*b*c^5*x^(11*n) +
1001*a^4*b^10*x^(10*n) + 18018*a^5*b^8*c*x^(10*n) + 84084*a^6*b^6*c^2*x^(10
*n) + 120120*a^7*b^4*c^3*x^(10*n) + 45045*a^8*b^2*c^4*x^(10*n) + 2002*a^9*c
^5*x^(10*n) + 2002*a^5*b^9*x^(9*n) + 24024*a^6*b^7*c*x^(9*n) + 72072*a^7*b^
5*c^2*x^(9*n) + 60060*a^8*b^3*c^3*x^(9*n) + 10010*a^9*b*c^4*x^(9*n) + 3003*
a^6*b^8*x^(8*n) + 24024*a^7*b^6*c*x^(8*n) + 45045*a^8*b^4*c^2*x^(8*n) + 200
20*a^9*b^2*c^3*x^(8*n) + 1001*a^10*c^4*x^(8*n) + 3432*a^7*b^7*x^(7*n) + 180
18*a^8*b^5*c*x^(7*n) + 20020*a^9*b^3*c^2*x^(7*n) + 4004*a^10*b*c^3*x^(7*n)
+ 3003*a^8*b^6*x^(6*n) + 10010*a^9*b^4*c*x^(6*n) + 6006*a^10*b^2*c^2*x^(6*n
) + 364*a^11*c^3*x^(6*n) + 2002*a^9*b^5*x^(5*n) + 4004*a^10*b^3*c*x^(5*n) +
 1092*a^11*b*c^2*x^(5*n) + 1001*a^10*b^4*x^(4*n) + 1092*a^11*b^2*c*x^(4*n)
+ 91*a^12*c^2*x^(4*n) + 364*a^11*b^3*x^(3*n) + 182*a^12*b*c*x^(3*n) + 91*a^
12*b^2*x^(2*n) + 14*a^13*c*x^(2*n) + 14*a^13*b*x^n)/n
```

3.97 $\int (b+2cx)\left(-a+bx+cx^2\right)^{13}\,dx$

Optimal. Leaf size=18

$$\frac{1}{14}\left(a-bx-cx^2\right)^{14}$$

[Out] (a - b*x - c*x^2)^14/14

Rubi [A] time = 0.0687475, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {629}

$$\frac{1}{14}\left(a-bx-cx^2\right)^{14}$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x)*(-a + b*x + c*x^2)^13,x]

[Out] (a - b*x - c*x^2)^14/14

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (b+2cx)\left(-a+bx+cx^2\right)^{13}\,dx=\frac{1}{14}\left(a-bx-cx^2\right)^{14}$$

Mathematica [B] time = 0.173047, size = 201, normalized size = 11.17

$$\frac{1}{14}x(b+cx)\left(-364a^{11}x^2(b+cx)^2+1001a^{10}x^3(b+cx)^3-2002a^9x^4(b+cx)^4+3003a^8x^5(b+cx)^5-3432a^7x^6(b+cx)^6+\cdots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)*(-a + b*x + c*x^2)^13,x]
```

```
[Out] (x*(b + c*x)*(-14*a^13 + 91*a^12*x*(b + c*x) - 364*a^11*x^2*(b + c*x)^2 + 1
001*a^10*x^3*(b + c*x)^3 - 2002*a^9*x^4*(b + c*x)^4 + 3003*a^8*x^5*(b + c*x
)^5 - 3432*a^7*x^6*(b + c*x)^6 + 3003*a^6*x^7*(b + c*x)^7 - 2002*a^5*x^8*(b
 + c*x)^8 + 1001*a^4*x^9*(b + c*x)^9 - 364*a^3*x^10*(b + c*x)^10 + 91*a^2*x
^11*(b + c*x)^11 - 14*a*x^12*(b + c*x)^12 + x^13*(b + c*x)^13))/14
```

Maple [B] time = 0.006, size = 47685, normalized size = 2649.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)*(c*x^2+b*x-a)^13,x)
```

```
[Out] result too large to display
```

Maxima [A] time = 1.2298, size = 22, normalized size = 1.22

$$\frac{1}{14}\left(cx^2+bx-a\right)^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x-a)^13,x, algorithm="maxima")
```

```
[Out] 1/14*(c*x^2 + b*x - a)^14
```

Fricas [B] time = 0.807591, size = 3474, normalized size = 193.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x-a)^13,x, algorithm="fricas")
```

```
[Out] 1/14*x^28*c^14 + x^27*c^13*b + 13/2*x^26*c^12*b^2 - x^26*c^13*a + 26*x^25*c
^11*b^3 - 13*x^25*c^12*b*a + 143/2*x^24*c^10*b^4 - 78*x^24*c^11*b^2*a + 13/
2*x^24*c^12*a^2 + 143*x^23*c^9*b^5 - 286*x^23*c^10*b^3*a + 78*x^23*c^11*b*a
^2 + 429/2*x^22*c^8*b^6 - 715*x^22*c^9*b^4*a + 429*x^22*c^10*b^2*a^2 - 26*x
^22*c^11*a^3 + 1716/7*x^21*c^7*b^7 - 1287*x^21*c^8*b^5*a + 1430*x^21*c^9*b^
3*a^2 - 286*x^21*c^10*b*a^3 + 429/2*x^20*c^6*b^8 - 1716*x^20*c^7*b^6*a + 64
35/2*x^20*c^8*b^4*a^2 - 1430*x^20*c^9*b^2*a^3 + 143/2*x^20*c^10*a^4 + 143*x
^19*c^5*b^9 - 1716*x^19*c^6*b^7*a + 5148*x^19*c^7*b^5*a^2 - 4290*x^19*c^8*b
^3*a^3 + 715*x^19*c^9*b*a^4 + 143/2*x^18*c^4*b^10 - 1287*x^18*c^5*b^8*a + 6
006*x^18*c^6*b^6*a^2 - 8580*x^18*c^7*b^4*a^3 + 6435/2*x^18*c^8*b^2*a^4 - 14
3*x^18*c^9*a^5 + 26*x^17*c^3*b^11 - 715*x^17*c^4*b^9*a + 5148*x^17*c^5*b^7*
a^2 - 12012*x^17*c^6*b^5*a^3 + 8580*x^17*c^7*b^3*a^4 - 1287*x^17*c^8*b*a^5
+ 13/2*x^16*c^2*b^12 - 286*x^16*c^3*b^10*a + 6435/2*x^16*c^4*b^8*a^2 - 1201
2*x^16*c^5*b^6*a^3 + 15015*x^16*c^6*b^4*a^4 - 5148*x^16*c^7*b^2*a^5 + 429/2
*x^16*c^8*a^6 + x^15*c*b^13 - 78*x^15*c^2*b^11*a + 1430*x^15*c^3*b^9*a^2 -
8580*x^15*c^4*b^7*a^3 + 18018*x^15*c^5*b^5*a^4 - 12012*x^15*c^6*b^3*a^5 + 1
716*x^15*c^7*b*a^6 + 1/14*x^14*b^14 - 13*x^14*c*b^12*a + 429*x^14*c^2*b^10*
a^2 - 4290*x^14*c^3*b^8*a^3 + 15015*x^14*c^4*b^6*a^4 - 18018*x^14*c^5*b^4*a
^5 + 6006*x^14*c^6*b^2*a^6 - 1716/7*x^14*c^7*a^7 - x^13*b^13*a + 78*x^13*c*
b^11*a^2 - 1430*x^13*c^2*b^9*a^3 + 8580*x^13*c^3*b^7*a^4 - 18018*x^13*c^4*b
^5*a^5 + 12012*x^13*c^5*b^3*a^6 - 1716*x^13*c^6*b*a^7 + 13/2*x^12*b^12*a^2
- 286*x^12*c*b^10*a^3 + 6435/2*x^12*c^2*b^8*a^4 - 12012*x^12*c^3*b^6*a^5 +
15015*x^12*c^4*b^4*a^6 - 5148*x^12*c^5*b^2*a^7 + 429/2*x^12*c^6*a^8 - 26*x^
11*b^11*a^3 + 715*x^11*c*b^9*a^4 - 5148*x^11*c^2*b^7*a^5 + 12012*x^11*c^3*b
^5*a^6 - 8580*x^11*c^4*b^3*a^7 + 1287*x^11*c^5*b*a^8 + 143/2*x^10*b^10*a^4
- 1287*x^10*c*b^8*a^5 + 6006*x^10*c^2*b^6*a^6 - 8580*x^10*c^3*b^4*a^7 + 643
5/2*x^10*c^4*b^2*a^8 - 143*x^10*c^5*a^9 - 143*x^9*b^9*a^5 + 1716*x^9*c*b^7*
a^6 - 5148*x^9*c^2*b^5*a^7 + 4290*x^9*c^3*b^3*a^8 - 715*x^9*c^4*b*a^9 + 429
/2*x^8*b^8*a^6 - 1716*x^8*c*b^6*a^7 + 6435/2*x^8*c^2*b^4*a^8 - 1430*x^8*c^3
*b^2*a^9 + 143/2*x^8*c^4*a^10 - 1716/7*x^7*b^7*a^7 + 1287*x^7*c*b^5*a^8 - 1
430*x^7*c^2*b^3*a^9 + 286*x^7*c^3*b*a^10 + 429/2*x^6*b^6*a^8 - 715*x^6*c*b^
4*a^9 + 429*x^6*c^2*b^2*a^10 - 26*x^6*c^3*a^11 - 143*x^5*b^5*a^9 + 286*x^5*
c*b^3*a^10 - 78*x^5*c^2*b*a^11 + 143/2*x^4*b^4*a^10 - 78*x^4*c*b^2*a^11 + 1
3/2*x^4*c^2*a^12 - 26*x^3*b^3*a^11 + 13*x^3*c*b*a^12 + 13/2*x^2*b^2*a^12 -
x^2*c*a^13 - x*b*a^13
```

Sympy [B] time = 0.312619, size = 1326, normalized size = 73.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x**2+b*x-a)**13,x)
```

```
[Out] -a**13*b*x + b*c**13*x**27 + c**14*x**28/14 + x**26*(-a*c**13 + 13*b**2*c**
12/2) + x**25*(-13*a*b*c**12 + 26*b**3*c**11) + x**24*(13*a**2*c**12/2 - 78
*a*b**2*c**11 + 143*b**4*c**10/2) + x**23*(78*a**2*b*c**11 - 286*a*b**3*c**
10 + 143*b**5*c**9) + x**22*(-26*a**3*c**11 + 429*a**2*b**2*c**10 - 715*a*b
**4*c**9 + 429*b**6*c**8/2) + x**21*(-286*a**3*b*c**10 + 1430*a**2*b**3*c**
9 - 1287*a*b**5*c**8 + 1716*b**7*c**7/7) + x**20*(143*a**4*c**10/2 - 1430*a
**3*b**2*c**9 + 6435*a**2*b**4*c**8/2 - 1716*a*b**6*c**7 + 429*b**8*c**6/2)
 + x**19*(715*a**4*b*c**9 - 4290*a**3*b**3*c**8 + 5148*a**2*b**5*c**7 - 171
6*a*b**7*c**6 + 143*b**9*c**5) + x**18*(-143*a**5*c**9 + 6435*a**4*b**2*c**
8/2 - 8580*a**3*b**4*c**7 + 6006*a**2*b**6*c**6 - 1287*a*b**8*c**5 + 143*b*
*10*c**4/2) + x**17*(-1287*a**5*b*c**8 + 8580*a**4*b**3*c**7 - 12012*a**3*b
**5*c**6 + 5148*a**2*b**7*c**5 - 715*a*b**9*c**4 + 26*b**11*c**3) + x**16*(
429*a**6*c**8/2 - 5148*a**5*b**2*c**7 + 15015*a**4*b**4*c**6 - 12012*a**3*b
**6*c**5 + 6435*a**2*b**8*c**4/2 - 286*a*b**10*c**3 + 13*b**12*c**2/2) + x*
*15*(1716*a**6*b*c**7 - 12012*a**5*b**3*c**6 + 18018*a**4*b**5*c**5 - 8580*
a**3*b**7*c**4 + 1430*a**2*b**9*c**3 - 78*a*b**11*c**2 + b**13*c) + x**14*(
-1716*a**7*c**7/7 + 6006*a**6*b**2*c**6 - 18018*a**5*b**4*c**5 + 15015*a**4
*b**6*c**4 - 4290*a**3*b**8*c**3 + 429*a**2*b**10*c**2 - 13*a*b**12*c + b**
14/14) + x**13*(-1716*a**7*b*c**6 + 12012*a**6*b**3*c**5 - 18018*a**5*b**5*
c**4 + 8580*a**4*b**7*c**3 - 1430*a**3*b**9*c**2 + 78*a**2*b**11*c - a*b**1
3) + x**12*(429*a**8*c**6/2 - 5148*a**7*b**2*c**5 + 15015*a**6*b**4*c**4 -
12012*a**5*b**6*c**3 + 6435*a**4*b**8*c**2/2 - 286*a**3*b**10*c + 13*a**2*b
**12/2) + x**11*(1287*a**8*b*c**5 - 8580*a**7*b**3*c**4 + 12012*a**6*b**5*c
**3 - 5148*a**5*b**7*c**2 + 715*a**4*b**9*c - 26*a**3*b**11) + x**10*(-143*
a**9*c**5 + 6435*a**8*b**2*c**4/2 - 8580*a**7*b**4*c**3 + 6006*a**6*b**6*c*
*2 - 1287*a**5*b**8*c + 143*a**4*b**10/2) + x**9*(-715*a**9*b*c**4 + 4290*a
**8*b**3*c**3 - 5148*a**7*b**5*c**2 + 1716*a**6*b**7*c - 143*a**5*b**9) + x
**8*(143*a**10*c**4/2 - 1430*a**9*b**2*c**3 + 6435*a**8*b**4*c**2/2 - 1716*
a**7*b**6*c + 429*a**6*b**8/2) + x**7*(286*a**10*b*c**3 - 1430*a**9*b**3*c*
*2 + 1287*a**8*b**5*c - 1716*a**7*b**7/7) + x**6*(-26*a**11*c**3 + 429*a**1
0*b**2*c**2 - 715*a**9*b**4*c + 429*a**8*b**6/2) + x**5*(-78*a**11*b*c**2 +
 286*a**10*b**3*c - 143*a**9*b**5) + x**4*(13*a**12*c**2/2 - 78*a**11*b**2*
c + 143*a**10*b**4/2) + x**3*(13*a**12*b*c - 26*a**11*b**3) + x**2*(-a**13*
c + 13*a**12*b**2/2)
```

Giac [B] time = 1.11006, size = 1958, normalized size = 108.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x-a)^13,x, algorithm="giac")

[Out] 1/14*c^14*x^28 + b*c^13*x^27 + 13/2*b^2*c^12*x^26 - a*c^13*x^26 + 26*b^3*c^
11*x^25 - 13*a*b*c^12*x^25 + 143/2*b^4*c^10*x^24 - 78*a*b^2*c^11*x^24 + 13/
2*a^2*c^12*x^24 + 143*b^5*c^9*x^23 - 286*a*b^3*c^10*x^23 + 78*a^2*b*c^11*x^
23 + 429/2*b^6*c^8*x^22 - 715*a*b^4*c^9*x^22 + 429*a^2*b^2*c^10*x^22 - 26*a
^3*c^11*x^22 + 1716/7*b^7*c^7*x^21 - 1287*a*b^5*c^8*x^21 + 1430*a^2*b^3*c^9
*x^21 - 286*a^3*b*c^10*x^21 + 429/2*b^8*c^6*x^20 - 1716*a*b^6*c^7*x^20 + 64
35/2*a^2*b^4*c^8*x^20 - 1430*a^3*b^2*c^9*x^20 + 143/2*a^4*c^10*x^20 + 143*b
^9*c^5*x^19 - 1716*a*b^7*c^6*x^19 + 5148*a^2*b^5*c^7*x^19 - 4290*a^3*b^3*c^
8*x^19 + 715*a^4*b*c^9*x^19 + 143/2*b^10*c^4*x^18 - 1287*a*b^8*c^5*x^18 + 6
006*a^2*b^6*c^6*x^18 - 8580*a^3*b^4*c^7*x^18 + 6435/2*a^4*b^2*c^8*x^18 - 14
3*a^5*c^9*x^18 + 26*b^11*c^3*x^17 - 715*a*b^9*c^4*x^17 + 5148*a^2*b^7*c^5*x
^17 - 12012*a^3*b^5*c^6*x^17 + 8580*a^4*b^3*c^7*x^17 - 1287*a^5*b*c^8*x^17
+ 13/2*b^12*c^2*x^16 - 286*a*b^10*c^3*x^16 + 6435/2*a^2*b^8*c^4*x^16 - 1201
2*a^3*b^6*c^5*x^16 + 15015*a^4*b^4*c^6*x^16 - 5148*a^5*b^2*c^7*x^16 + 429/2
*a^6*c^8*x^16 + b^13*c*x^15 - 78*a*b^11*c^2*x^15 + 1430*a^2*b^9*c^3*x^15 -
8580*a^3*b^7*c^4*x^15 + 18018*a^4*b^5*c^5*x^15 - 12012*a^5*b^3*c^6*x^15 + 1
716*a^6*b*c^7*x^15 + 1/14*b^14*x^14 - 13*a*b^12*c*x^14 + 429*a^2*b^10*c^2*x
^14 - 4290*a^3*b^8*c^3*x^14 + 15015*a^4*b^6*c^4*x^14 - 18018*a^5*b^4*c^5*x^
14 + 6006*a^6*b^2*c^6*x^14 - 1716/7*a^7*c^7*x^14 - a*b^13*x^13 + 78*a^2*b^1
1*c*x^13 - 1430*a^3*b^9*c^2*x^13 + 8580*a^4*b^7*c^3*x^13 - 18018*a^5*b^5*c^
4*x^13 + 12012*a^6*b^3*c^5*x^13 - 1716*a^7*b*c^6*x^13 + 13/2*a^2*b^12*x^12
- 286*a^3*b^10*c*x^12 + 6435/2*a^4*b^8*c^2*x^12 - 12012*a^5*b^6*c^3*x^12 +
15015*a^6*b^4*c^4*x^12 - 5148*a^7*b^2*c^5*x^12 + 429/2*a^8*c^6*x^12 - 26*a^
3*b^11*x^11 + 715*a^4*b^9*c*x^11 - 5148*a^5*b^7*c^2*x^11 + 12012*a^6*b^5*c^
3*x^11 - 8580*a^7*b^3*c^4*x^11 + 1287*a^8*b*c^5*x^11 + 143/2*a^4*b^10*x^10
- 1287*a^5*b^8*c*x^10 + 6006*a^6*b^6*c^2*x^10 - 8580*a^7*b^4*c^3*x^10 + 643
5/2*a^8*b^2*c^4*x^10 - 143*a^9*c^5*x^10 - 143*a^5*b^9*x^9 + 1716*a^6*b^7*c*
x^9 - 5148*a^7*b^5*c^2*x^9 + 4290*a^8*b^3*c^3*x^9 - 715*a^9*b*c^4*x^9 + 429
/2*a^6*b^8*x^8 - 1716*a^7*b^6*c*x^8 + 6435/2*a^8*b^4*c^2*x^8 - 1430*a^9*b^2
*c^3*x^8 + 143/2*a^10*c^4*x^8 - 1716/7*a^7*b^7*x^7 + 1287*a^8*b^5*c*x^7 - 1
430*a^9*b^3*c^2*x^7 + 286*a^10*b*c^3*x^7 + 429/2*a^8*b^6*x^6 - 715*a^9*b^4*
c*x^6 + 429*a^10*b^2*c^2*x^6 - 26*a^11*c^3*x^6 - 143*a^9*b^5*x^5 + 286*a^10
*b^3*c*x^5 - 78*a^11*b*c^2*x^5 + 143/2*a^10*b^4*x^4 - 78*a^11*b^2*c*x^4 + 1
3/2*a^12*c^2*x^4 - 26*a^11*b^3*x^3 + 13*a^12*b*c*x^3 + 13/2*a^12*b^2*x^2 -
a^13*c*x^2 - a^13*b*x
```

3.98 $\int x\left(b+2cx^2\right)\left(-a+bx^2+cx^4\right)^{13}\,dx$

Optimal. Leaf size=20

$$\frac{1}{28}\left(a-bx^2-cx^4\right)^{14}$$

```
[Out] (a - b*x^2 - c*x^4)^14/28
```

Rubi [A] time = 0.32171, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1247, 629}

$$\frac{1}{28}\left(a-bx^2-cx^4\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[x*(b + 2*c*x^2)*(-a + b*x^2 + c*x^4)^13,x]
```

```
[Out] (a - b*x^2 - c*x^4)^14/28
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int x\left(b+2cx^2\right)\left(-a+bx^2+cx^4\right)^{13}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int (b+2cx)\left(-a+bx+cx^2\right)^{13}\,dx, x, x^2\right)\\ &= \frac{1}{28}\left(a-bx^2-cx^4\right)^{14}\end{aligned}$$

Mathematica [B] time = 0.168198, size = 233, normalized size = 11.65

$$\frac{1}{28}x^2\left(b+cx^2\right)\left(91a^2x^{22}\left(b+cx^2\right)^{11}-364a^3x^{20}\left(b+cx^2\right)^{10}+1001a^4x^{18}\left(b+cx^2\right)^9-2002a^5x^{16}\left(b+cx^2\right)^8+3003a^6x^{14}\left(\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(b + 2*c*x^2)*(-a + b*x^2 + c*x^4)^13,x]
```

```
[Out] (x^2*(b + c*x^2)*(-14*a^13 + 91*a^12*x^2*(b + c*x^2) - 364*a^11*x^4*(b + c*
x^2)^2 + 1001*a^10*x^6*(b + c*x^2)^3 - 2002*a^9*x^8*(b + c*x^2)^4 + 3003*a^
8*x^10*(b + c*x^2)^5 - 3432*a^7*x^12*(b + c*x^2)^6 + 3003*a^6*x^14*(b + c*x
^2)^7 - 2002*a^5*x^16*(b + c*x^2)^8 + 1001*a^4*x^18*(b + c*x^2)^9 - 364*a^3
*x^20*(b + c*x^2)^10 + 91*a^2*x^22*(b + c*x^2)^11 - 14*a*x^24*(b + c*x^2)^1
2 + x^26*(b + c*x^2)^13))/28
```

Maple [B] time = 0.002, size = 47688, normalized size = 2384.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^13,x)
```

```
[Out] result too large to display
```

Maxima [B] time = 1.05417, size = 1677, normalized size = 83.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^13,x, algorithm="maxima")
```

```
[Out] 1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 1/4*(13*b^2*c^12 - 2*a*c^13)*x^52 + 13/2
*(2*b^3*c^11 - a*b*c^12)*x^50 + 13/4*(11*b^4*c^10 - 12*a*b^2*c^11 + a^2*c^1
2)*x^48 + 13/2*(11*b^5*c^9 - 22*a*b^3*c^10 + 6*a^2*b*c^11)*x^46 + 13/4*(33*
b^6*c^8 - 110*a*b^4*c^9 + 66*a^2*b^2*c^10 - 4*a^3*c^11)*x^44 + 143/14*(12*b
^7*c^7 - 63*a*b^5*c^8 + 70*a^2*b^3*c^9 - 14*a^3*b*c^10)*x^42 + 143/4*(3*b^8
```

```
*c^6 - 24*a*b^6*c^7 + 45*a^2*b^4*c^8 - 20*a^3*b^2*c^9 + a^4*c^10)*x^40 + 14
3/2*(b^9*c^5 - 12*a*b^7*c^6 + 36*a^2*b^5*c^7 - 30*a^3*b^3*c^8 + 5*a^4*b*c^9
)*x^38 + 143/4*(b^10*c^4 - 18*a*b^8*c^5 + 84*a^2*b^6*c^6 - 120*a^3*b^4*c^7
+ 45*a^4*b^2*c^8 - 2*a^5*c^9)*x^36 + 13/2*(2*b^11*c^3 - 55*a*b^9*c^4 + 396*
a^2*b^7*c^5 - 924*a^3*b^5*c^6 + 660*a^4*b^3*c^7 - 99*a^5*b*c^8)*x^34 + 13/4
*(b^12*c^2 - 44*a*b^10*c^3 + 495*a^2*b^8*c^4 - 1848*a^3*b^6*c^5 + 2310*a^4*
b^4*c^6 - 792*a^5*b^2*c^7 + 33*a^6*c^8)*x^32 + 1/2*(b^13*c - 78*a*b^11*c^2
+ 1430*a^2*b^9*c^3 - 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 - 12012*a^5*b^3*c
^6 + 1716*a^6*b*c^7)*x^30 + 1/28*(b^14 - 182*a*b^12*c + 6006*a^2*b^10*c^2 -
 60060*a^3*b^8*c^3 + 210210*a^4*b^6*c^4 - 252252*a^5*b^4*c^5 + 84084*a^6*b^
2*c^6 - 3432*a^7*c^7)*x^28 - 1/2*(a*b^13 - 78*a^2*b^11*c + 1430*a^3*b^9*c^2
 - 8580*a^4*b^7*c^3 + 18018*a^5*b^5*c^4 - 12012*a^6*b^3*c^5 + 1716*a^7*b*c^
6)*x^26 + 13/4*(a^2*b^12 - 44*a^3*b^10*c + 495*a^4*b^8*c^2 - 1848*a^5*b^6*c
^3 + 2310*a^6*b^4*c^4 - 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^24 - 13/2*(2*a^3*b^
11 - 55*a^4*b^9*c + 396*a^5*b^7*c^2 - 924*a^6*b^5*c^3 + 660*a^7*b^3*c^4 - 9
9*a^8*b*c^5)*x^22 + 143/4*(a^4*b^10 - 18*a^5*b^8*c + 84*a^6*b^6*c^2 - 120*a
^7*b^4*c^3 + 45*a^8*b^2*c^4 - 2*a^9*c^5)*x^20 - 143/2*(a^5*b^9 - 12*a^6*b^7
*c + 36*a^7*b^5*c^2 - 30*a^8*b^3*c^3 + 5*a^9*b*c^4)*x^18 + 143/4*(3*a^6*b^8
 - 24*a^7*b^6*c + 45*a^8*b^4*c^2 - 20*a^9*b^2*c^3 + a^10*c^4)*x^16 - 1/2*a^
13*b*x^2 - 143/14*(12*a^7*b^7 - 63*a^8*b^5*c + 70*a^9*b^3*c^2 - 14*a^10*b*c
^3)*x^14 + 13/4*(33*a^8*b^6 - 110*a^9*b^4*c + 66*a^10*b^2*c^2 - 4*a^11*c^3)
*x^12 - 13/2*(11*a^9*b^5 - 22*a^10*b^3*c + 6*a^11*b*c^2)*x^10 + 13/4*(11*a^
10*b^4 - 12*a^11*b^2*c + a^12*c^2)*x^8 - 13/2*(2*a^11*b^3 - a^12*b*c)*x^6 +
 1/4*(13*a^12*b^2 - 2*a^13*c)*x^4
```

Fricas [B] time = 0.859741, size = 3578, normalized size = 178.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^13,x, algorithm="fricas")
```

```
[Out] 1/28*x^56*c^14 + 1/2*x^54*c^13*b + 13/4*x^52*c^12*b^2 - 1/2*x^52*c^13*a + 1
3*x^50*c^11*b^3 - 13/2*x^50*c^12*b*a + 143/4*x^48*c^10*b^4 - 39*x^48*c^11*b
^2*a + 13/4*x^48*c^12*a^2 + 143/2*x^46*c^9*b^5 - 143*x^46*c^10*b^3*a + 39*x
^46*c^11*b*a^2 + 429/4*x^44*c^8*b^6 - 715/2*x^44*c^9*b^4*a + 429/2*x^44*c^1
0*b^2*a^2 - 13*x^44*c^11*a^3 + 858/7*x^42*c^7*b^7 - 1287/2*x^42*c^8*b^5*a +
 715*x^42*c^9*b^3*a^2 - 143*x^42*c^10*b*a^3 + 429/4*x^40*c^6*b^8 - 858*x^40
*c^7*b^6*a + 6435/4*x^40*c^8*b^4*a^2 - 715*x^40*c^9*b^2*a^3 + 143/4*x^40*c^
10*a^4 + 143/2*x^38*c^5*b^9 - 858*x^38*c^6*b^7*a + 2574*x^38*c^7*b^5*a^2 -
2145*x^38*c^8*b^3*a^3 + 715/2*x^38*c^9*b*a^4 + 143/4*x^36*c^4*b^10 - 1287/2
```

```
*x^36*c^5*b^8*a + 3003*x^36*c^6*b^6*a^2 - 4290*x^36*c^7*b^4*a^3 + 6435/4*x^
36*c^8*b^2*a^4 - 143/2*x^36*c^9*a^5 + 13*x^34*c^3*b^11 - 715/2*x^34*c^4*b^9
*a + 2574*x^34*c^5*b^7*a^2 - 6006*x^34*c^6*b^5*a^3 + 4290*x^34*c^7*b^3*a^4
- 1287/2*x^34*c^8*b*a^5 + 13/4*x^32*c^2*b^12 - 143*x^32*c^3*b^10*a + 6435/4
*x^32*c^4*b^8*a^2 - 6006*x^32*c^5*b^6*a^3 + 15015/2*x^32*c^6*b^4*a^4 - 2574
*x^32*c^7*b^2*a^5 + 429/4*x^32*c^8*a^6 + 1/2*x^30*c*b^13 - 39*x^30*c^2*b^11
*a + 715*x^30*c^3*b^9*a^2 - 4290*x^30*c^4*b^7*a^3 + 9009*x^30*c^5*b^5*a^4 -
 6006*x^30*c^6*b^3*a^5 + 858*x^30*c^7*b*a^6 + 1/28*x^28*b^14 - 13/2*x^28*c*
b^12*a + 429/2*x^28*c^2*b^10*a^2 - 2145*x^28*c^3*b^8*a^3 + 15015/2*x^28*c^4
*b^6*a^4 - 9009*x^28*c^5*b^4*a^5 + 3003*x^28*c^6*b^2*a^6 - 858/7*x^28*c^7*a
^7 - 1/2*x^26*b^13*a + 39*x^26*c*b^11*a^2 - 715*x^26*c^2*b^9*a^3 + 4290*x^2
6*c^3*b^7*a^4 - 9009*x^26*c^4*b^5*a^5 + 6006*x^26*c^5*b^3*a^6 - 858*x^26*c^
6*b*a^7 + 13/4*x^24*b^12*a^2 - 143*x^24*c*b^10*a^3 + 6435/4*x^24*c^2*b^8*a^
4 - 6006*x^24*c^3*b^6*a^5 + 15015/2*x^24*c^4*b^4*a^6 - 2574*x^24*c^5*b^2*a^
7 + 429/4*x^24*c^6*a^8 - 13*x^22*b^11*a^3 + 715/2*x^22*c*b^9*a^4 - 2574*x^2
2*c^2*b^7*a^5 + 6006*x^22*c^3*b^5*a^6 - 4290*x^22*c^4*b^3*a^7 + 1287/2*x^22
*c^5*b*a^8 + 143/4*x^20*b^10*a^4 - 1287/2*x^20*c*b^8*a^5 + 3003*x^20*c^2*b^
6*a^6 - 4290*x^20*c^3*b^4*a^7 + 6435/4*x^20*c^4*b^2*a^8 - 143/2*x^20*c^5*a^
9 - 143/2*x^18*b^9*a^5 + 858*x^18*c*b^7*a^6 - 2574*x^18*c^2*b^5*a^7 + 2145*
x^18*c^3*b^3*a^8 - 715/2*x^18*c^4*b*a^9 + 429/4*x^16*b^8*a^6 - 858*x^16*c*b
^6*a^7 + 6435/4*x^16*c^2*b^4*a^8 - 715*x^16*c^3*b^2*a^9 + 143/4*x^16*c^4*a^
10 - 858/7*x^14*b^7*a^7 + 1287/2*x^14*c*b^5*a^8 - 715*x^14*c^2*b^3*a^9 + 14
3*x^14*c^3*b*a^10 + 429/4*x^12*b^6*a^8 - 715/2*x^12*c*b^4*a^9 + 429/2*x^12*
c^2*b^2*a^10 - 13*x^12*c^3*a^11 - 143/2*x^10*b^5*a^9 + 143*x^10*c*b^3*a^10
- 39*x^10*c^2*b*a^11 + 143/4*x^8*b^4*a^10 - 39*x^8*c*b^2*a^11 + 13/4*x^8*c^
2*a^12 - 13*x^6*b^3*a^11 + 13/2*x^6*c*b*a^12 + 13/4*x^4*b^2*a^12 - 1/2*x^4*
c*a^13 - 1/2*x^2*b*a^13
```

Sympy [B] time = 0.311278, size = 1384, normalized size = 69.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x**2+b)*(c*x**4+b*x**2-a)**13,x)
```

```
[Out] -a**13*b*x**2/2 + b*c**13*x**54/2 + c**14*x**56/28 + x**52*(-a*c**13/2 + 13
*b**2*c**12/4) + x**50*(-13*a*b*c**12/2 + 13*b**3*c**11) + x**48*(13*a**2*c
**12/4 - 39*a*b**2*c**11 + 143*b**4*c**10/4) + x**46*(39*a**2*b*c**11 - 143
*a*b**3*c**10 + 143*b**5*c**9/2) + x**44*(-13*a**3*c**11 + 429*a**2*b**2*c*
*10/2 - 715*a*b**4*c**9/2 + 429*b**6*c**8/4) + x**42*(-143*a**3*b*c**10 + 7
15*a**2*b**3*c**9 - 1287*a*b**5*c**8/2 + 858*b**7*c**7/7) + x**40*(143*a**4
```

```
*c**10/4 - 715*a**3*b**2*c**9 + 6435*a**2*b**4*c**8/4 - 858*a*b**6*c**7 + 4
29*b**8*c**6/4) + x**38*(715*a**4*b*c**9/2 - 2145*a**3*b**3*c**8 + 2574*a**
2*b**5*c**7 - 858*a*b**7*c**6 + 143*b**9*c**5/2) + x**36*(-143*a**5*c**9/2
+ 6435*a**4*b**2*c**8/4 - 4290*a**3*b**4*c**7 + 3003*a**2*b**6*c**6 - 1287*
a*b**8*c**5/2 + 143*b**10*c**4/4) + x**34*(-1287*a**5*b*c**8/2 + 4290*a**4*
b**3*c**7 - 6006*a**3*b**5*c**6 + 2574*a**2*b**7*c**5 - 715*a*b**9*c**4/2 +
 13*b**11*c**3) + x**32*(429*a**6*c**8/4 - 2574*a**5*b**2*c**7 + 15015*a**4
*b**4*c**6/2 - 6006*a**3*b**6*c**5 + 6435*a**2*b**8*c**4/4 - 143*a*b**10*c*
*3 + 13*b**12*c**2/4) + x**30*(858*a**6*b*c**7 - 6006*a**5*b**3*c**6 + 9009
*a**4*b**5*c**5 - 4290*a**3*b**7*c**4 + 715*a**2*b**9*c**3 - 39*a*b**11*c**
2 + b**13*c/2) + x**28*(-858*a**7*c**7/7 + 3003*a**6*b**2*c**6 - 9009*a**5*
b**4*c**5 + 15015*a**4*b**6*c**4/2 - 2145*a**3*b**8*c**3 + 429*a**2*b**10*c
**2/2 - 13*a*b**12*c/2 + b**14/28) + x**26*(-858*a**7*b*c**6 + 6006*a**6*b*
*3*c**5 - 9009*a**5*b**5*c**4 + 4290*a**4*b**7*c**3 - 715*a**3*b**9*c**2 +
39*a**2*b**11*c - a*b**13/2) + x**24*(429*a**8*c**6/4 - 2574*a**7*b**2*c**5
 + 15015*a**6*b**4*c**4/2 - 6006*a**5*b**6*c**3 + 6435*a**4*b**8*c**2/4 - 1
43*a**3*b**10*c + 13*a**2*b**12/4) + x**22*(1287*a**8*b*c**5/2 - 4290*a**7*
b**3*c**4 + 6006*a**6*b**5*c**3 - 2574*a**5*b**7*c**2 + 715*a**4*b**9*c/2 -
 13*a**3*b**11) + x**20*(-143*a**9*c**5/2 + 6435*a**8*b**2*c**4/4 - 4290*a*
*7*b**4*c**3 + 3003*a**6*b**6*c**2 - 1287*a**5*b**8*c/2 + 143*a**4*b**10/4)
 + x**18*(-715*a**9*b*c**4/2 + 2145*a**8*b**3*c**3 - 2574*a**7*b**5*c**2 +
858*a**6*b**7*c - 143*a**5*b**9/2) + x**16*(143*a**10*c**4/4 - 715*a**9*b**
2*c**3 + 6435*a**8*b**4*c**2/4 - 858*a**7*b**6*c + 429*a**6*b**8/4) + x**14
*(143*a**10*b*c**3 - 715*a**9*b**3*c**2 + 1287*a**8*b**5*c/2 - 858*a**7*b**
7/7) + x**12*(-13*a**11*c**3 + 429*a**10*b**2*c**2/2 - 715*a**9*b**4*c/2 +
429*a**8*b**6/4) + x**10*(-39*a**11*b*c**2 + 143*a**10*b**3*c - 143*a**9*b*
*5/2) + x**8*(13*a**12*c**2/4 - 39*a**11*b**2*c + 143*a**10*b**4/4) + x**6*
(13*a**12*b*c/2 - 13*a**11*b**3) + x**4*(-a**13*c/2 + 13*a**12*b**2/4)
```

Giac [B] time = 1.14587, size = 1963, normalized size = 98.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^13,x, algorithm="giac")
```

```
[Out] 1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 13/4*b^2*c^12*x^52 - 1/2*a*c^13*x^52 + 1
3*b^3*c^11*x^50 - 13/2*a*b*c^12*x^50 + 143/4*b^4*c^10*x^48 - 39*a*b^2*c^11*
x^48 + 13/4*a^2*c^12*x^48 + 143/2*b^5*c^9*x^46 - 143*a*b^3*c^10*x^46 + 39*a
^2*b*c^11*x^46 + 429/4*b^6*c^8*x^44 - 715/2*a*b^4*c^9*x^44 + 429/2*a^2*b^2*
c^10*x^44 - 13*a^3*c^11*x^44 + 858/7*b^7*c^7*x^42 - 1287/2*a*b^5*c^8*x^42 +
```

```
 715*a^2*b^3*c^9*x^42 - 143*a^3*b*c^10*x^42 + 429/4*b^8*c^6*x^40 - 858*a*b^
6*c^7*x^40 + 6435/4*a^2*b^4*c^8*x^40 - 715*a^3*b^2*c^9*x^40 + 143/4*a^4*c^1
0*x^40 + 143/2*b^9*c^5*x^38 - 858*a*b^7*c^6*x^38 + 2574*a^2*b^5*c^7*x^38 -
2145*a^3*b^3*c^8*x^38 + 715/2*a^4*b*c^9*x^38 + 143/4*b^10*c^4*x^36 - 1287/2
*a*b^8*c^5*x^36 + 3003*a^2*b^6*c^6*x^36 - 4290*a^3*b^4*c^7*x^36 + 6435/4*a^
4*b^2*c^8*x^36 - 143/2*a^5*c^9*x^36 + 13*b^11*c^3*x^34 - 715/2*a*b^9*c^4*x^
34 + 2574*a^2*b^7*c^5*x^34 - 6006*a^3*b^5*c^6*x^34 + 4290*a^4*b^3*c^7*x^34
- 1287/2*a^5*b*c^8*x^34 + 13/4*b^12*c^2*x^32 - 143*a*b^10*c^3*x^32 + 6435/4
*a^2*b^8*c^4*x^32 - 6006*a^3*b^6*c^5*x^32 + 15015/2*a^4*b^4*c^6*x^32 - 2574
*a^5*b^2*c^7*x^32 + 429/4*a^6*c^8*x^32 + 1/2*b^13*c*x^30 - 39*a*b^11*c^2*x^
30 + 715*a^2*b^9*c^3*x^30 - 4290*a^3*b^7*c^4*x^30 + 9009*a^4*b^5*c^5*x^30 -
 6006*a^5*b^3*c^6*x^30 + 858*a^6*b*c^7*x^30 + 1/28*b^14*x^28 - 13/2*a*b^12*
c*x^28 + 429/2*a^2*b^10*c^2*x^28 - 2145*a^3*b^8*c^3*x^28 + 15015/2*a^4*b^6*
c^4*x^28 - 9009*a^5*b^4*c^5*x^28 + 3003*a^6*b^2*c^6*x^28 - 858/7*a^7*c^7*x^
28 - 1/2*a*b^13*x^26 + 39*a^2*b^11*c*x^26 - 715*a^3*b^9*c^2*x^26 + 4290*a^4
*b^7*c^3*x^26 - 9009*a^5*b^5*c^4*x^26 + 6006*a^6*b^3*c^5*x^26 - 858*a^7*b*c
^6*x^26 + 13/4*a^2*b^12*x^24 - 143*a^3*b^10*c*x^24 + 6435/4*a^4*b^8*c^2*x^2
4 - 6006*a^5*b^6*c^3*x^24 + 15015/2*a^6*b^4*c^4*x^24 - 2574*a^7*b^2*c^5*x^2
4 + 429/4*a^8*c^6*x^24 - 13*a^3*b^11*x^22 + 715/2*a^4*b^9*c*x^22 - 2574*a^5
*b^7*c^2*x^22 + 6006*a^6*b^5*c^3*x^22 - 4290*a^7*b^3*c^4*x^22 + 1287/2*a^8*
b*c^5*x^22 + 143/4*a^4*b^10*x^20 - 1287/2*a^5*b^8*c*x^20 + 3003*a^6*b^6*c^2
*x^20 - 4290*a^7*b^4*c^3*x^20 + 6435/4*a^8*b^2*c^4*x^20 - 143/2*a^9*c^5*x^2
0 - 143/2*a^5*b^9*x^18 + 858*a^6*b^7*c*x^18 - 2574*a^7*b^5*c^2*x^18 + 2145*
a^8*b^3*c^3*x^18 - 715/2*a^9*b*c^4*x^18 + 429/4*a^6*b^8*x^16 - 858*a^7*b^6*
c*x^16 + 6435/4*a^8*b^4*c^2*x^16 - 715*a^9*b^2*c^3*x^16 + 143/4*a^10*c^4*x^
16 - 858/7*a^7*b^7*x^14 + 1287/2*a^8*b^5*c*x^14 - 715*a^9*b^3*c^2*x^14 + 14
3*a^10*b*c^3*x^14 + 429/4*a^8*b^6*x^12 - 715/2*a^9*b^4*c*x^12 + 429/2*a^10*
b^2*c^2*x^12 - 13*a^11*c^3*x^12 - 143/2*a^9*b^5*x^10 + 143*a^10*b^3*c*x^10
- 39*a^11*b*c^2*x^10 + 143/4*a^10*b^4*x^8 - 39*a^11*b^2*c*x^8 + 13/4*a^12*c
^2*x^8 - 13*a^11*b^3*x^6 + 13/2*a^12*b*c*x^6 + 13/4*a^12*b^2*x^4 - 1/2*a^13
*c*x^4 - 1/2*a^13*b*x^2
```

3.99 $\int x^2 \left(b+2 c x^3\right) \left(-a+b x^3+c x^6\right)^{13} \, dx$

Optimal. Leaf size=20

$$\frac{1}{42} \left(a-b x^3-c x^6\right)^{14}$$

```
[Out] (a - b*x^3 - c*x^6)^14/42
```

Rubi [A] time = 0.310304, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1468, 629}

$$\frac{1}{42} \left(a-b x^3-c x^6\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(b + 2*c*x^3)*(-a + b*x^3 + c*x^6)^13,x]

[Out] (a - b*x^3 - c*x^6)^14/42
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned} \int x^2 \left(b+2 c x^3\right) \left(-a+b x^3+c x^6\right)^{13} \, dx &= \frac{1}{3} \text{Subst}\left(\int (b+2 c x) \left(-a+b x+c x^2\right)^{13} \, dx,x,x^3\right)\\ &= \frac{1}{42} \left(a-b x^3-c x^6\right)^{14} \end{aligned}$$

Mathematica [B] time = 0.164877, size = 233, normalized size = 11.65

$$\frac{1}{42}x^3\left(b+cx^3\right)\left(91a^2x^{33}\left(b+cx^3\right)^{11}-364a^3x^{30}\left(b+cx^3\right)^{10}+1001a^4x^{27}\left(b+cx^3\right)^9-2002a^5x^{24}\left(b+cx^3\right)^8+3003a^6x^{21}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(b + 2*c*x^3)*(-a + b*x^3 + c*x^6)^13,x]
```

```
[Out] (x^3*(b + c*x^3)*(-14*a^13 + 91*a^12*x^3*(b + c*x^3) - 364*a^11*x^6*(b + c*
x^3)^2 + 1001*a^10*x^9*(b + c*x^3)^3 - 2002*a^9*x^12*(b + c*x^3)^4 + 3003*a
^8*x^15*(b + c*x^3)^5 - 3432*a^7*x^18*(b + c*x^3)^6 + 3003*a^6*x^21*(b + c*
x^3)^7 - 2002*a^5*x^24*(b + c*x^3)^8 + 1001*a^4*x^27*(b + c*x^3)^9 - 364*a^
3*x^30*(b + c*x^3)^10 + 91*a^2*x^33*(b + c*x^3)^11 - 14*a*x^36*(b + c*x^3)^
12 + x^39*(b + c*x^3)^13))/42
```

Maple [B] time = 0.001, size = 47688, normalized size = 2384.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^13,x)
```

```
[Out] result too large to display
```

Maxima [B] time = 1.06222, size = 1677, normalized size = 83.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^13,x, algorithm="maxima")
```

```
[Out] 1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 1/6*(13*b^2*c^12 - 2*a*c^13)*x^78 + 13/3
*(2*b^3*c^11 - a*b*c^12)*x^75 + 13/6*(11*b^4*c^10 - 12*a*b^2*c^11 + a^2*c^1
2)*x^72 + 13/3*(11*b^5*c^9 - 22*a*b^3*c^10 + 6*a^2*b*c^11)*x^69 + 13/6*(33*
b^6*c^8 - 110*a*b^4*c^9 + 66*a^2*b^2*c^10 - 4*a^3*c^11)*x^66 + 143/21*(12*b
^7*c^7 - 63*a*b^5*c^8 + 70*a^2*b^3*c^9 - 14*a^3*b*c^10)*x^63 + 143/6*(3*b^8
```

```
*c^6 - 24*a*b^6*c^7 + 45*a^2*b^4*c^8 - 20*a^3*b^2*c^9 + a^4*c^10)*x^60 + 14
3/3*(b^9*c^5 - 12*a*b^7*c^6 + 36*a^2*b^5*c^7 - 30*a^3*b^3*c^8 + 5*a^4*b*c^9
)*x^57 + 143/6*(b^10*c^4 - 18*a*b^8*c^5 + 84*a^2*b^6*c^6 - 120*a^3*b^4*c^7
+ 45*a^4*b^2*c^8 - 2*a^5*c^9)*x^54 + 13/3*(2*b^11*c^3 - 55*a*b^9*c^4 + 396*
a^2*b^7*c^5 - 924*a^3*b^5*c^6 + 660*a^4*b^3*c^7 - 99*a^5*b*c^8)*x^51 + 13/6
*(b^12*c^2 - 44*a*b^10*c^3 + 495*a^2*b^8*c^4 - 1848*a^3*b^6*c^5 + 2310*a^4*
b^4*c^6 - 792*a^5*b^2*c^7 + 33*a^6*c^8)*x^48 + 1/3*(b^13*c - 78*a*b^11*c^2
+ 1430*a^2*b^9*c^3 - 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 - 12012*a^5*b^3*c
^6 + 1716*a^6*b*c^7)*x^45 + 1/42*(b^14 - 182*a*b^12*c + 6006*a^2*b^10*c^2 -
 60060*a^3*b^8*c^3 + 210210*a^4*b^6*c^4 - 252252*a^5*b^4*c^5 + 84084*a^6*b^
2*c^6 - 3432*a^7*c^7)*x^42 - 1/3*(a*b^13 - 78*a^2*b^11*c + 1430*a^3*b^9*c^2
 - 8580*a^4*b^7*c^3 + 18018*a^5*b^5*c^4 - 12012*a^6*b^3*c^5 + 1716*a^7*b*c^
6)*x^39 + 13/6*(a^2*b^12 - 44*a^3*b^10*c + 495*a^4*b^8*c^2 - 1848*a^5*b^6*c
^3 + 2310*a^6*b^4*c^4 - 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^36 - 13/3*(2*a^3*b^
11 - 55*a^4*b^9*c + 396*a^5*b^7*c^2 - 924*a^6*b^5*c^3 + 660*a^7*b^3*c^4 - 9
9*a^8*b*c^5)*x^33 + 143/6*(a^4*b^10 - 18*a^5*b^8*c + 84*a^6*b^6*c^2 - 120*a
^7*b^4*c^3 + 45*a^8*b^2*c^4 - 2*a^9*c^5)*x^30 - 143/3*(a^5*b^9 - 12*a^6*b^7
*c + 36*a^7*b^5*c^2 - 30*a^8*b^3*c^3 + 5*a^9*b*c^4)*x^27 + 143/6*(3*a^6*b^8
 - 24*a^7*b^6*c + 45*a^8*b^4*c^2 - 20*a^9*b^2*c^3 + a^10*c^4)*x^24 - 143/21
*(12*a^7*b^7 - 63*a^8*b^5*c + 70*a^9*b^3*c^2 - 14*a^10*b*c^3)*x^21 + 13/6*(
33*a^8*b^6 - 110*a^9*b^4*c + 66*a^10*b^2*c^2 - 4*a^11*c^3)*x^18 - 1/3*a^13*
b*x^3 - 13/3*(11*a^9*b^5 - 22*a^10*b^3*c + 6*a^11*b*c^2)*x^15 + 13/6*(11*a^
10*b^4 - 12*a^11*b^2*c + a^12*c^2)*x^12 - 13/3*(2*a^11*b^3 - a^12*b*c)*x^9
+ 1/6*(13*a^12*b^2 - 2*a^13*c)*x^6
```

Fricas [B] time = 0.967692, size = 3594, normalized size = 179.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^13,x, algorithm="fricas")
```

```
[Out] 1/42*x^84*c^14 + 1/3*x^81*c^13*b + 13/6*x^78*c^12*b^2 - 1/3*x^78*c^13*a + 2
6/3*x^75*c^11*b^3 - 13/3*x^75*c^12*b*a + 143/6*x^72*c^10*b^4 - 26*x^72*c^11
*b^2*a + 13/6*x^72*c^12*a^2 + 143/3*x^69*c^9*b^5 - 286/3*x^69*c^10*b^3*a +
26*x^69*c^11*b*a^2 + 143/2*x^66*c^8*b^6 - 715/3*x^66*c^9*b^4*a + 143*x^66*c
^10*b^2*a^2 - 26/3*x^66*c^11*a^3 + 572/7*x^63*c^7*b^7 - 429*x^63*c^8*b^5*a
+ 1430/3*x^63*c^9*b^3*a^2 - 286/3*x^63*c^10*b*a^3 + 143/2*x^60*c^6*b^8 - 57
2*x^60*c^7*b^6*a + 2145/2*x^60*c^8*b^4*a^2 - 1430/3*x^60*c^9*b^2*a^3 + 143/
6*x^60*c^10*a^4 + 143/3*x^57*c^5*b^9 - 572*x^57*c^6*b^7*a + 1716*x^57*c^7*b
^5*a^2 - 1430*x^57*c^8*b^3*a^3 + 715/3*x^57*c^9*b*a^4 + 143/6*x^54*c^4*b^10
```

```
 - 429*x^54*c^5*b^8*a + 2002*x^54*c^6*b^6*a^2 - 2860*x^54*c^7*b^4*a^3 + 214
5/2*x^54*c^8*b^2*a^4 - 143/3*x^54*c^9*a^5 + 26/3*x^51*c^3*b^11 - 715/3*x^51
*c^4*b^9*a + 1716*x^51*c^5*b^7*a^2 - 4004*x^51*c^6*b^5*a^3 + 2860*x^51*c^7*
b^3*a^4 - 429*x^51*c^8*b*a^5 + 13/6*x^48*c^2*b^12 - 286/3*x^48*c^3*b^10*a +
 2145/2*x^48*c^4*b^8*a^2 - 4004*x^48*c^5*b^6*a^3 + 5005*x^48*c^6*b^4*a^4 -
1716*x^48*c^7*b^2*a^5 + 143/2*x^48*c^8*a^6 + 1/3*x^45*c*b^13 - 26*x^45*c^2*
b^11*a + 1430/3*x^45*c^3*b^9*a^2 - 2860*x^45*c^4*b^7*a^3 + 6006*x^45*c^5*b^
5*a^4 - 4004*x^45*c^6*b^3*a^5 + 572*x^45*c^7*b*a^6 + 1/42*x^42*b^14 - 13/3*
x^42*c*b^12*a + 143*x^42*c^2*b^10*a^2 - 1430*x^42*c^3*b^8*a^3 + 5005*x^42*c
^4*b^6*a^4 - 6006*x^42*c^5*b^4*a^5 + 2002*x^42*c^6*b^2*a^6 - 572/7*x^42*c^7
*a^7 - 1/3*x^39*b^13*a + 26*x^39*c*b^11*a^2 - 1430/3*x^39*c^2*b^9*a^3 + 286
0*x^39*c^3*b^7*a^4 - 6006*x^39*c^4*b^5*a^5 + 4004*x^39*c^5*b^3*a^6 - 572*x^
39*c^6*b*a^7 + 13/6*x^36*b^12*a^2 - 286/3*x^36*c*b^10*a^3 + 2145/2*x^36*c^2
*b^8*a^4 - 4004*x^36*c^3*b^6*a^5 + 5005*x^36*c^4*b^4*a^6 - 1716*x^36*c^5*b^
2*a^7 + 143/2*x^36*c^6*a^8 - 26/3*x^33*b^11*a^3 + 715/3*x^33*c*b^9*a^4 - 17
16*x^33*c^2*b^7*a^5 + 4004*x^33*c^3*b^5*a^6 - 2860*x^33*c^4*b^3*a^7 + 429*x
^33*c^5*b*a^8 + 143/6*x^30*b^10*a^4 - 429*x^30*c*b^8*a^5 + 2002*x^30*c^2*b^
6*a^6 - 2860*x^30*c^3*b^4*a^7 + 2145/2*x^30*c^4*b^2*a^8 - 143/3*x^30*c^5*a^
9 - 143/3*x^27*b^9*a^5 + 572*x^27*c*b^7*a^6 - 1716*x^27*c^2*b^5*a^7 + 1430*
x^27*c^3*b^3*a^8 - 715/3*x^27*c^4*b*a^9 + 143/2*x^24*b^8*a^6 - 572*x^24*c*b
^6*a^7 + 2145/2*x^24*c^2*b^4*a^8 - 1430/3*x^24*c^3*b^2*a^9 + 143/6*x^24*c^4
*a^10 - 572/7*x^21*b^7*a^7 + 429*x^21*c*b^5*a^8 - 1430/3*x^21*c^2*b^3*a^9 +
 286/3*x^21*c^3*b*a^10 + 143/2*x^18*b^6*a^8 - 715/3*x^18*c*b^4*a^9 + 143*x^
18*c^2*b^2*a^10 - 26/3*x^18*c^3*a^11 - 143/3*x^15*b^5*a^9 + 286/3*x^15*c*b^
3*a^10 - 26*x^15*c^2*b*a^11 + 143/6*x^12*b^4*a^10 - 26*x^12*c*b^2*a^11 + 13
/6*x^12*c^2*a^12 - 26/3*x^9*b^3*a^11 + 13/3*x^9*c*b*a^12 + 13/6*x^6*b^2*a^1
2 - 1/3*x^6*c*a^13 - 1/3*x^3*b*a^13
```

Sympy [B] time = 0.321249, size = 1394, normalized size = 69.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*c*x**3+b)*(c*x**6+b*x**3-a)**13,x)
```

```
[Out] -a**13*b*x**3/3 + b*c**13*x**81/3 + c**14*x**84/42 + x**78*(-a*c**13/3 + 13
*b**2*c**12/6) + x**75*(-13*a*b*c**12/3 + 26*b**3*c**11/3) + x**72*(13*a**2
*c**12/6 - 26*a*b**2*c**11 + 143*b**4*c**10/6) + x**69*(26*a**2*b*c**11 - 2
86*a*b**3*c**10/3 + 143*b**5*c**9/3) + x**66*(-26*a**3*c**11/3 + 143*a**2*b
**2*c**10 - 715*a*b**4*c**9/3 + 143*b**6*c**8/2) + x**63*(-286*a**3*b*c**10
/3 + 1430*a**2*b**3*c**9/3 - 429*a*b**5*c**8 + 572*b**7*c**7/7) + x**60*(14
```

```
3*a**4*c**10/6 - 1430*a**3*b**2*c**9/3 + 2145*a**2*b**4*c**8/2 - 572*a*b**6
*c**7 + 143*b**8*c**6/2) + x**57*(715*a**4*b*c**9/3 - 1430*a**3*b**3*c**8 +
 1716*a**2*b**5*c**7 - 572*a*b**7*c**6 + 143*b**9*c**5/3) + x**54*(-143*a**
5*c**9/3 + 2145*a**4*b**2*c**8/2 - 2860*a**3*b**4*c**7 + 2002*a**2*b**6*c**
6 - 429*a*b**8*c**5 + 143*b**10*c**4/6) + x**51*(-429*a**5*b*c**8 + 2860*a*
*4*b**3*c**7 - 4004*a**3*b**5*c**6 + 1716*a**2*b**7*c**5 - 715*a*b**9*c**4/
3 + 26*b**11*c**3/3) + x**48*(143*a**6*c**8/2 - 1716*a**5*b**2*c**7 + 5005*
a**4*b**4*c**6 - 4004*a**3*b**6*c**5 + 2145*a**2*b**8*c**4/2 - 286*a*b**10*
c**3/3 + 13*b**12*c**2/6) + x**45*(572*a**6*b*c**7 - 4004*a**5*b**3*c**6 +
6006*a**4*b**5*c**5 - 2860*a**3*b**7*c**4 + 1430*a**2*b**9*c**3/3 - 26*a*b*
*11*c**2 + b**13*c/3) + x**42*(-572*a**7*c**7/7 + 2002*a**6*b**2*c**6 - 600
6*a**5*b**4*c**5 + 5005*a**4*b**6*c**4 - 1430*a**3*b**8*c**3 + 143*a**2*b**
10*c**2 - 13*a*b**12*c/3 + b**14/42) + x**39*(-572*a**7*b*c**6 + 4004*a**6*
b**3*c**5 - 6006*a**5*b**5*c**4 + 2860*a**4*b**7*c**3 - 1430*a**3*b**9*c**2
/3 + 26*a**2*b**11*c - a*b**13/3) + x**36*(143*a**8*c**6/2 - 1716*a**7*b**2
*c**5 + 5005*a**6*b**4*c**4 - 4004*a**5*b**6*c**3 + 2145*a**4*b**8*c**2/2 -
 286*a**3*b**10*c/3 + 13*a**2*b**12/6) + x**33*(429*a**8*b*c**5 - 2860*a**7
*b**3*c**4 + 4004*a**6*b**5*c**3 - 1716*a**5*b**7*c**2 + 715*a**4*b**9*c/3
- 26*a**3*b**11/3) + x**30*(-143*a**9*c**5/3 + 2145*a**8*b**2*c**4/2 - 2860
*a**7*b**4*c**3 + 2002*a**6*b**6*c**2 - 429*a**5*b**8*c + 143*a**4*b**10/6)
 + x**27*(-715*a**9*b*c**4/3 + 1430*a**8*b**3*c**3 - 1716*a**7*b**5*c**2 +
572*a**6*b**7*c - 143*a**5*b**9/3) + x**24*(143*a**10*c**4/6 - 1430*a**9*b*
*2*c**3/3 + 2145*a**8*b**4*c**2/2 - 572*a**7*b**6*c + 143*a**6*b**8/2) + x*
*21*(286*a**10*b*c**3/3 - 1430*a**9*b**3*c**2/3 + 429*a**8*b**5*c - 572*a**
7*b**7/7) + x**18*(-26*a**11*c**3/3 + 143*a**10*b**2*c**2 - 715*a**9*b**4*c
/3 + 143*a**8*b**6/2) + x**15*(-26*a**11*b*c**2 + 286*a**10*b**3*c/3 - 143*
a**9*b**5/3) + x**12*(13*a**12*c**2/6 - 26*a**11*b**2*c + 143*a**10*b**4/6)
 + x**9*(13*a**12*b*c/3 - 26*a**11*b**3/3) + x**6*(-a**13*c/3 + 13*a**12*b*
*2/6)
```

Giac [B] time = 1.14009, size = 1963, normalized size = 98.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^13,x, algorithm="giac")
```

```
[Out] 1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 13/6*b^2*c^12*x^78 - 1/3*a*c^13*x^78 + 2
6/3*b^3*c^11*x^75 - 13/3*a*b*c^12*x^75 + 143/6*b^4*c^10*x^72 - 26*a*b^2*c^1
1*x^72 + 13/6*a^2*c^12*x^72 + 143/3*b^5*c^9*x^69 - 286/3*a*b^3*c^10*x^69 +
26*a^2*b*c^11*x^69 + 143/2*b^6*c^8*x^66 - 715/3*a*b^4*c^9*x^66 + 143*a^2*b^
```

2*c^10*x^66 - 26/3*a^3*c^11*x^66 + 572/7*b^7*c^7*x^63 - 429*a*b^5*c^8*x^63 + 1430/3*a^2*b^3*c^9*x^63 - 286/3*a^3*b*c^10*x^63 + 143/2*b^8*c^6*x^60 - 572*a*b^6*c^7*x^60 + 2145/2*a^2*b^4*c^8*x^60 - 1430/3*a^3*b^2*c^9*x^60 + 143/6*a^4*c^10*x^60 + 143/3*b^9*c^5*x^57 - 572*a*b^7*c^6*x^57 + 1716*a^2*b^5*c^7*x^57 - 1430*a^3*b^3*c^8*x^57 + 715/3*a^4*b*c^9*x^57 + 143/6*b^10*c^4*x^54 - 429*a*b^8*c^5*x^54 + 2002*a^2*b^6*c^6*x^54 - 2860*a^3*b^4*c^7*x^54 + 2145/2*a^4*b^2*c^8*x^54 - 143/3*a^5*c^9*x^54 + 26/3*b^11*c^3*x^51 - 715/3*a*b^9*c^4*x^51 + 1716*a^2*b^7*c^5*x^51 - 4004*a^3*b^5*c^6*x^51 + 2860*a^4*b^3*c^7*x^51 - 429*a^5*b*c^8*x^51 + 13/6*b^12*c^2*x^48 - 286/3*a*b^10*c^3*x^48 + 2145/2*a^2*b^8*c^4*x^48 - 4004*a^3*b^6*c^5*x^48 + 5005*a^4*b^4*c^6*x^48 - 1716*a^5*b^2*c^7*x^48 + 143/2*a^6*c^8*x^48 + 1/3*b^13*c*x^45 - 26*a*b^11*c^2*x^45 + 1430/3*a^2*b^9*c^3*x^45 - 2860*a^3*b^7*c^4*x^45 + 6006*a^4*b^5*c^5*x^45 - 4004*a^5*b^3*c^6*x^45 + 572*a^6*b*c^7*x^45 + 1/42*b^14*x^42 - 13/3*a*b^12*c*x^42 + 143*a^2*b^10*c^2*x^42 - 1430*a^3*b^8*c^3*x^42 + 5005*a^4*b^6*c^4*x^42 - 6006*a^5*b^4*c^5*x^42 + 2002*a^6*b^2*c^6*x^42 - 572/7*a^7*c^7*x^42 - 1/3*a*b^13*x^39 + 26*a^2*b^11*c*x^39 - 1430/3*a^3*b^9*c^2*x^39 + 2860*a^4*b^7*c^3*x^39 - 6006*a^5*b^5*c^4*x^39 + 4004*a^6*b^3*c^5*x^39 - 572*a^7*b*c^6*x^39 + 13/6*a^2*b^12*x^36 - 286/3*a^3*b^10*c*x^36 + 2145/2*a^4*b^8*c^2*x^36 - 4004*a^5*b^6*c^3*x^36 + 5005*a^6*b^4*c^4*x^36 - 1716*a^7*b^2*c^5*x^36 + 143/2*a^8*c^6*x^36 - 26/3*a^3*b^11*x^33 + 715/3*a^4*b^9*c*x^33 - 1716*a^5*b^7*c^2*x^33 + 4004*a^6*b^5*c^3*x^33 - 2860*a^7*b^3*c^4*x^33 + 429*a^8*b*c^5*x^33 + 143/6*a^4*b^10*x^30 - 429*a^5*b^8*c*x^30 + 2002*a^6*b^6*c^2*x^30 - 2860*a^7*b^4*c^3*x^30 + 2145/2*a^8*b^2*c^4*x^30 - 143/3*a^9*c^5*x^30 - 143/3*a^5*b^9*x^27 + 572*a^6*b^7*c*x^27 - 1716*a^7*b^5*c^2*x^27 + 1430*a^8*b^3*c^3*x^27 - 715/3*a^9*b*c^4*x^27 + 143/2*a^6*b^8*x^24 - 572*a^7*b^6*c*x^24 + 2145/2*a^8*b^4*c^2*x^24 - 1430/3*a^9*b^2*c^3*x^24 + 143/6*a^10*c^4*x^24 - 572/7*a^7*b^7*x^21 + 429*a^8*b^5*c*x^21 - 1430/3*a^9*b^3*c^2*x^21 + 286/3*a^10*b*c^3*x^21 + 143/2*a^8*b^6*x^18 - 715/3*a^9*b^4*c*x^18 + 143*a^10*b^2*c^2*x^18 - 26/3*a^11*c^3*x^18 - 143/3*a^9*b^5*x^15 + 286/3*a^10*b^3*c*x^15 - 26*a^11*b*c^2*x^15 + 143/6*a^10*b^4*x^12 - 26*a^11*b^2*c*x^12 + 13/6*a^12*c^2*x^12 - 26/3*a^11*b^3*x^9 + 13/3*a^12*b*c*x^9 + 13/6*a^12*b^2*x^6 - 1/3*a^13*c*x^6 - 1/3*a^13*b*x^3

3.100 $\int x^{-1+n}\left(b+2cx^n\right)\left(-a+bx^n+cx^{2n}\right)^{13}\,dx$

Optimal. Leaf size=25

$$\frac{\left(a-bx^n-cx^{2n}\right)^{14}}{14n}$$

```
[Out] (a - b*x^n - c*x^(2*n))^14/(14*n)
```

Rubi [A] time = 0.05999, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1468, 629}

$$\frac{\left(a-bx^n-cx^{2n}\right)^{14}}{14n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*(b + 2*c*x^n)*(-a + b*x^n + c*x^(2*n))^13,x]
```

```
[Out] (a - b*x^n - c*x^(2*n))^14/(14*n)
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int x^{-1+n}\left(b+2cx^n\right)\left(-a+bx^n+cx^{2n}\right)^{13}\,dx=\frac{\text{Subst}\left(\int(b+2cx)\left(-a+bx+cx^2\right)^{13}\,dx,x,x^n\right)}{n}$$
$$=\frac{\left(a-bx^n-cx^{2n}\right)^{14}}{14n}$$

Mathematica [A] time = 0.0555686, size = 24, normalized size = 0.96

$$\frac{\left(x^n\left(b+cx^n\right)-a\right)^{14}}{14n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*(b + 2*c*x^n)*(-a + b*x^n + c*x^(2*n))^13,x]
```

```
[Out] (-a + x^n*(b + c*x^n))^14/(14*n)
```

Maple [B] time = 0.06, size = 2046, normalized size = 81.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n)*(b+2*c*x^n)*(-a+b*x^n+c*x^(2*n))^13,x)
```

```
[Out] 1716/7*b^7*c^7/n*(x^n)^21+143*b^5*c^9/n*(x^n)^23+26*b^3*c^11/n*(x^n)^25-26*
a^11*b^3/n*(x^n)^3-143*b^5*a^9/n*(x^n)^5-1716/7*b^7*a^7/n*(x^n)^7-143*a^5*b
^9/n*(x^n)^9-26*b^11*a^3/n*(x^n)^11-a*b^13/n*(x^n)^13+b^13*c/n*(x^n)^15+26*
b^11*c^3/n*(x^n)^17+143*b^9*c^5/n*(x^n)^19-a^13/n*(x^n)^2*c+13/2*a^12/n*(x^
n)^2*b^2-1716/7/n*(x^n)^14*a^7*c^7-c^13/n*(x^n)^26*a+13/2*c^12/n*(x^n)^26*b
^2+143/2*c^10/n*(x^n)^24*b^4+429/2*c^6/n*(x^n)^20*b^8-26*c^11/n*(x^n)^22*a^
3+429/2*c^8/n*(x^n)^22*b^6+13/2*c^12/n*(x^n)^24*a^2-143*a^9/n*(x^n)^10*c^5+
143/2*a^4/n*(x^n)^10*b^10+429/2*a^8/n*(x^n)^12*c^6+13/2*a^2/n*(x^n)^12*b^12
-143*c^9/n*(x^n)^18*a^5+143/2*c^4/n*(x^n)^18*b^10+429/2*c^8/n*(x^n)^16*a^6+
13/2*c^2/n*(x^n)^16*b^12+143/2*c^10/n*(x^n)^20*a^4+13/2*a^12/n*(x^n)^4*c^2+
143/2*a^10/n*(x^n)^4*b^4+143/2*a^10/n*(x^n)^8*c^4+429/2*a^6/n*(x^n)^8*b^8-2
6*a^11/n*(x^n)^6*c^3+429/2*a^8/n*(x^n)^6*b^6+b*c^13/n*(x^n)^27-b*a^13/n*x^n
+1/14/n*(x^n)^14*b^14-8580*a^7/n*(x^n)^10*b^4*c^3+6006*a^6/n*(x^n)^10*b^6*c
^2-1287*a^5/n*(x^n)^10*b^8*c-5148*a^7/n*(x^n)^12*b^2*c^5+15015*a^6/n*(x^n)^
```

```
12*b^4*c^4-12012*a^5/n*(x^n)^12*b^6*c^3+6435/2*a^4/n*(x^n)^12*b^8*c^2+6435/
2*a^8/n*(x^n)^8*b^4*c^2-1716*a^7/n*(x^n)^8*b^6*c+429*a^10/n*(x^n)^6*b^2*c^2
-715*a^9/n*(x^n)^6*b^4*c+6435/2*a^8/n*(x^n)^10*b^2*c^4+78*b*c^11/n*(x^n)^23
*a^2-286*b^3*c^10/n*(x^n)^23*a-13*b*c^12/n*(x^n)^25*a+715*b*c^9/n*(x^n)^19*
a^4-4290*b^3*c^8/n*(x^n)^19*a^3+5148*b^5*c^7/n*(x^n)^19*a^2-1716*b^7*c^6/n*
(x^n)^19*a-286*b*c^10/n*(x^n)^21*a^3+1430*b^3*c^9/n*(x^n)^21*a^2-1287*b^5*c
^8/n*(x^n)^21*a-1287*b*c^8/n*(x^n)^17*a^5+8580*b^3*c^7/n*(x^n)^17*a^4-12012
*b^5*c^6/n*(x^n)^17*a^3+5148*b^7*c^5/n*(x^n)^17*a^2-715*b^9*c^4/n*(x^n)^17*
a+8580*a^4*b^7/n*(x^n)^13*c^3-1430*a^3*b^9/n*(x^n)^13*c^2+78*a^2*b^11/n*(x^
n)^13*c+1716*b*c^7/n*(x^n)^15*a^6-12012*b^3*c^6/n*(x^n)^15*a^5+18018*b^5*c^
5/n*(x^n)^15*a^4-8580*b^7*c^4/n*(x^n)^15*a^3+1430*b^9*c^3/n*(x^n)^15*a^2-78
*b^11*c^2/n*(x^n)^15*a-715*a^9*b/n*(x^n)^9*c^4+4290*a^8*b^3/n*(x^n)^9*c^3-5
148*a^7*b^5/n*(x^n)^9*c^2+1716*a^6*b^7/n*(x^n)^9*c+1287*b*a^8/n*(x^n)^11*c^
5-8580*b^3*a^7/n*(x^n)^11*c^4+12012*b^5*a^6/n*(x^n)^11*c^3-5148*b^7*a^5/n*(
x^n)^11*c^2+715*b^9*a^4/n*(x^n)^11*c-1716*a^7*b/n*(x^n)^13*c^6+12012*a^6*b^
3/n*(x^n)^13*c^5-18018*a^5*b^5/n*(x^n)^13*c^4-78*b*a^11/n*(x^n)^5*c^2+286*b
^3*a^10/n*(x^n)^5*c+286*b*a^10/n*(x^n)^7*c^3-1430*b^3*a^9/n*(x^n)^7*c^2+128
7*b^5*a^8/n*(x^n)^7*c+6006/n*(x^n)^14*a^6*b^2*c^6-18018/n*(x^n)^14*a^5*b^4*
c^5+15015/n*(x^n)^14*a^4*b^6*c^4-4290/n*(x^n)^14*a^3*b^8*c^3+429/n*(x^n)^14
*a^2*b^10*c^2-13/n*(x^n)^14*a*b^12*c+13*a^12*b/n*(x^n)^3*c-1430*c^9/n*(x^n)
^20*a^3*b^2+6435/2*c^8/n*(x^n)^20*a^2*b^4-1716*c^7/n*(x^n)^20*a*b^6+429*c^1
0/n*(x^n)^22*a^2*b^2-715*c^9/n*(x^n)^22*a*b^4-78*c^11/n*(x^n)^24*a*b^2-286*
a^3/n*(x^n)^12*b^10*c+6435/2*c^8/n*(x^n)^18*a^4*b^2-8580*c^7/n*(x^n)^18*a^3
*b^4+6006*c^6/n*(x^n)^18*a^2*b^6-1287*c^5/n*(x^n)^18*a*b^8-5148*c^7/n*(x^n)
^16*a^5*b^2+15015*c^6/n*(x^n)^16*a^4*b^4-12012*c^5/n*(x^n)^16*a^3*b^6+6435/
2*c^4/n*(x^n)^16*a^2*b^8-286*c^3/n*(x^n)^16*a*b^10+1/14*c^14/n*(x^n)^28-78*
a^11/n*(x^n)^4*b^2*c-1430*a^9/n*(x^n)^8*b^2*c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(-a+b*x^n+c*x^(2*n))^13,x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.35823, size = 2969, normalized size = 118.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(-a+b*x^n+c*x^(2*n))^13,x, algorithm="fricas
")
```

```
[Out] 1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) - 14*a^13*b*x^n + 7*(13*b^2*c^12 -
 2*a*c^13)*x^(26*n) + 182*(2*b^3*c^11 - a*b*c^12)*x^(25*n) + 91*(11*b^4*c^1
0 - 12*a*b^2*c^11 + a^2*c^12)*x^(24*n) + 182*(11*b^5*c^9 - 22*a*b^3*c^10 +
6*a^2*b*c^11)*x^(23*n) + 91*(33*b^6*c^8 - 110*a*b^4*c^9 + 66*a^2*b^2*c^10 -
 4*a^3*c^11)*x^(22*n) + 286*(12*b^7*c^7 - 63*a*b^5*c^8 + 70*a^2*b^3*c^9 - 1
4*a^3*b*c^10)*x^(21*n) + 1001*(3*b^8*c^6 - 24*a*b^6*c^7 + 45*a^2*b^4*c^8 -
20*a^3*b^2*c^9 + a^4*c^10)*x^(20*n) + 2002*(b^9*c^5 - 12*a*b^7*c^6 + 36*a^2
*b^5*c^7 - 30*a^3*b^3*c^8 + 5*a^4*b*c^9)*x^(19*n) + 1001*(b^10*c^4 - 18*a*b
^8*c^5 + 84*a^2*b^6*c^6 - 120*a^3*b^4*c^7 + 45*a^4*b^2*c^8 - 2*a^5*c^9)*x^(
18*n) + 182*(2*b^11*c^3 - 55*a*b^9*c^4 + 396*a^2*b^7*c^5 - 924*a^3*b^5*c^6
+ 660*a^4*b^3*c^7 - 99*a^5*b*c^8)*x^(17*n) + 91*(b^12*c^2 - 44*a*b^10*c^3 +
 495*a^2*b^8*c^4 - 1848*a^3*b^6*c^5 + 2310*a^4*b^4*c^6 - 792*a^5*b^2*c^7 +
33*a^6*c^8)*x^(16*n) + 14*(b^13*c - 78*a*b^11*c^2 + 1430*a^2*b^9*c^3 - 8580
*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 - 12012*a^5*b^3*c^6 + 1716*a^6*b*c^7)*x^(1
5*n) + (b^14 - 182*a*b^12*c + 6006*a^2*b^10*c^2 - 60060*a^3*b^8*c^3 + 21021
0*a^4*b^6*c^4 - 252252*a^5*b^4*c^5 + 84084*a^6*b^2*c^6 - 3432*a^7*c^7)*x^(1
4*n) - 14*(a*b^13 - 78*a^2*b^11*c + 1430*a^3*b^9*c^2 - 8580*a^4*b^7*c^3 + 1
8018*a^5*b^5*c^4 - 12012*a^6*b^3*c^5 + 1716*a^7*b*c^6)*x^(13*n) + 91*(a^2*b
^12 - 44*a^3*b^10*c + 495*a^4*b^8*c^2 - 1848*a^5*b^6*c^3 + 2310*a^6*b^4*c^4
 - 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^(12*n) - 182*(2*a^3*b^11 - 55*a^4*b^9*c
+ 396*a^5*b^7*c^2 - 924*a^6*b^5*c^3 + 660*a^7*b^3*c^4 - 99*a^8*b*c^5)*x^(11
*n) + 1001*(a^4*b^10 - 18*a^5*b^8*c + 84*a^6*b^6*c^2 - 120*a^7*b^4*c^3 + 45
*a^8*b^2*c^4 - 2*a^9*c^5)*x^(10*n) - 2002*(a^5*b^9 - 12*a^6*b^7*c + 36*a^7*
b^5*c^2 - 30*a^8*b^3*c^3 + 5*a^9*b*c^4)*x^(9*n) + 1001*(3*a^6*b^8 - 24*a^7*
b^6*c + 45*a^8*b^4*c^2 - 20*a^9*b^2*c^3 + a^10*c^4)*x^(8*n) - 286*(12*a^7*b
^7 - 63*a^8*b^5*c + 70*a^9*b^3*c^2 - 14*a^10*b*c^3)*x^(7*n) + 91*(33*a^8*b^
6 - 110*a^9*b^4*c + 66*a^10*b^2*c^2 - 4*a^11*c^3)*x^(6*n) - 182*(11*a^9*b^5
 - 22*a^10*b^3*c + 6*a^11*b*c^2)*x^(5*n) + 91*(11*a^10*b^4 - 12*a^11*b^2*c
+ a^12*c^2)*x^(4*n) - 182*(2*a^11*b^3 - a^12*b*c)*x^(3*n) + 7*(13*a^12*b^2
- 2*a^13*c)*x^(2*n))/n
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)*(-a+b*x**n+c*x**(2*n))**13,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.2868, size = 2286, normalized size = 91.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(-a+b*x^n+c*x^(2*n))^13,x, algorithm="giac")
```

```
[Out] 1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 91*b^2*c^12*x^(26*n) - 14*a*c^13
*x^(26*n) + 364*b^3*c^11*x^(25*n) - 182*a*b*c^12*x^(25*n) + 1001*b^4*c^10*x
^(24*n) - 1092*a*b^2*c^11*x^(24*n) + 91*a^2*c^12*x^(24*n) + 2002*b^5*c^9*x^
(23*n) - 4004*a*b^3*c^10*x^(23*n) + 1092*a^2*b*c^11*x^(23*n) + 3003*b^6*c^8
*x^(22*n) - 10010*a*b^4*c^9*x^(22*n) + 6006*a^2*b^2*c^10*x^(22*n) - 364*a^3
*c^11*x^(22*n) + 3432*b^7*c^7*x^(21*n) - 18018*a*b^5*c^8*x^(21*n) + 20020*a
^2*b^3*c^9*x^(21*n) - 4004*a^3*b*c^10*x^(21*n) + 3003*b^8*c^6*x^(20*n) - 24
024*a*b^6*c^7*x^(20*n) + 45045*a^2*b^4*c^8*x^(20*n) - 20020*a^3*b^2*c^9*x^(
20*n) + 1001*a^4*c^10*x^(20*n) + 2002*b^9*c^5*x^(19*n) - 24024*a*b^7*c^6*x^
(19*n) + 72072*a^2*b^5*c^7*x^(19*n) - 60060*a^3*b^3*c^8*x^(19*n) + 10010*a^
4*b*c^9*x^(19*n) + 1001*b^10*c^4*x^(18*n) - 18018*a*b^8*c^5*x^(18*n) + 8408
4*a^2*b^6*c^6*x^(18*n) - 120120*a^3*b^4*c^7*x^(18*n) + 45045*a^4*b^2*c^8*x^
(18*n) - 2002*a^5*c^9*x^(18*n) + 364*b^11*c^3*x^(17*n) - 10010*a*b^9*c^4*x^
(17*n) + 72072*a^2*b^7*c^5*x^(17*n) - 168168*a^3*b^5*c^6*x^(17*n) + 120120*
a^4*b^3*c^7*x^(17*n) - 18018*a^5*b*c^8*x^(17*n) + 91*b^12*c^2*x^(16*n) - 40
04*a*b^10*c^3*x^(16*n) + 45045*a^2*b^8*c^4*x^(16*n) - 168168*a^3*b^6*c^5*x^
(16*n) + 210210*a^4*b^4*c^6*x^(16*n) - 72072*a^5*b^2*c^7*x^(16*n) + 3003*a^
6*c^8*x^(16*n) + 14*b^13*c*x^(15*n) - 1092*a*b^11*c^2*x^(15*n) + 20020*a^2*
b^9*c^3*x^(15*n) - 120120*a^3*b^7*c^4*x^(15*n) + 252252*a^4*b^5*c^5*x^(15*n
) - 168168*a^5*b^3*c^6*x^(15*n) + 24024*a^6*b*c^7*x^(15*n) + b^14*x^(14*n)
- 182*a*b^12*c*x^(14*n) + 6006*a^2*b^10*c^2*x^(14*n) - 60060*a^3*b^8*c^3*x^
(14*n) + 210210*a^4*b^6*c^4*x^(14*n) - 252252*a^5*b^4*c^5*x^(14*n) + 84084*
a^6*b^2*c^6*x^(14*n) - 3432*a^7*c^7*x^(14*n) - 14*a*b^13*x^(13*n) + 1092*a^
2*b^11*c*x^(13*n) - 20020*a^3*b^9*c^2*x^(13*n) + 120120*a^4*b^7*c^3*x^(13*n
) - 252252*a^5*b^5*c^4*x^(13*n) + 168168*a^6*b^3*c^5*x^(13*n) - 24024*a^7*b
*c^6*x^(13*n) + 91*a^2*b^12*x^(12*n) - 4004*a^3*b^10*c*x^(12*n) + 45045*a^4
*b^8*c^2*x^(12*n) - 168168*a^5*b^6*c^3*x^(12*n) + 210210*a^6*b^4*c^4*x^(12*
n) - 72072*a^7*b^2*c^5*x^(12*n) + 3003*a^8*c^6*x^(12*n) - 364*a^3*b^11*x^(1
```

```
1*n) + 10010*a^4*b^9*c*x^(11*n) - 72072*a^5*b^7*c^2*x^(11*n) + 168168*a^6*b
^5*c^3*x^(11*n) - 120120*a^7*b^3*c^4*x^(11*n) + 18018*a^8*b*c^5*x^(11*n) +
1001*a^4*b^10*x^(10*n) - 18018*a^5*b^8*c*x^(10*n) + 84084*a^6*b^6*c^2*x^(10
*n) - 120120*a^7*b^4*c^3*x^(10*n) + 45045*a^8*b^2*c^4*x^(10*n) - 2002*a^9*c
^5*x^(10*n) - 2002*a^5*b^9*x^(9*n) + 24024*a^6*b^7*c*x^(9*n) - 72072*a^7*b^
5*c^2*x^(9*n) + 60060*a^8*b^3*c^3*x^(9*n) - 10010*a^9*b*c^4*x^(9*n) + 3003*
a^6*b^8*x^(8*n) - 24024*a^7*b^6*c*x^(8*n) + 45045*a^8*b^4*c^2*x^(8*n) - 200
20*a^9*b^2*c^3*x^(8*n) + 1001*a^10*c^4*x^(8*n) - 3432*a^7*b^7*x^(7*n) + 180
18*a^8*b^5*c*x^(7*n) - 20020*a^9*b^3*c^2*x^(7*n) + 4004*a^10*b*c^3*x^(7*n)
+ 3003*a^8*b^6*x^(6*n) - 10010*a^9*b^4*c*x^(6*n) + 6006*a^10*b^2*c^2*x^(6*n
) - 364*a^11*c^3*x^(6*n) - 2002*a^9*b^5*x^(5*n) + 4004*a^10*b^3*c*x^(5*n) -
 1092*a^11*b*c^2*x^(5*n) + 1001*a^10*b^4*x^(4*n) - 1092*a^11*b^2*c*x^(4*n)
+ 91*a^12*c^2*x^(4*n) - 364*a^11*b^3*x^(3*n) + 182*a^12*b*c*x^(3*n) + 91*a^
12*b^2*x^(2*n) - 14*a^13*c*x^(2*n) - 14*a^13*b*x^n)/n
```

3.101 $\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx$

Optimal. Leaf size=15

$$\frac{1}{14}\left(bx+cx^2\right)^{14}$$

```
[Out] (b*x + c*x^2)^14/14
```

Rubi [A] time = 0.0138468, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {629}

$$\frac{1}{14}\left(bx+cx^2\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)*(b*x + c*x^2)^13,x]
```

```
[Out] (b*x + c*x^2)^14/14
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx=\frac{1}{14}\left(bx+cx^2\right)^{14}$$

Mathematica [B] time = 0.0052051, size = 172, normalized size = 11.47

$$\frac{13}{2}b^2c^{12}x^{26}+26b^3c^{11}x^{25}+\frac{143}{2}b^4c^{10}x^{24}+143b^5c^9x^{23}+\frac{429}{2}b^6c^8x^{22}+\frac{1716}{7}b^7c^7x^{21}+\frac{429}{2}b^8c^6x^{20}+143b^9c^5x^{19}+\frac{143}{2}b$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)*(b*x + c*x^2)^13,x]
```

```
[Out] (b^14*x^14)/14 + b^13*c*x^15 + (13*b^12*c^2*x^16)/2 + 26*b^11*c^3*x^17 + (1
43*b^10*c^4*x^18)/2 + 143*b^9*c^5*x^19 + (429*b^8*c^6*x^20)/2 + (1716*b^7*c
^7*x^21)/7 + (429*b^6*c^8*x^22)/2 + 143*b^5*c^9*x^23 + (143*b^4*c^10*x^24)/
2 + 26*b^3*c^11*x^25 + (13*b^2*c^12*x^26)/2 + b*c^13*x^27 + (c^14*x^28)/14
```

Maple [B] time = 0.003, size = 155, normalized size = 10.3

$$\frac{c^{14}x^{28}}{14}+bc^{13}x^{27}+\frac{13\,b^{2}c^{12}x^{26}}{2}+26\,b^{3}c^{11}x^{25}+\frac{143\,b^{4}c^{10}x^{24}}{2}+143\,b^{5}c^{9}x^{23}+\frac{429\,b^{6}c^{8}x^{22}}{2}+\frac{1716\,b^{7}c^{7}x^{21}}{7}+\frac{429\,b^{8}c^{6}x^{20}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)*(c*x^2+b*x)^13,x)
```

```
[Out] 1/14*c^14*x^28+b*c^13*x^27+13/2*b^2*c^12*x^26+26*b^3*c^11*x^25+143/2*b^4*c^
10*x^24+143*b^5*c^9*x^23+429/2*b^6*c^8*x^22+1716/7*b^7*c^7*x^21+429/2*b^8*c
^6*x^20+143*b^9*c^5*x^19+143/2*b^10*c^4*x^18+26*b^11*c^3*x^17+13/2*b^12*c^2
*x^16+b^13*c*x^15+1/14*b^14*x^14
```

Maxima [A] time = 1.00223, size = 18, normalized size = 1.2

$$\frac{1}{14}\left(cx^{2}+bx\right)^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^13,x, algorithm="maxima")
```

```
[Out] 1/14*(c*x^2 + b*x)^14
```

Fricas [B] time = 0.936906, size = 387, normalized size = 25.8

$$\frac{1}{14}x^{28}c^{14}+x^{27}c^{13}b+\frac{13}{2}x^{26}c^{12}b^{2}+26x^{25}c^{11}b^{3}+\frac{143}{2}x^{24}c^{10}b^{4}+143x^{23}c^{9}b^{5}+\frac{429}{2}x^{22}c^{8}b^{6}+\frac{1716}{7}x^{21}c^{7}b^{7}+\frac{429}{2}x^{20}c^{6}b^{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^13,x, algorithm="fricas")
```

```
[Out] 1/14*x^28*c^14 + x^27*c^13*b + 13/2*x^26*c^12*b^2 + 26*x^25*c^11*b^3 + 143/
2*x^24*c^10*b^4 + 143*x^23*c^9*b^5 + 429/2*x^22*c^8*b^6 + 1716/7*x^21*c^7*b
^7 + 429/2*x^20*c^6*b^8 + 143*x^19*c^5*b^9 + 143/2*x^18*c^4*b^10 + 26*x^17*
c^3*b^11 + 13/2*x^16*c^2*b^12 + x^15*c*b^13 + 1/14*x^14*b^14
```

Sympy [B] time = 0.117751, size = 175, normalized size = 11.67

$$\frac{b^{14}x^{14}}{14} + b^{13}cx^{15} + \frac{13b^{12}c^2x^{16}}{2} + 26b^{11}c^3x^{17} + \frac{143b^{10}c^4x^{18}}{2} + 143b^9c^5x^{19} + \frac{429b^8c^6x^{20}}{2} + \frac{1716b^7c^7x^{21}}{7} + \frac{429b^6c^8x^{22}}{2} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x**2+b*x)**13,x)
```

```
[Out] b**14*x**14/14 + b**13*c*x**15 + 13*b**12*c**2*x**16/2 + 26*b**11*c**3*x**1
7 + 143*b**10*c**4*x**18/2 + 143*b**9*c**5*x**19 + 429*b**8*c**6*x**20/2 +
1716*b**7*c**7*x**21/7 + 429*b**6*c**8*x**22/2 + 143*b**5*c**9*x**23 + 143*
b**4*c**10*x**24/2 + 26*b**3*c**11*x**25 + 13*b**2*c**12*x**26/2 + b*c**13*
x**27 + c**14*x**28/14
```

Giac [B] time = 1.08786, size = 208, normalized size = 13.87

$$\frac{1}{14}c^{14}x^{28} + bc^{13}x^{27} + \frac{13}{2}b^2c^{12}x^{26} + 26\,b^3c^{11}x^{25} + \frac{143}{2}b^4c^{10}x^{24} + 143\,b^5c^9x^{23} + \frac{429}{2}b^6c^8x^{22} + \frac{1716}{7}b^7c^7x^{21} + \frac{429}{2}b^8c^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^13,x, algorithm="giac")
```

```
[Out] 1/14*c^14*x^28 + b*c^13*x^27 + 13/2*b^2*c^12*x^26 + 26*b^3*c^11*x^25 + 143/
2*b^4*c^10*x^24 + 143*b^5*c^9*x^23 + 429/2*b^6*c^8*x^22 + 1716/7*b^7*c^7*x^
21 + 429/2*b^8*c^6*x^20 + 143*b^9*c^5*x^19 + 143/2*b^10*c^4*x^18 + 26*b^11*
c^3*x^17 + 13/2*b^12*c^2*x^16 + b^13*c*x^15 + 1/14*b^14*x^14
```

3.102 $\int x\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{13}\,dx$

Optimal. Leaf size=16

$$\frac{1}{28}x^{28}\left(b+cx^2\right)^{14}$$

```
[Out] (x^28*(b + c*x^2)^14)/28
```

Rubi [A] time = 0.0541807, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {1584, 446, 74}

$$\frac{1}{28}x^{28}\left(b+cx^2\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[x*(b + 2*c*x^2)*(b*x^2 + c*x^4)^13,x]
```

```
[Out] (x^28*(b + c*x^2)^14)/28
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\int x\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{13}\,dx=\int x^{27}\left(b+cx^2\right)^{13}\left(b+2cx^2\right)\,dx$$
$$=\frac{1}{2}\,\text{Subst}\left(\int x^{13}(b+cx)^{13}(b+2cx)\,dx,x,x^2\right)$$
$$=\frac{1}{28}x^{28}\left(b+cx^2\right)^{14}$$

Mathematica [B] time = 0.0058507, size = 182, normalized size = 11.38

$$\frac{13}{4}b^2c^{12}x^{52}+13b^3c^{11}x^{50}+\frac{143}{4}b^4c^{10}x^{48}+\frac{143}{2}b^5c^9x^{46}+\frac{429}{4}b^6c^8x^{44}+\frac{858}{7}b^7c^7x^{42}+\frac{429}{4}b^8c^6x^{40}+\frac{143}{2}b^9c^5x^{38}+\frac{143}{4}b$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(b + 2*c*x^2)*(b*x^2 + c*x^4)^13,x]
```

```
[Out] (b^14*x^28)/28 + (b^13*c*x^30)/2 + (13*b^12*c^2*x^32)/4 + 13*b^11*c^3*x^34
+ (143*b^10*c^4*x^36)/4 + (143*b^9*c^5*x^38)/2 + (429*b^8*c^6*x^40)/4 + (85
8*b^7*c^7*x^42)/7 + (429*b^6*c^8*x^44)/4 + (143*b^5*c^9*x^46)/2 + (143*b^4*
c^10*x^48)/4 + 13*b^3*c^11*x^50 + (13*b^2*c^12*x^52)/4 + (b*c^13*x^54)/2 +
(c^14*x^56)/28
```

Maple [B] time = 0.004, size = 157, normalized size = 9.8

$$\frac{c^{14}x^{56}}{28}+\frac{bc^{13}x^{54}}{2}+\frac{13\,b^2c^{12}x^{52}}{4}+13\,b^3c^{11}x^{50}+\frac{143\,b^4c^{10}x^{48}}{4}+\frac{143\,b^5c^9x^{46}}{2}+\frac{429\,b^6c^8x^{44}}{4}+\frac{858\,b^7c^7x^{42}}{7}+\frac{429\,b^8c^6x^{40}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c*x^2+b)*(c*x^4+b*x^2)^13,x)
```

```
[Out] 1/28*c^14*x^56+1/2*b*c^13*x^54+13/4*b^2*c^12*x^52+13*b^3*c^11*x^50+143/4*b^
4*c^10*x^48+143/2*b^5*c^9*x^46+429/4*b^6*c^8*x^44+858/7*b^7*c^7*x^42+429/4*
b^8*c^6*x^40+143/2*b^9*c^5*x^38+143/4*b^10*c^4*x^36+13*b^11*c^3*x^34+13/4*b
^12*c^2*x^32+1/2*b^13*c*x^30+1/28*b^14*x^28
```

Maxima [B] time = 1.13387, size = 211, normalized size = 13.19

$$\frac{1}{28}\,c^{14}x^{56} + \frac{1}{2}\,bc^{13}x^{54} + \frac{13}{4}\,b^2c^{12}x^{52} + 13\,b^3c^{11}x^{50} + \frac{143}{4}\,b^4c^{10}x^{48} + \frac{143}{2}\,b^5c^9x^{46} + \frac{429}{4}\,b^6c^8x^{44} + \frac{858}{7}\,b^7c^7x^{42} + \frac{429}{4}\,b^8c^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2)^13,x, algorithm="maxima")
```

```
[Out] 1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 13/4*b^2*c^12*x^52 + 13*b^3*c^11*x^50 +
143/4*b^4*c^10*x^48 + 143/2*b^5*c^9*x^46 + 429/4*b^6*c^8*x^44 + 858/7*b^7*c
^7*x^42 + 429/4*b^8*c^6*x^40 + 143/2*b^9*c^5*x^38 + 143/4*b^10*c^4*x^36 + 1
3*b^11*c^3*x^34 + 13/4*b^12*c^2*x^32 + 1/2*b^13*c*x^30 + 1/28*b^14*x^28
```

Fricas [B] time = 0.897156, size = 402, normalized size = 25.12

$$\frac{1}{28}x^{56}c^{14} + \frac{1}{2}x^{54}c^{13}b + \frac{13}{4}x^{52}c^{12}b^2 + 13x^{50}c^{11}b^3 + \frac{143}{4}x^{48}c^{10}b^4 + \frac{143}{2}x^{46}c^9b^5 + \frac{429}{4}x^{44}c^8b^6 + \frac{858}{7}x^{42}c^7b^7 + \frac{429}{4}x^{40}c^6b^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2)^13,x, algorithm="fricas")
```

```
[Out] 1/28*x^56*c^14 + 1/2*x^54*c^13*b + 13/4*x^52*c^12*b^2 + 13*x^50*c^11*b^3 +
143/4*x^48*c^10*b^4 + 143/2*x^46*c^9*b^5 + 429/4*x^44*c^8*b^6 + 858/7*x^42*
c^7*b^7 + 429/4*x^40*c^6*b^8 + 143/2*x^38*c^5*b^9 + 143/4*x^36*c^4*b^10 + 1
3*x^34*c^3*b^11 + 13/4*x^32*c^2*b^12 + 1/2*x^30*c*b^13 + 1/28*x^28*b^14
```

Sympy [B] time = 0.156746, size = 182, normalized size = 11.38

$$\frac{b^{14}x^{28}}{28} + \frac{b^{13}cx^{30}}{2} + \frac{13b^{12}c^2x^{32}}{4} + 13b^{11}c^3x^{34} + \frac{143b^{10}c^4x^{36}}{4} + \frac{143b^9c^5x^{38}}{2} + \frac{429b^8c^6x^{40}}{4} + \frac{858b^7c^7x^{42}}{7} + \frac{429b^6c^8x^{44}}{4} + $$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x**2+b)*(c*x**4+b*x**2)**13,x)
```

```
[Out] b**14*x**28/28 + b**13*c*x**30/2 + 13*b**12*c**2*x**32/4 + 13*b**11*c**3*x*
*34 + 143*b**10*c**4*x**36/4 + 143*b**9*c**5*x**38/2 + 429*b**8*c**6*x**40/
4 + 858*b**7*c**7*x**42/7 + 429*b**6*c**8*x**44/4 + 143*b**5*c**9*x**46/2 +
```

```
 143*b**4*c**10*x**48/4 + 13*b**3*c**11*x**50 + 13*b**2*c**12*x**52/4 + b*c
**13*x**54/2 + c**14*x**56/28
```

Giac [B] time = 1.08891, size = 211, normalized size = 13.19

$$\frac{1}{28}c^{14}x^{56} + \frac{1}{2}bc^{13}x^{54} + \frac{13}{4}b^2c^{12}x^{52} + 13\,b^3c^{11}x^{50} + \frac{143}{4}b^4c^{10}x^{48} + \frac{143}{2}b^5c^9x^{46} + \frac{429}{4}b^6c^8x^{44} + \frac{858}{7}b^7c^7x^{42} + \frac{429}{4}b^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2)^13,x, algorithm="giac")
```

```
[Out] 1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 13/4*b^2*c^12*x^52 + 13*b^3*c^11*x^50 +
143/4*b^4*c^10*x^48 + 143/2*b^5*c^9*x^46 + 429/4*b^6*c^8*x^44 + 858/7*b^7*c
^7*x^42 + 429/4*b^8*c^6*x^40 + 143/2*b^9*c^5*x^38 + 143/4*b^10*c^4*x^36 + 1
3*b^11*c^3*x^34 + 13/4*b^12*c^2*x^32 + 1/2*b^13*c*x^30 + 1/28*b^14*x^28
```

3.103 $\int x^2\left(b+2cx^3\right)\left(bx^3+cx^6\right)^{13}\,dx$

Optimal. Leaf size=16

$$\frac{1}{42}x^{42}\left(b+cx^3\right)^{14}$$

```
[Out] (x^42*(b + c*x^3)^14)/42
```

Rubi [A] time = 0.0557805, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {1584, 446, 74}

$$\frac{1}{42}x^{42}\left(b+cx^3\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(b + 2*c*x^3)*(b*x^3 + c*x^6)^13,x]
```

```
[Out] (x^42*(b + c*x^3)^14)/42
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\int x^2 \left(b+2cx^3\right) \left(bx^3+cx^6\right)^{13} \, dx = \int x^{41} \left(b+cx^3\right)^{13} \left(b+2cx^3\right) \, dx$$
$$= \frac{1}{3} \text{Subst}\left(\int x^{13}(b+cx)^{13}(b+2cx) \, dx, x, x^3\right)$$
$$= \frac{1}{42} x^{42} \left(b+cx^3\right)^{14}$$

Mathematica [B] time = 0.006226, size = 186, normalized size = 11.62

$$\frac{13}{6}b^2c^{12}x^{78} + \frac{26}{3}b^3c^{11}x^{75} + \frac{143}{6}b^4c^{10}x^{72} + \frac{143}{3}b^5c^9x^{69} + \frac{143}{2}b^6c^8x^{66} + \frac{572}{7}b^7c^7x^{63} + \frac{143}{2}b^8c^6x^{60} + \frac{143}{3}b^9c^5x^{57} + \frac{143}{6}b$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(b + 2*c*x^3)*(b*x^3 + c*x^6)^13,x]
```

```
[Out] (b^14*x^42)/42 + (b^13*c*x^45)/3 + (13*b^12*c^2*x^48)/6 + (26*b^11*c^3*x^51
)/3 + (143*b^10*c^4*x^54)/6 + (143*b^9*c^5*x^57)/3 + (143*b^8*c^6*x^60)/2 +
 (572*b^7*c^7*x^63)/7 + (143*b^6*c^8*x^66)/2 + (143*b^5*c^9*x^69)/3 + (143*
b^4*c^10*x^72)/6 + (26*b^3*c^11*x^75)/3 + (13*b^2*c^12*x^78)/6 + (b*c^13*x^
81)/3 + (c^14*x^84)/42
```

Maple [B] time = 0.004, size = 157, normalized size = 9.8

$$\frac{c^{14}x^{84}}{42} + \frac{bc^{13}x^{81}}{3} + \frac{13\,b^2c^{12}x^{78}}{6} + \frac{26\,b^3c^{11}x^{75}}{3} + \frac{143\,b^4c^{10}x^{72}}{6} + \frac{143\,b^5c^9x^{69}}{3} + \frac{143\,b^6c^8x^{66}}{2} + \frac{572\,b^7c^7x^{63}}{7} + \frac{143\,b^8c^6x^6}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^13,x)
```

```
[Out] 1/42*c^14*x^84+1/3*b*c^13*x^81+13/6*b^2*c^12*x^78+26/3*b^3*c^11*x^75+143/6*
b^4*c^10*x^72+143/3*b^5*c^9*x^69+143/2*b^6*c^8*x^66+572/7*b^7*c^7*x^63+143/
2*b^8*c^6*x^60+143/3*b^9*c^5*x^57+143/6*b^10*c^4*x^54+26/3*b^11*c^3*x^51+13
/6*b^12*c^2*x^48+1/3*b^13*c*x^45+1/42*b^14*x^42
```

Maxima [B] time = 1.17024, size = 211, normalized size = 13.19

$$\frac{1}{42}\,c^{14}x^{84} + \frac{1}{3}\,bc^{13}x^{81} + \frac{13}{6}\,b^{2}c^{12}x^{78} + \frac{26}{3}\,b^{3}c^{11}x^{75} + \frac{143}{6}\,b^{4}c^{10}x^{72} + \frac{143}{3}\,b^{5}c^{9}x^{69} + \frac{143}{2}\,b^{6}c^{8}x^{66} + \frac{572}{7}\,b^{7}c^{7}x^{63} + \frac{143}{2}\,b^{8}c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^13,x, algorithm="maxima")
```

```
[Out] 1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 13/6*b^2*c^12*x^78 + 26/3*b^3*c^11*x^75
+ 143/6*b^4*c^10*x^72 + 143/3*b^5*c^9*x^69 + 143/2*b^6*c^8*x^66 + 572/7*b^7
*c^7*x^63 + 143/2*b^8*c^6*x^60 + 143/3*b^9*c^5*x^57 + 143/6*b^10*c^4*x^54 +
 26/3*b^11*c^3*x^51 + 13/6*b^12*c^2*x^48 + 1/3*b^13*c*x^45 + 1/42*b^14*x^42
```

Fricas [B] time = 0.818319, size = 408, normalized size = 25.5

$$\frac{1}{42}x^{84}c^{14} + \frac{1}{3}x^{81}c^{13}b + \frac{13}{6}x^{78}c^{12}b^{2} + \frac{26}{3}x^{75}c^{11}b^{3} + \frac{143}{6}x^{72}c^{10}b^{4} + \frac{143}{3}x^{69}c^{9}b^{5} + \frac{143}{2}x^{66}c^{8}b^{6} + \frac{572}{7}x^{63}c^{7}b^{7} + \frac{143}{2}x^{60}c^{6}b^{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^13,x, algorithm="fricas")
```

```
[Out] 1/42*x^84*c^14 + 1/3*x^81*c^13*b + 13/6*x^78*c^12*b^2 + 26/3*x^75*c^11*b^3
+ 143/6*x^72*c^10*b^4 + 143/3*x^69*c^9*b^5 + 143/2*x^66*c^8*b^6 + 572/7*x^6
3*c^7*b^7 + 143/2*x^60*c^6*b^8 + 143/3*x^57*c^5*b^9 + 143/6*x^54*c^4*b^10 +
 26/3*x^51*c^3*b^11 + 13/6*x^48*c^2*b^12 + 1/3*x^45*c*b^13 + 1/42*x^42*b^14
```

Sympy [B] time = 0.120845, size = 185, normalized size = 11.56

$$\frac{b^{14}x^{42}}{42} + \frac{b^{13}cx^{45}}{3} + \frac{13b^{12}c^{2}x^{48}}{6} + \frac{26b^{11}c^{3}x^{51}}{3} + \frac{143b^{10}c^{4}x^{54}}{6} + \frac{143b^{9}c^{5}x^{57}}{3} + \frac{143b^{8}c^{6}x^{60}}{2} + \frac{572b^{7}c^{7}x^{63}}{7} + \frac{143b^{6}c^{8}x^{66}}{2} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*c*x**3+b)*(c*x**6+b*x**3)**13,x)
```

```
[Out] b**14*x**42/42 + b**13*c*x**45/3 + 13*b**12*c**2*x**48/6 + 26*b**11*c**3*x*
*51/3 + 143*b**10*c**4*x**54/6 + 143*b**9*c**5*x**57/3 + 143*b**8*c**6*x**6
0/2 + 572*b**7*c**7*x**63/7 + 143*b**6*c**8*x**66/2 + 143*b**5*c**9*x**69/3
```

```
+ 143*b**4*c**10*x**72/6 + 26*b**3*c**11*x**75/3 + 13*b**2*c**12*x**78/6 +
b*c**13*x**81/3 + c**14*x**84/42
```

Giac [B] time = 1.0941, size = 211, normalized size = 13.19

$$\frac{1}{42}c^{14}x^{84} + \frac{1}{3}bc^{13}x^{81} + \frac{13}{6}b^2c^{12}x^{78} + \frac{26}{3}b^3c^{11}x^{75} + \frac{143}{6}b^4c^{10}x^{72} + \frac{143}{3}b^5c^9x^{69} + \frac{143}{2}b^6c^8x^{66} + \frac{572}{7}b^7c^7x^{63} + \frac{143}{2}b^8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^13,x, algorithm="giac")
```

```
[Out] 1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 13/6*b^2*c^12*x^78 + 26/3*b^3*c^11*x^75
+ 143/6*b^4*c^10*x^72 + 143/3*b^5*c^9*x^69 + 143/2*b^6*c^8*x^66 + 572/7*b^7
*c^7*x^63 + 143/2*b^8*c^6*x^60 + 143/3*b^9*c^5*x^57 + 143/6*b^10*c^4*x^54 +
 26/3*b^11*c^3*x^51 + 13/6*b^12*c^2*x^48 + 1/3*b^13*c*x^45 + 1/42*b^14*x^42
```

3.104 $\int x^{-1+n}(b+2cx^n)\left(bx^n+cx^{2n}\right)^{13}\,dx$

Optimal. Leaf size=21

$$\frac{x^{14n}(b+cx^n)^{14}}{14n}$$

[Out] (x^(14*n)*(b + c*x^n)^14)/(14*n)

Rubi [A] time = 0.0327483, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1584, 446, 74}

$$\frac{x^{14n}(b+cx^n)^{14}}{14n}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + n)*(b + 2*c*x^n)*(b*x^n + c*x^(2*n))^13,x]

[Out] (x^(14*n)*(b + c*x^n)^14)/(14*n)

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\int x^{-1+n}(b+2cx^n)\left(bx^n+cx^{2n}\right)^{13}dx=\int x^{-1+14n}(b+cx^n)^{13}(b+2cx^n)\,dx$$
$$=\frac{\text{Subst}\left(\int x^{13}(b+cx)^{13}(b+2cx)\,dx,x,x^n\right)}{n}$$
$$=\frac{x^{14n}(b+cx^n)^{14}}{14n}$$

Mathematica [A] time = 0.118071, size = 21, normalized size = 1.

$$\frac{x^{14n}(b+cx^n)^{14}}{14n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*(b + 2*c*x^n)*(b*x^n + c*x^(2*n))^13,x]
```

```
[Out] (x^(14*n)*(b + c*x^n)^14)/(14*n)
```

Maple [B] time = 0.033, size = 230, normalized size = 11.

$$\frac{c^{14}(x^n)^{28}}{14n}+\frac{bc^{13}(x^n)^{27}}{n}+\frac{13c^{12}(x^n)^{26}b^2}{2n}+26\frac{b^3c^{11}(x^n)^{25}}{n}+\frac{143c^{10}(x^n)^{24}b^4}{2n}+143\frac{b^5c^9(x^n)^{23}}{n}+\frac{429c^8(x^n)^{22}b^6}{2n}+\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^13,x)
```

```
[Out] 1/14*c^14/n*(x^n)^28+b*c^13/n*(x^n)^27+13/2*c^12/n*(x^n)^26*b^2+26*b^3*c^11
/n*(x^n)^25+143/2*c^10/n*(x^n)^24*b^4+143*b^5*c^9/n*(x^n)^23+429/2*c^8/n*(x
^n)^22*b^6+1716/7*b^7*c^7/n*(x^n)^21+429/2*c^6/n*(x^n)^20*b^8+143*b^9*c^5/n
*(x^n)^19+143/2*c^4/n*(x^n)^18*b^10+26*b^11*c^3/n*(x^n)^17+13/2*c^2/n*(x^n)
^16*b^12+b^13*c/n*(x^n)^15+1/14/n*(x^n)^14*b^14
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^13,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.10325, size = 467, normalized size = 22.24

$$\frac{c^{14}x^{28\,n} + 14\,bc^{13}x^{27\,n} + 91\,b^{2}c^{12}x^{26\,n} + 364\,b^{3}c^{11}x^{25\,n} + 1001\,b^{4}c^{10}x^{24\,n} + 2002\,b^{5}c^{9}x^{23\,n} + 3003\,b^{6}c^{8}x^{22\,n} + 3432\,b^{7}c^{7}x^{21\,n}}{14\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^13,x, algorithm="fricas")
```

```
[Out] 1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 91*b^2*c^12*x^(26*n) + 364*b^3*c
^11*x^(25*n) + 1001*b^4*c^10*x^(24*n) + 2002*b^5*c^9*x^(23*n) + 3003*b^6*c^
8*x^(22*n) + 3432*b^7*c^7*x^(21*n) + 3003*b^8*c^6*x^(20*n) + 2002*b^9*c^5*x
^(19*n) + 1001*b^10*c^4*x^(18*n) + 364*b^11*c^3*x^(17*n) + 91*b^12*c^2*x^(1
6*n) + 14*b^13*c*x^(15*n) + b^14*x^(14*n))/n
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)*(b*x**n+c*x**(2*n))**13,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.13235, size = 255, normalized size = 12.14

$$\frac{c^{14}x^{28\,n} + 14\,bc^{13}x^{27\,n} + 91\,b^{2}c^{12}x^{26\,n} + 364\,b^{3}c^{11}x^{25\,n} + 1001\,b^{4}c^{10}x^{24\,n} + 2002\,b^{5}c^{9}x^{23\,n} + 3003\,b^{6}c^{8}x^{22\,n} + 3432\,b^{7}c^{7}x^{21\,n}}{14\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^13,x, algorithm="giac")
```

```
[Out] 1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 91*b^2*c^12*x^(26*n) + 364*b^3*c
^11*x^(25*n) + 1001*b^4*c^10*x^(24*n) + 2002*b^5*c^9*x^(23*n) + 3003*b^6*c^
8*x^(22*n) + 3432*b^7*c^7*x^(21*n) + 3003*b^8*c^6*x^(20*n) + 2002*b^9*c^5*x
^(19*n) + 1001*b^10*c^4*x^(18*n) + 364*b^11*c^3*x^(17*n) + 91*b^12*c^2*x^(1
6*n) + 14*b^13*c*x^(15*n) + b^14*x^(14*n))/n
```

3.105 $\int \frac{b+2cx}{a+bx+cx^2} \, dx$

Optimal. Leaf size=11

$$\log \left(a+bx+cx^2\right)$$

[Out] Log[a + b*x + c*x^2]

Rubi [A] time = 0.0040946, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {628}

$$\log \left(a+bx+cx^2\right)$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x)/(a + b*x + c*x^2),x]

[Out] Log[a + b*x + c*x^2]

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{b+2cx}{a+bx+cx^2} \, dx = \log \left(a+bx+cx^2\right)$$

Mathematica [A] time = 0.0029445, size = 10, normalized size = 0.91

$$\log (a+x(b+cx))$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*c*x)/(a + b*x + c*x^2),x]

```
[Out] Log[a + x*(b + c*x)]
```

Maple [A] time = 0.001, size = 12, normalized size = 1.1

$$\ln\left(cx^2 + bx + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)/(c*x^2+b*x+a),x)
```

```
[Out] ln(c*x^2+b*x+a)
```

Maxima [A] time = 1.13787, size = 15, normalized size = 1.36

$$\log\left(cx^2 + bx + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] log(c*x^2 + b*x + a)
```

Fricas [A] time = 0.979356, size = 30, normalized size = 2.73

$$\log\left(cx^2 + bx + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] log(c*x^2 + b*x + a)
```

Sympy [A] time = 0.297273, size = 10, normalized size = 0.91

$$\log\left(a + bx + cx^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x**2+b*x+a),x)
```

```
[Out] log(a + b*x + c*x**2)
```

Giac [A] time = 1.09568, size = 15, normalized size = 1.36

$$\log\left(cx^2 + bx + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] log(c*x^2 + b*x + a)
```

3.106 $\int \frac{x\left(b+2cx^2\right)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=17

$$\frac{1}{2}\log\left(a+bx^2+cx^4\right)$$

```
[Out] Log[a + b*x^2 + c*x^4]/2
```

Rubi [A] time = 0.0186135, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1247, 628}

$$\frac{1}{2}\log\left(a+bx^2+cx^4\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x*(b + 2*c*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] Log[a + b*x^2 + c*x^4]/2
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(b+2cx^2\right)}{a+bx^2+cx^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)\\ &= \frac{1}{2}\log\left(a+bx^2+cx^4\right)\end{aligned}$$

Mathematica [A] time = 0.0060359, size = 17, normalized size = 1.

$$\frac{1}{2}\log\left(a+bx^2+cx^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(b + 2*c*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] Log[a + b*x^2 + c*x^4]/2
```

Maple [A] time = 0., size = 16, normalized size = 0.9

$$\frac{\ln\left(cx^4+bx^2+a\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c*x^2+b)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*ln(c*x^4+b*x^2+a)
```

Maxima [A] time = 1.13879, size = 20, normalized size = 1.18

$$\frac{1}{2}\log\left(cx^4+bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*log(c*x^4 + b*x^2 + a)
```

Fricas [A] time = 0.995402, size = 38, normalized size = 2.24

$$\frac{1}{2}\log\left(cx^4+bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*log(c*x^4 + b*x^2 + a)
```

Sympy [A] time = 0.417882, size = 14, normalized size = 0.82

$$\frac{\log\left(a + bx^2 + cx^4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x**2+b)/(c*x**4+b*x**2+a),x)
```

```
[Out] log(a + b*x**2 + c*x**4)/2
```

Giac [A] time = 1.13661, size = 20, normalized size = 1.18

$$\frac{1}{2}\log\left(cx^4 + bx^2 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*log(c*x^4 + b*x^2 + a)
```

3.107 $\int \frac{x^2(b+2cx^3)}{a+bx^3+cx^6} \, dx$

Optimal. Leaf size=17

$$\frac{1}{3}\log\left(a+bx^3+cx^6\right)$$

```
[Out] Log[a + b*x^3 + c*x^6]/3
```

Rubi [A] time = 0.0237446, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1468, 628}

$$\frac{1}{3}\log\left(a+bx^3+cx^6\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(b + 2*c*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] Log[a + b*x^3 + c*x^6]/3
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^2 \left(b+2cx^3\right)}{a+bx^3+cx^6}\,dx = \frac{1}{3}\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^3\right)$$
$$= \frac{1}{3}\log\left(a+bx^3+cx^6\right)$$

Mathematica [A] time = 0.0067678, size = 17, normalized size = 1.

$$\frac{1}{3}\log\left(a+bx^3+cx^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(b + 2*c*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] Log[a + b*x^3 + c*x^6]/3
```

Maple [A] time = 0.002, size = 16, normalized size = 0.9

$$\frac{\ln\left(cx^6+bx^3+a\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*c*x^3+b)/(c*x^6+b*x^3+a),x)
```

```
[Out] 1/3*ln(c*x^6+b*x^3+a)
```

Maxima [A] time = 1.02964, size = 20, normalized size = 1.18

$$\frac{1}{3}\,\log\left(cx^6+bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*log(c*x^6 + b*x^3 + a)
```

Fricas [A] time = 0.98849, size = 38, normalized size = 2.24

$$\frac{1}{3}\log\left(cx^6 + bx^3 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/3*log(c*x^6 + b*x^3 + a)
```

Sympy [A] time = 0.529788, size = 14, normalized size = 0.82

$$\frac{\log\left(a + bx^3 + cx^6\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*c*x**3+b)/(c*x**6+b*x**3+a),x)
```

```
[Out] log(a + b*x**3 + c*x**6)/3
```

Giac [A] time = 1.36809, size = 20, normalized size = 1.18

$$\frac{1}{3}\log\left(cx^6 + bx^3 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*log(c*x^6 + b*x^3 + a)
```

3.108 $\int \frac{x^{-1+n}(b+2cx^n)}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=19

$$\frac{\log\left(a+bx^n+cx^{2n}\right)}{n}$$

```
[Out] Log[a + b*x^n + c*x^(2*n)]/n
```

Rubi [A] time = 0.0269621, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1468, 628}

$$\frac{\log\left(a+bx^n+cx^{2n}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + n)*(b + 2*c*x^n))/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] Log[a + b*x^n + c*x^(2*n)]/n
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^{-1+n}(b+2cx^n)}{a+bx^n+cx^{2n}}\,dx = \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^n\right)}{n}$$
$$= \frac{\log\left(a+bx^n+cx^{2n}\right)}{n}$$

Mathematica [A] time = 0.109311, size = 19, normalized size = 1.

$$\frac{\log\left(a+bx^n+cx^{2n}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + n)*(b + 2*c*x^n))/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] Log[a + b*x^n + c*x^(2*n)]/n
```

Maple [A] time = 0.019, size = 24, normalized size = 1.3

$$\frac{\ln\left(a+be^{n\ln(x)}+c\left(e^{n\ln(x)}\right)^2\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n)*(b+2*c*x^n)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] 1/n*ln(a+b*exp(n*ln(x))+c*exp(n*ln(x))^2)
```

Maxima [A] time = 1.16091, size = 31, normalized size = 1.63

$$\frac{\log\left(\frac{cx^{2n}+bx^n+a}{c}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] log((c*x^(2*n) + b*x^n + a)/c)/n
```

Fricas [A] time = 1.10865, size = 41, normalized size = 2.16

$$\frac{\log\left(cx^{2\,n}+bx^{n}+a\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] log(c*x^(2*n) + b*x^n + a)/n
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.08196, size = 26, normalized size = 1.37

$$\frac{\log\left(cx^{2\,n}+bx^{n}+a\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] log(c*x^(2*n) + b*x^n + a)/n
```

3.109 $\int \frac{b+2cx}{(a+bx+cx^2)^8} dx$

Optimal. Leaf size=16

$$-\frac{1}{7(a+bx+cx^2)^7}$$

[Out] -1/(7*(a + b*x + c*x^2)^7)

Rubi [A] time = 0.0045067, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {629}

$$-\frac{1}{7(a+bx+cx^2)^7}$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x)/(a + b*x + c*x^2)^8,x]

[Out] -1/(7*(a + b*x + c*x^2)^7)

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{b+2cx}{(a+bx+cx^2)^8} dx = -\frac{1}{7(a+bx+cx^2)^7}$$

Mathematica [A] time = 0.0112869, size = 15, normalized size = 0.94

$$-\frac{1}{7(a+x(b+cx))^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)/(a + b*x + c*x^2)^8,x]
```

```
[Out] -1/(7*(a + x*(b + c*x))^7)
```

Maple [A] time = 0., size = 15, normalized size = 0.9

$$-\frac{1}{7\left(cx^2+bx+a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)/(c*x^2+b*x+a)^8,x)
```

```
[Out] -1/7/(c*x^2+b*x+a)^7
```

Maxima [A] time = 1.03121, size = 19, normalized size = 1.19

$$-\frac{1}{7\left(cx^2+bx+a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x+a)^8,x, algorithm="maxima")
```

```
[Out] -1/7/(c*x^2 + b*x + a)^7
```

Fricas [B] time = 1.23583, size = 737, normalized size = 46.06

$$-\frac{}{7\left(c^7x^{14}+7\,bc^6x^{13}+7\left(3\,b^2c^5+ac^6\right)x^{12}+7\left(5\,b^3c^4+6\,abc^5\right)x^{11}+7\left(5\,b^4c^3+15\,ab^2c^4+3\,a^2c^5\right)x^{10}+7\left(3\,b^5c^2+20\,a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x+a)^8,x, algorithm="fricas")
```

```
[Out] -1/7/(c^7*x^14 + 7*b*c^6*x^13 + 7*(3*b^2*c^5 + a*c^6)*x^12 + 7*(5*b^3*c^4 +
 6*a*b*c^5)*x^11 + 7*(5*b^4*c^3 + 15*a*b^2*c^4 + 3*a^2*c^5)*x^10 + 7*(3*b^5
*c^2 + 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^9 + 7*(b^6*c + 15*a*b^4*c^2 + 30*a^2*
b^2*c^3 + 5*a^3*c^4)*x^8 + 7*a^6*b*x + (b^7 + 42*a*b^5*c + 210*a^2*b^3*c^2
+ 140*a^3*b*c^3)*x^7 + a^7 + 7*(a*b^6 + 15*a^2*b^4*c + 30*a^3*b^2*c^2 + 5*a
^4*c^3)*x^6 + 7*(3*a^2*b^5 + 20*a^3*b^3*c + 15*a^4*b*c^2)*x^5 + 7*(5*a^3*b^
4 + 15*a^4*b^2*c + 3*a^5*c^2)*x^4 + 7*(5*a^4*b^3 + 6*a^5*b*c)*x^3 + 7*(3*a^
5*b^2 + a^6*c)*x^2)
```

Sympy [B] time = 51.4507, size = 359, normalized size = 22.44

$$-\frac{}{7a^7 + 49a^6bx + 49bc^6x^{13} + 7c^7x^{14} + x^{12}\left(49ac^6 + 147b^2c^5\right) + x^{11}\left(294abc^5 + 245b^3c^4\right) + x^{10}\left(147a^2c^5 + 735ab^2c^4 + 245\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x**2+b*x+a)**8,x)
```

```
[Out] -1/(7*a**7 + 49*a**6*b*x + 49*b*c**6*x**13 + 7*c**7*x**14 + x**12*(49*a*c**
6 + 147*b**2*c**5) + x**11*(294*a*b*c**5 + 245*b**3*c**4) + x**10*(147*a**2
*c**5 + 735*a*b**2*c**4 + 245*b**4*c**3) + x**9*(735*a**2*b*c**4 + 980*a*b*
*3*c**3 + 147*b**5*c**2) + x**8*(245*a**3*c**4 + 1470*a**2*b**2*c**3 + 735*
a*b**4*c**2 + 49*b**6*c) + x**7*(980*a**3*b*c**3 + 1470*a**2*b**3*c**2 + 29
4*a*b**5*c + 7*b**7) + x**6*(245*a**4*c**3 + 1470*a**3*b**2*c**2 + 735*a**2
*b**4*c + 49*a*b**6) + x**5*(735*a**4*b*c**2 + 980*a**3*b**3*c + 147*a**2*b
**5) + x**4*(147*a**5*c**2 + 735*a**4*b**2*c + 245*a**3*b**4) + x**3*(294*a
**5*b*c + 245*a**4*b**3) + x**2*(49*a**6*c + 147*a**5*b**2))
```

Giac [A] time = 1.14072, size = 19, normalized size = 1.19

$$-\frac{1}{7\left(cx^2 + bx + a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x+a)^8,x, algorithm="giac")
```

```
[Out] -1/7/(c*x^2 + b*x + a)^7
```

3.110 $\int \frac{x(b+2cx^2)}{(a+bx^2+cx^4)^8}\,dx$

Optimal. Leaf size=18

$$-\frac{1}{14\left(a+bx^2+cx^4\right)^7}$$

```
[Out] -1/(14*(a + b*x^2 + c*x^4)^7)
```

Rubi [A] time = 0.019708, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1247, 629}

$$-\frac{1}{14\left(a+bx^2+cx^4\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(b + 2*c*x^2))/(a + b*x^2 + c*x^4)^8,x]
```

```
[Out] -1/(14*(a + b*x^2 + c*x^4)^7)
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\left(b+2cx^2\right)}{\left(a+bx^2+cx^4\right)^8}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{b+2cx}{\left(a+bx+cx^2\right)^8}\,dx, x, x^2\right)$$
$$= -\frac{1}{14\left(a+bx^2+cx^4\right)^7}$$

Mathematica [A] time = 0.0135346, size = 18, normalized size = 1.

$$-\frac{1}{14\left(a+bx^2+cx^4\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(b + 2*c*x^2))/(a + b*x^2 + c*x^4)^8,x]
```

```
[Out] -1/(14*(a + b*x^2 + c*x^4)^7)
```

Maple [A] time = 0.002, size = 17, normalized size = 0.9

$$-\frac{1}{14\,\left(cx^4+bx^2+a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c*x^2+b)/(c*x^4+b*x^2+a)^8,x)
```

```
[Out] -1/14/(c*x^4+b*x^2+a)^7
```

Maxima [B] time = 1.67348, size = 475, normalized size = 26.39

$$-\frac{}{14\left(c^7x^{28}+7\,bc^6x^{26}+7\left(3\,b^2c^5+ac^6\right)x^{24}+7\left(5\,b^3c^4+6\,abc^5\right)x^{22}+7\left(5\,b^4c^3+15\,ab^2c^4+3\,a^2c^5\right)x^{20}+7\left(3\,b^5c^2+20\,a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2+a)^8,x, algorithm="maxima")
```

```
[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 7*(3*b^2*c^5 + a*c^6)*x^24 + 7*(5*b^3*c^4
+ 6*a*b*c^5)*x^22 + 7*(5*b^4*c^3 + 15*a*b^2*c^4 + 3*a^2*c^5)*x^20 + 7*(3*b^
5*c^2 + 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^18 + 7*(b^6*c + 15*a*b^4*c^2 + 30*a^
2*b^2*c^3 + 5*a^3*c^4)*x^16 + (b^7 + 42*a*b^5*c + 210*a^2*b^3*c^2 + 140*a^3
*b*c^3)*x^14 + 7*(a*b^6 + 15*a^2*b^4*c + 30*a^3*b^2*c^2 + 5*a^4*c^3)*x^12 +
 7*(3*a^2*b^5 + 20*a^3*b^3*c + 15*a^4*b*c^2)*x^10 + 7*a^6*b*x^2 + 7*(5*a^3*
b^4 + 15*a^4*b^2*c + 3*a^5*c^2)*x^8 + a^7 + 7*(5*a^4*b^3 + 6*a^5*b*c)*x^6 +
 7*(3*a^5*b^2 + a^6*c)*x^4)
```

Fricas [B] time = 1.24354, size = 748, normalized size = 41.56

$$-\frac{}{14\left(c^{7}x^{28}+7\,bc^{6}x^{26}+7\left(3\,b^{2}c^{5}+ac^{6}\right)x^{24}+7\left(5\,b^{3}c^{4}+6\,abc^{5}\right)x^{22}+7\left(5\,b^{4}c^{3}+15\,ab^{2}c^{4}+3\,a^{2}c^{5}\right)x^{20}+7\left(3\,b^{5}c^{2}+20\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2+a)^8,x, algorithm="fricas")
```

```
[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 7*(3*b^2*c^5 + a*c^6)*x^24 + 7*(5*b^3*c^4
+ 6*a*b*c^5)*x^22 + 7*(5*b^4*c^3 + 15*a*b^2*c^4 + 3*a^2*c^5)*x^20 + 7*(3*b^
5*c^2 + 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^18 + 7*(b^6*c + 15*a*b^4*c^2 + 30*a^
2*b^2*c^3 + 5*a^3*c^4)*x^16 + (b^7 + 42*a*b^5*c + 210*a^2*b^3*c^2 + 140*a^3
*b*c^3)*x^14 + 7*(a*b^6 + 15*a^2*b^4*c + 30*a^3*b^2*c^2 + 5*a^4*c^3)*x^12 +
 7*(3*a^2*b^5 + 20*a^3*b^3*c + 15*a^4*b*c^2)*x^10 + 7*a^6*b*x^2 + 7*(5*a^3*
b^4 + 15*a^4*b^2*c + 3*a^5*c^2)*x^8 + a^7 + 7*(5*a^4*b^3 + 6*a^5*b*c)*x^6 +
 7*(3*a^5*b^2 + a^6*c)*x^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x**2+b)/(c*x**4+b*x**2+a)**8,x)
```

```
[Out] Timed out
```

Giac [A] time = 50.3184, size = 22, normalized size = 1.22

$$-\frac{1}{14\left(cx^4+bx^2+a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2+a)^8,x, algorithm="giac")
```

```
[Out] -1/14/(c*x^4 + b*x^2 + a)^7
```

3.111 $\int \frac{x^2(b+2cx^3)}{(a+bx^3+cx^6)^8} \, dx$

Optimal. Leaf size=18

$$-\frac{1}{21\left(a+bx^3+cx^6\right)^7}$$

```
[Out] -1/(21*(a + b*x^3 + c*x^6)^7)
```

Rubi [A] time = 0.023087, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1468, 629}

$$-\frac{1}{21\left(a+bx^3+cx^6\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(b + 2*c*x^3))/(a + b*x^3 + c*x^6)^8,x]
```

```
[Out] -1/(21*(a + b*x^3 + c*x^6)^7)
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^2(b+2cx^3)}{(a+bx^3+cx^6)^8}\,dx = \frac{1}{3}\operatorname{Subst}\left(\int \frac{b+2cx}{(a+bx+cx^2)^8}\,dx, x, x^3\right)$$
$$= -\frac{1}{21(a+bx^3+cx^6)^7}$$

Mathematica [A] time = 0.0130597, size = 18, normalized size = 1.

$$-\frac{1}{21(a+bx^3+cx^6)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(b + 2*c*x^3))/(a + b*x^3 + c*x^6)^8,x]
```

```
[Out] -1/(21*(a + b*x^3 + c*x^6)^7)
```

Maple [A] time = 0.001, size = 17, normalized size = 0.9

$$-\frac{1}{21(cx^6+bx^3+a)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*c*x^3+b)/(c*x^6+b*x^3+a)^8,x)
```

```
[Out] -1/21/(c*x^6+b*x^3+a)^7
```

Maxima [B] time = 1.70515, size = 475, normalized size = 26.39

$$-\frac{}{21\left(c^7x^{42}+7\,bc^6x^{39}+7\left(3\,b^2c^5+ac^6\right)x^{36}+7\left(5\,b^3c^4+6\,abc^5\right)x^{33}+7\left(5\,b^4c^3+15\,ab^2c^4+3\,a^2c^5\right)x^{30}+7\left(3\,b^5c^2+20\,a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3+a)^8,x, algorithm="maxima")
```

```
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 7*(3*b^2*c^5 + a*c^6)*x^36 + 7*(5*b^3*c^4
+ 6*a*b*c^5)*x^33 + 7*(5*b^4*c^3 + 15*a*b^2*c^4 + 3*a^2*c^5)*x^30 + 7*(3*b^
5*c^2 + 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^27 + 7*(b^6*c + 15*a*b^4*c^2 + 30*a^
2*b^2*c^3 + 5*a^3*c^4)*x^24 + (b^7 + 42*a*b^5*c + 210*a^2*b^3*c^2 + 140*a^3
*b*c^3)*x^21 + 7*(a*b^6 + 15*a^2*b^4*c + 30*a^3*b^2*c^2 + 5*a^4*c^3)*x^18 +
 7*(3*a^2*b^5 + 20*a^3*b^3*c + 15*a^4*b*c^2)*x^15 + 7*(5*a^3*b^4 + 15*a^4*b
^2*c + 3*a^5*c^2)*x^12 + 7*a^6*b*x^3 + 7*(5*a^4*b^3 + 6*a^5*b*c)*x^9 + a^7
+ 7*(3*a^5*b^2 + a^6*c)*x^6)
```

Fricas [B] time = 1.26847, size = 749, normalized size = 41.61

$$-\frac{}{21\left(c^7x^{42}+7\,bc^6x^{39}+7\left(3\,b^2c^5+ac^6\right)x^{36}+7\left(5\,b^3c^4+6\,abc^5\right)x^{33}+7\left(5\,b^4c^3+15\,ab^2c^4+3\,a^2c^5\right)x^{30}+7\left(3\,b^5c^2+20\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3+a)^8,x, algorithm="fricas")
```

```
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 7*(3*b^2*c^5 + a*c^6)*x^36 + 7*(5*b^3*c^4
+ 6*a*b*c^5)*x^33 + 7*(5*b^4*c^3 + 15*a*b^2*c^4 + 3*a^2*c^5)*x^30 + 7*(3*b^
5*c^2 + 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^27 + 7*(b^6*c + 15*a*b^4*c^2 + 30*a^
2*b^2*c^3 + 5*a^3*c^4)*x^24 + (b^7 + 42*a*b^5*c + 210*a^2*b^3*c^2 + 140*a^3
*b*c^3)*x^21 + 7*(a*b^6 + 15*a^2*b^4*c + 30*a^3*b^2*c^2 + 5*a^4*c^3)*x^18 +
 7*(3*a^2*b^5 + 20*a^3*b^3*c + 15*a^4*b*c^2)*x^15 + 7*(5*a^3*b^4 + 15*a^4*b
^2*c + 3*a^5*c^2)*x^12 + 7*a^6*b*x^3 + 7*(5*a^4*b^3 + 6*a^5*b*c)*x^9 + a^7
+ 7*(3*a^5*b^2 + a^6*c)*x^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*c*x**3+b)/(c*x**6+b*x**3+a)**8,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3+a)^8,x, algorithm="giac")
```

```
[Out] Timed out
```

3.112 $\int \frac{x^{-1+n}(b+2cx^n)}{\left(a+bx^n+cx^{2n}\right)^8}\,dx$

Optimal. Leaf size=23

$$-\frac{1}{7n\left(a+bx^n+cx^{2n}\right)^7}$$

```
[Out] -1/(7*n*(a + b*x^n + c*x^(2*n))^7)
```

Rubi [A] time = 0.0266705, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1468, 629}

$$-\frac{1}{7n\left(a+bx^n+cx^{2n}\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + n)*(b + 2*c*x^n))/(a + b*x^n + c*x^(2*n))^8,x]
```

```
[Out] -1/(7*n*(a + b*x^n + c*x^(2*n))^7)
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^{-1+n}(b+2cx^n)}{\left(a+bx^n+cx^{2n}\right)^8}\,dx = \frac{\text{Subst}\left(\int \frac{b+2cx}{\left(a+bx+cx^2\right)^8}\,dx, x, x^n\right)}{n}$$
$$= -\frac{1}{7n\left(a+bx^n+cx^{2n}\right)^7}$$

Mathematica [A] time = 0.0621506, size = 22, normalized size = 0.96

$$-\frac{1}{7n\left(a+x^n\left(b+cx^n\right)\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + n)*(b + 2*c*x^n))/(a + b*x^n + c*x^(2*n))^8,x]
```

```
[Out] -1/(7*n*(a + x^n*(b + c*x^n))^7)
```

Maple [A] time = 0.059, size = 22, normalized size = 1.

$$-\frac{1}{7\,n\left(a+bx^n+c\left(x^n\right)^2\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n)*(b+2*c*x^n)/(a+b*x^n+c*x^(2*n))^8,x)
```

```
[Out] -1/7/n/(a+b*x^n+c*(x^n)^2)^7
```

Maxima [B] time = 3.20277, size = 562, normalized size = 24.43

$$-\frac{}{7\left(c^7nx^{14n}+7\,bc^6nx^{13n}+7\,a^6bnx^n+a^7n+7\left(3\,b^2c^5n+ac^6n\right)x^{12n}+7\left(5\,b^3c^4n+6\,abc^5n\right)x^{11n}+7\left(5\,b^4c^3n+15\,ab^2c^4\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(a+b*x^n+c*x^(2*n))^8,x, algorithm="maxima")
```

```
[Out] -1/7/(c^7*n*x^(14*n) + 7*b*c^6*n*x^(13*n) + 7*a^6*b*n*x^n + a^7*n + 7*(3*b^
2*c^5*n + a*c^6*n)*x^(12*n) + 7*(5*b^3*c^4*n + 6*a*b*c^5*n)*x^(11*n) + 7*(5
*b^4*c^3*n + 15*a*b^2*c^4*n + 3*a^2*c^5*n)*x^(10*n) + 7*(3*b^5*c^2*n + 20*a
*b^3*c^3*n + 15*a^2*b*c^4*n)*x^(9*n) + 7*(b^6*c*n + 15*a*b^4*c^2*n + 30*a^2
*b^2*c^3*n + 5*a^3*c^4*n)*x^(8*n) + (b^7*n + 42*a*b^5*c*n + 210*a^2*b^3*c^2
*n + 140*a^3*b*c^3*n)*x^(7*n) + 7*(a*b^6*n + 15*a^2*b^4*c*n + 30*a^3*b^2*c^
2*n + 5*a^4*c^3*n)*x^(6*n) + 7*(3*a^2*b^5*n + 20*a^3*b^3*c*n + 15*a^4*b*c^2
*n)*x^(5*n) + 7*(5*a^3*b^4*n + 15*a^4*b^2*c*n + 3*a^5*c^2*n)*x^(4*n) + 7*(5
*a^4*b^3*n + 6*a^5*b*c*n)*x^(3*n) + 7*(3*a^5*b^2*n + a^6*c*n)*x^(2*n))
```

Fricas [B] time = 1.35582, size = 851, normalized size = 37.

$$-\frac{}{7\left(c^7nx^{14n} + 7\,bc^6nx^{13n} + 7\,a^6bnx^n + a^7n + 7\left(3\,b^2c^5 + ac^6\right)nx^{12n} + 7\left(5\,b^3c^4 + 6\,abc^5\right)nx^{11n} + 7\left(5\,b^4c^3 + 15\,ab^2c^4 + \right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(a+b*x^n+c*x^(2*n))^8,x, algorithm="fricas")
```

```
[Out] -1/7/(c^7*n*x^(14*n) + 7*b*c^6*n*x^(13*n) + 7*a^6*b*n*x^n + a^7*n + 7*(3*b^
2*c^5 + a*c^6)*n*x^(12*n) + 7*(5*b^3*c^4 + 6*a*b*c^5)*n*x^(11*n) + 7*(5*b^4
*c^3 + 15*a*b^2*c^4 + 3*a^2*c^5)*n*x^(10*n) + 7*(3*b^5*c^2 + 20*a*b^3*c^3 +
 15*a^2*b*c^4)*n*x^(9*n) + 7*(b^6*c + 15*a*b^4*c^2 + 30*a^2*b^2*c^3 + 5*a^3
*c^4)*n*x^(8*n) + (b^7 + 42*a*b^5*c + 210*a^2*b^3*c^2 + 140*a^3*b*c^3)*n*x^
(7*n) + 7*(a*b^6 + 15*a^2*b^4*c + 30*a^3*b^2*c^2 + 5*a^4*c^3)*n*x^(6*n) + 7
*(3*a^2*b^5 + 20*a^3*b^3*c + 15*a^4*b*c^2)*n*x^(5*n) + 7*(5*a^3*b^4 + 15*a^
4*b^2*c + 3*a^5*c^2)*n*x^(4*n) + 7*(5*a^4*b^3 + 6*a^5*b*c)*n*x^(3*n) + 7*(3
*a^5*b^2 + a^6*c)*n*x^(2*n))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)/(a+b*x**n+c*x**(2*n))**8,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18251, size = 28, normalized size = 1.22

$$-\frac{1}{7\left(cx^{2n}+bx^{n}+a\right)^{7}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(a+b*x^n+c*x^(2*n))^8,x, algorithm="giac")
```

```
[Out] -1/7/((c*x^(2*n) + b*x^n + a)^7*n)
```

3.113 $\int \frac{b+2cx}{-a+bx+cx^2} \, dx$

Optimal. Leaf size=13

$$\log \left(a-bx-cx^2\right)$$

```
[Out] Log[a - b*x - c*x^2]
```

Rubi [A] time = 0.0047574, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {628}

$$\log \left(a-bx-cx^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)/(-a + b*x + c*x^2),x]
```

```
[Out] Log[a - b*x - c*x^2]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{b+2cx}{-a+bx+cx^2} \, dx = \log \left(a-bx-cx^2\right)$$

Mathematica [A] time = 0.0045095, size = 12, normalized size = 0.92

$$\log (x(b+cx)-a)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)/(-a + b*x + c*x^2),x]
```

```
[Out] Log[-a + x*(b + c*x)]
```

Maple [A] time = 0., size = 14, normalized size = 1.1

$$\ln\left(cx^2 + bx - a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)/(c*x^2+b*x-a),x)
```

```
[Out] ln(c*x^2+b*x-a)
```

Maxima [A] time = 1.18616, size = 18, normalized size = 1.38

$$\log\left(cx^2 + bx - a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x-a),x, algorithm="maxima")
```

```
[Out] log(c*x^2 + b*x - a)
```

Fricas [A] time = 0.950366, size = 30, normalized size = 2.31

$$\log\left(cx^2 + bx - a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x-a),x, algorithm="fricas")
```

```
[Out] log(c*x^2 + b*x - a)
```

Sympy [A] time = 0.393052, size = 10, normalized size = 0.77

$$\log\left(-a + bx + cx^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x**2+b*x-a),x)
```

```
[Out] log(-a + b*x + c*x**2)
```

Giac [A] time = 1.12131, size = 18, normalized size = 1.38

$$\log\left(cx^2 + bx - a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x-a),x, algorithm="giac")
```

```
[Out] log(c*x^2 + b*x - a)
```

3.114 $\int \frac{x(b+2cx^2)}{-a+bx^2+cx^4} dx$

Optimal. Leaf size=19

$$\frac{1}{2}\log\left(a-bx^2-cx^4\right)$$

```
[Out] Log[a - b*x^2 - c*x^4]/2
```

Rubi [A] time = 0.0193331, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1247, 628}

$$\frac{1}{2}\log\left(a-bx^2-cx^4\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x*(b + 2*c*x^2))/(-a + b*x^2 + c*x^4),x]
```

```
[Out] Log[a - b*x^2 - c*x^4]/2
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(b+2cx^2\right)}{-a+bx^2+cx^4}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{b+2cx}{-a+bx+cx^2}\,dx, x, x^2\right)\\ &= \frac{1}{2}\log\left(a-bx^2-cx^4\right)\end{aligned}$$

Mathematica [A] time = 0.006661, size = 19, normalized size = 1.

$$\frac{1}{2}\log\left(-a+bx^2+cx^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(b + 2*c*x^2))/(-a + b*x^2 + c*x^4),x]
```

```
[Out] Log[-a + b*x^2 + c*x^4]/2
```

Maple [A] time = 0.003, size = 18, normalized size = 1.

$$\frac{\ln\left(cx^4+bx^2-a\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c*x^2+b)/(c*x^4+b*x^2-a),x)
```

```
[Out] 1/2*ln(c*x^4+b*x^2-a)
```

Maxima [A] time = 1.01428, size = 23, normalized size = 1.21

$$\frac{1}{2}\log\left(cx^4+bx^2-a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2-a),x, algorithm="maxima")
```

```
[Out] 1/2*log(c*x^4 + b*x^2 - a)
```

Fricas [A] time = 1.03838, size = 38, normalized size = 2.

$$\frac{1}{2}\log\left(cx^4+bx^2-a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2-a),x, algorithm="fricas")
```

```
[Out] 1/2*log(c*x^4 + b*x^2 - a)
```

Sympy [A] time = 0.418333, size = 14, normalized size = 0.74

$$\frac{\log\left(-a+bx^2+cx^4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x**2+b)/(c*x**4+b*x**2-a),x)
```

```
[Out] log(-a + b*x**2 + c*x**4)/2
```

Giac [A] time = 1.158, size = 23, normalized size = 1.21

$$\frac{1}{2}\log\left(cx^4+bx^2-a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2-a),x, algorithm="giac")
```

```
[Out] 1/2*log(c*x^4 + b*x^2 - a)
```

3.115 $\int \frac{x^2(b+2cx^3)}{-a+bx^3+cx^6} \, dx$

Optimal. Leaf size=19

$$\frac{1}{3}\log\left(a-bx^3-cx^6\right)$$

```
[Out] Log[a - b*x^3 - c*x^6]/3
```

Rubi [A] time = 0.0240185, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1468, 628}

$$\frac{1}{3}\log\left(a-bx^3-cx^6\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(b + 2*c*x^3))/(-a + b*x^3 + c*x^6),x]
```

```
[Out] Log[a - b*x^3 - c*x^6]/3
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^2 \left(b+2 c x^3\right)}{-a+b x^3+c x^6} \, dx = \frac{1}{3} \text{Subst}\left(\int \frac{b+2 c x}{-a+b x+c x^2} \, dx,x,x^3\right)$$
$$= \frac{1}{3} \log \left(a-b x^3-c x^6\right)$$

Mathematica [A] time = 0.0064442, size = 19, normalized size = 1.

$$\frac{1}{3} \log \left(-a+b x^3+c x^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(b + 2*c*x^3))/(-a + b*x^3 + c*x^6),x]
```

```
[Out] Log[-a + b*x^3 + c*x^6]/3
```

Maple [A] time = 0., size = 18, normalized size = 1.

$$\frac{\ln \left(c x^6+b x^3-a\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a),x)
```

```
[Out] 1/3*ln(c*x^6+b*x^3-a)
```

Maxima [A] time = 0.978929, size = 23, normalized size = 1.21

$$\frac{1}{3} \log \left(c x^6+b x^3-a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a),x, algorithm="maxima")
```

```
[Out] 1/3*log(c*x^6 + b*x^3 - a)
```

Fricas [A] time = 1.01625, size = 38, normalized size = 2.

$$\frac{1}{3}\log\left(cx^6+bx^3-a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a),x, algorithm="fricas")
```

```
[Out] 1/3*log(c*x^6 + b*x^3 - a)
```

Sympy [A] time = 0.512, size = 14, normalized size = 0.74

$$\frac{\log\left(-a+bx^3+cx^6\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*c*x**3+b)/(c*x**6+b*x**3-a),x)
```

```
[Out] log(-a + b*x**3 + c*x**6)/3
```

Giac [A] time = 1.33209, size = 23, normalized size = 1.21

$$\frac{1}{3}\log\left(cx^6+bx^3-a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a),x, algorithm="giac")
```

```
[Out] 1/3*log(c*x^6 + b*x^3 - a)
```

3.116 $\int \frac{x^{-1+n}(b+2cx^n)}{-a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=21

$$\frac{\log\left(a-bx^n-cx^{2n}\right)}{n}$$

```
[Out] Log[a - b*x^n - c*x^(2*n)]/n
```

Rubi [A] time = 0.0288613, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1468, 628}

$$\frac{\log\left(a-bx^n-cx^{2n}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + n)*(b + 2*c*x^n))/(-a + b*x^n + c*x^(2*n)),x]
```

```
[Out] Log[a - b*x^n - c*x^(2*n)]/n
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x^{-1+n}\left(b+2cx^n\right)}{-a+bx^n+cx^{2n}}\,dx = \frac{\text{Subst}\left(\int \frac{b+2cx}{-a+bx+cx^2}\,dx, x, x^n\right)}{n}$$
$$= \frac{\log\left(a-bx^n-cx^{2n}\right)}{n}$$

Mathematica [A] time = 0.11436, size = 21, normalized size = 1.

$$\frac{\log\left(a-bx^n-cx^{2n}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + n)*(b + 2*c*x^n))/(-a + b*x^n + c*x^(2*n)),x]
```

```
[Out] Log[a - b*x^n - c*x^(2*n)]/n
```

Maple [A] time = 0.02, size = 26, normalized size = 1.2

$$\frac{\ln\left(-c\left(e^{n\ln(x)}\right)^2-be^{n\ln(x)}+a\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n)*(b+2*c*x^n)/(-a+b*x^n+c*x^(2*n)),x)
```

```
[Out] 1/n*ln(-c*exp(n*ln(x))^2-b*exp(n*ln(x))+a)
```

Maxima [A] time = 1.14785, size = 34, normalized size = 1.62

$$\frac{\log\left(\frac{cx^{2n}+bx^n-a}{c}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(-a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] log((c*x^(2*n) + b*x^n - a)/c)/n
```

Fricas [A] time = 1.20768, size = 41, normalized size = 1.95

$$\frac{\log\left(cx^{2\,n}+bx^{n}-a\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(-a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] log(c*x^(2*n) + b*x^n - a)/n
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)/(-a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09546, size = 28, normalized size = 1.33

$$\frac{\log\left(cx^{2\,n}+bx^{n}-a\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(-a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] log(c*x^(2*n) + b*x^n - a)/n
```

3.117 $\int \frac{b+2cx}{\left(-a+bx+cx^2\right)^8}\,dx$

Optimal. Leaf size=18

$$\frac{1}{7\left(a-bx-cx^2\right)^7}$$

```
[Out] 1/(7*(a - b*x - c*x^2)^7)
```

Rubi [A] time = 0.004369, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {629}

$$\frac{1}{7\left(a-bx-cx^2\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)/(-a + b*x + c*x^2)^8,x]
```

```
[Out] 1/(7*(a - b*x - c*x^2)^7)
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{b+2cx}{\left(-a+bx+cx^2\right)^8}\,dx=\frac{1}{7\left(a-bx-cx^2\right)^7}$$

Mathematica [A] time = 0.0133975, size = 16, normalized size = 0.89

$$\frac{1}{7(a-x(b+cx))^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)/(-a + b*x + c*x^2)^8,x]
```

```
[Out] 1/(7*(a - x*(b + c*x))^7)
```

Maple [A] time = 0.002, size = 17, normalized size = 0.9

$$-\frac{1}{7\left(cx^{2}+bx-a\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)/(c*x^2+b*x-a)^8,x)
```

```
[Out] -1/7/(c*x^2+b*x-a)^7
```

Maxima [A] time = 1.00645, size = 22, normalized size = 1.22

$$-\frac{1}{7\left(cx^{2}+bx-a\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x-a)^8,x, algorithm="maxima")
```

```
[Out] -1/7/(c*x^2 + b*x - a)^7
```

Fricas [B] time = 1.31757, size = 737, normalized size = 40.94

$$-\frac{}{7\left(c^{7}x^{14}+7\,bc^{6}x^{13}+7\left(3\,b^{2}c^{5}-ac^{6}\right)x^{12}+7\left(5\,b^{3}c^{4}-6\,abc^{5}\right)x^{11}+7\left(5\,b^{4}c^{3}-15\,ab^{2}c^{4}+3\,a^{2}c^{5}\right)x^{10}+7\left(3\,b^{5}c^{2}-20\,ab^{3}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x-a)^8,x, algorithm="fricas")
```

```
[Out] -1/7/(c^7*x^14 + 7*b*c^6*x^13 + 7*(3*b^2*c^5 - a*c^6)*x^12 + 7*(5*b^3*c^4 -
 6*a*b*c^5)*x^11 + 7*(5*b^4*c^3 - 15*a*b^2*c^4 + 3*a^2*c^5)*x^10 + 7*(3*b^5
*c^2 - 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^9 + 7*(b^6*c - 15*a*b^4*c^2 + 30*a^2*
b^2*c^3 - 5*a^3*c^4)*x^8 + 7*a^6*b*x + (b^7 - 42*a*b^5*c + 210*a^2*b^3*c^2
- 140*a^3*b*c^3)*x^7 - a^7 - 7*(a*b^6 - 15*a^2*b^4*c + 30*a^3*b^2*c^2 - 5*a
^4*c^3)*x^6 + 7*(3*a^2*b^5 - 20*a^3*b^3*c + 15*a^4*b*c^2)*x^5 - 7*(5*a^3*b^
4 - 15*a^4*b^2*c + 3*a^5*c^2)*x^4 + 7*(5*a^4*b^3 - 6*a^5*b*c)*x^3 - 7*(3*a^
5*b^2 - a^6*c)*x^2)
```

Sympy [B] time = 55.2036, size = 359, normalized size = 19.94

$$-\frac{}{-7a^7 + 49a^6bx + 49bc^6x^{13} + 7c^7x^{14} + x^{12}\left(-49ac^6 + 147b^2c^5\right) + x^{11}\left(-294abc^5 + 245b^3c^4\right) + x^{10}\left(147a^2c^5 - 735ab^2c^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x**2+b*x-a)**8,x)
```

```
[Out] -1/(-7*a**7 + 49*a**6*b*x + 49*b*c**6*x**13 + 7*c**7*x**14 + x**12*(-49*a*c
**6 + 147*b**2*c**5) + x**11*(-294*a*b*c**5 + 245*b**3*c**4) + x**10*(147*a
**2*c**5 - 735*a*b**2*c**4 + 245*b**4*c**3) + x**9*(735*a**2*b*c**4 - 980*a
*b**3*c**3 + 147*b**5*c**2) + x**8*(-245*a**3*c**4 + 1470*a**2*b**2*c**3 -
735*a*b**4*c**2 + 49*b**6*c) + x**7*(-980*a**3*b*c**3 + 1470*a**2*b**3*c**2
 - 294*a*b**5*c + 7*b**7) + x**6*(245*a**4*c**3 - 1470*a**3*b**2*c**2 + 735
*a**2*b**4*c - 49*a*b**6) + x**5*(735*a**4*b*c**2 - 980*a**3*b**3*c + 147*a
**2*b**5) + x**4*(-147*a**5*c**2 + 735*a**4*b**2*c - 245*a**3*b**4) + x**3*
(-294*a**5*b*c + 245*a**4*b**3) + x**2*(49*a**6*c - 147*a**5*b**2))
```

Giac [A] time = 1.10916, size = 22, normalized size = 1.22

$$-\frac{1}{7\left(cx^2 + bx - a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x-a)^8,x, algorithm="giac")
```

```
[Out] -1/7/(c*x^2 + b*x - a)^7
```

3.118 $\int \frac{x(b+2cx^2)}{(-a+bx^2+cx^4)^8} dx$

Optimal. Leaf size=20

$$\frac{1}{14(a-bx^2-cx^4)^7}$$

```
[Out] 1/(14*(a - b*x^2 - c*x^4)^7)
```

Rubi [A] time = 0.0195398, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1247, 629}

$$\frac{1}{14(a-bx^2-cx^4)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(b + 2*c*x^2))/(-a + b*x^2 + c*x^4)^8,x]
```

```
[Out] 1/(14*(a - b*x^2 - c*x^4)^7)
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x(b+2cx^2)}{(-a+bx^2+cx^4)^8}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{b+2cx}{(-a+bx+cx^2)^8}\,dx, x, x^2\right)$$
$$= \frac{1}{14\left(a-bx^2-cx^4\right)^7}$$

Mathematica [A] time = 0.0169894, size = 20, normalized size = 1.

$$-\frac{1}{14\left(-a+bx^2+cx^4\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(b + 2*c*x^2))/(-a + b*x^2 + c*x^4)^8,x]
```

```
[Out] -1/(14*(-a + b*x^2 + c*x^4)^7)
```

Maple [A] time = 0.001, size = 19, normalized size = 1.

$$-\frac{1}{14\left(cx^4+bx^2-a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c*x^2+b)/(c*x^4+b*x^2-a)^8,x)
```

```
[Out] -1/14/(c*x^4+b*x^2-a)^7
```

Maxima [B] time = 1.48109, size = 481, normalized size = 24.05

$$-\frac{}{14\left(c^7x^{28}+7\,bc^6x^{26}+7\left(3\,b^2c^5-ac^6\right)x^{24}+7\left(5\,b^3c^4-6\,abc^5\right)x^{22}+7\left(5\,b^4c^3-15\,ab^2c^4+3\,a^2c^5\right)x^{20}+7\left(3\,b^5c^2-20\,a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2-a)^8,x, algorithm="maxima")
```

```
[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 7*(3*b^2*c^5 - a*c^6)*x^24 + 7*(5*b^3*c^4
- 6*a*b*c^5)*x^22 + 7*(5*b^4*c^3 - 15*a*b^2*c^4 + 3*a^2*c^5)*x^20 + 7*(3*b^
5*c^2 - 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^18 + 7*(b^6*c - 15*a*b^4*c^2 + 30*a^
2*b^2*c^3 - 5*a^3*c^4)*x^16 + (b^7 - 42*a*b^5*c + 210*a^2*b^3*c^2 - 140*a^3
*b*c^3)*x^14 - 7*(a*b^6 - 15*a^2*b^4*c + 30*a^3*b^2*c^2 - 5*a^4*c^3)*x^12 +
 7*(3*a^2*b^5 - 20*a^3*b^3*c + 15*a^4*b*c^2)*x^10 + 7*a^6*b*x^2 - 7*(5*a^3*
b^4 - 15*a^4*b^2*c + 3*a^5*c^2)*x^8 - a^7 + 7*(5*a^4*b^3 - 6*a^5*b*c)*x^6 -
 7*(3*a^5*b^2 - a^6*c)*x^4)
```

Fricas [B] time = 1.32395, size = 748, normalized size = 37.4

$$-\frac{}{14\left(c^7x^{28}+7\,bc^6x^{26}+7\left(3\,b^2c^5-ac^6\right)x^{24}+7\left(5\,b^3c^4-6\,abc^5\right)x^{22}+7\left(5\,b^4c^3-15\,ab^2c^4+3\,a^2c^5\right)x^{20}+7\left(3\,b^5c^2-20\,ab\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2-a)^8,x, algorithm="fricas")
```

```
[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 7*(3*b^2*c^5 - a*c^6)*x^24 + 7*(5*b^3*c^4
- 6*a*b*c^5)*x^22 + 7*(5*b^4*c^3 - 15*a*b^2*c^4 + 3*a^2*c^5)*x^20 + 7*(3*b^
5*c^2 - 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^18 + 7*(b^6*c - 15*a*b^4*c^2 + 30*a^
2*b^2*c^3 - 5*a^3*c^4)*x^16 + (b^7 - 42*a*b^5*c + 210*a^2*b^3*c^2 - 140*a^3
*b*c^3)*x^14 - 7*(a*b^6 - 15*a^2*b^4*c + 30*a^3*b^2*c^2 - 5*a^4*c^3)*x^12 +
 7*(3*a^2*b^5 - 20*a^3*b^3*c + 15*a^4*b*c^2)*x^10 + 7*a^6*b*x^2 - 7*(5*a^3*
b^4 - 15*a^4*b^2*c + 3*a^5*c^2)*x^8 - a^7 + 7*(5*a^4*b^3 - 6*a^5*b*c)*x^6 -
 7*(3*a^5*b^2 - a^6*c)*x^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x**2+b)/(c*x**4+b*x**2-a)**8,x)
```

```
[Out] Timed out
```

Giac [A] time = 55.4883, size = 24, normalized size = 1.2

$$-\frac{1}{14\left(cx^4+bx^2-a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2-a)^8,x, algorithm="giac")
```

```
[Out] -1/14/(c*x^4 + b*x^2 - a)^7
```

3.119 $\int \frac{x^2(b+2cx^3)}{(-a+bx^3+cx^6)^8}\,dx$

Optimal. Leaf size=20

$$\frac{1}{21\left(a-bx^3-cx^6\right)^7}$$

```
[Out] 1/(21*(a - b*x^3 - c*x^6)^7)
```

Rubi [A] time = 0.0244405, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1468, 629}

$$\frac{1}{21\left(a-bx^3-cx^6\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(b + 2*c*x^3))/(-a + b*x^3 + c*x^6)^8,x]
```

```
[Out] 1/(21*(a - b*x^3 - c*x^6)^7)
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^2 \left(b+2cx^3\right)}{\left(-a+bx^3+cx^6\right)^8}\,dx = \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{b+2cx}{\left(-a+bx+cx^2\right)^8}\,dx, x, x^3\right)$$
$$= \frac{1}{21\left(a-bx^3-cx^6\right)^7}$$

Mathematica [A] time = 0.0173717, size = 20, normalized size = 1.

$$-\frac{1}{21\left(-a+bx^3+cx^6\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(b + 2*c*x^3))/(-a + b*x^3 + c*x^6)^8,x]
```

```
[Out] -1/(21*(-a + b*x^3 + c*x^6)^7)
```

Maple [A] time = 0., size = 19, normalized size = 1.

$$-\frac{1}{21\left(cx^6+bx^3-a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a)^8,x)
```

```
[Out] -1/21/(c*x^6+b*x^3-a)^7
```

Maxima [B] time = 1.53085, size = 481, normalized size = 24.05

$$-\frac{}{21\left(c^7x^{42}+7\,bc^6x^{39}+7\left(3\,b^2c^5-ac^6\right)x^{36}+7\left(5\,b^3c^4-6\,abc^5\right)x^{33}+7\left(5\,b^4c^3-15\,ab^2c^4+3\,a^2c^5\right)x^{30}+7\left(3\,b^5c^2-20\,\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a)^8,x, algorithm="maxima")
```

```
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 7*(3*b^2*c^5 - a*c^6)*x^36 + 7*(5*b^3*c^4
- 6*a*b*c^5)*x^33 + 7*(5*b^4*c^3 - 15*a*b^2*c^4 + 3*a^2*c^5)*x^30 + 7*(3*b^
5*c^2 - 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^27 + 7*(b^6*c - 15*a*b^4*c^2 + 30*a^
2*b^2*c^3 - 5*a^3*c^4)*x^24 + (b^7 - 42*a*b^5*c + 210*a^2*b^3*c^2 - 140*a^3
*b*c^3)*x^21 - 7*(a*b^6 - 15*a^2*b^4*c + 30*a^3*b^2*c^2 - 5*a^4*c^3)*x^18 +
 7*(3*a^2*b^5 - 20*a^3*b^3*c + 15*a^4*b*c^2)*x^15 - 7*(5*a^3*b^4 - 15*a^4*b
^2*c + 3*a^5*c^2)*x^12 + 7*a^6*b*x^3 + 7*(5*a^4*b^3 - 6*a^5*b*c)*x^9 - a^7
- 7*(3*a^5*b^2 - a^6*c)*x^6)
```

Fricas [B] time = 1.32698, size = 749, normalized size = 37.45

$$-\frac{}{21\left(c^{7}x^{42}+7\,bc^{6}x^{39}+7\left(3\,b^{2}c^{5}-ac^{6}\right)x^{36}+7\left(5\,b^{3}c^{4}-6\,abc^{5}\right)x^{33}+7\left(5\,b^{4}c^{3}-15\,ab^{2}c^{4}+3\,a^{2}c^{5}\right)x^{30}+7\left(3\,b^{5}c^{2}-20\,ab\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a)^8,x, algorithm="fricas")
```

```
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 7*(3*b^2*c^5 - a*c^6)*x^36 + 7*(5*b^3*c^4
- 6*a*b*c^5)*x^33 + 7*(5*b^4*c^3 - 15*a*b^2*c^4 + 3*a^2*c^5)*x^30 + 7*(3*b^
5*c^2 - 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^27 + 7*(b^6*c - 15*a*b^4*c^2 + 30*a^
2*b^2*c^3 - 5*a^3*c^4)*x^24 + (b^7 - 42*a*b^5*c + 210*a^2*b^3*c^2 - 140*a^3
*b*c^3)*x^21 - 7*(a*b^6 - 15*a^2*b^4*c + 30*a^3*b^2*c^2 - 5*a^4*c^3)*x^18 +
 7*(3*a^2*b^5 - 20*a^3*b^3*c + 15*a^4*b*c^2)*x^15 - 7*(5*a^3*b^4 - 15*a^4*b
^2*c + 3*a^5*c^2)*x^12 + 7*a^6*b*x^3 + 7*(5*a^4*b^3 - 6*a^5*b*c)*x^9 - a^7
- 7*(3*a^5*b^2 - a^6*c)*x^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*c*x**3+b)/(c*x**6+b*x**3-a)**8,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a)^8,x, algorithm="giac")
```

```
[Out] Timed out
```

3.120 $\int \frac{x^{-1+n}(b+2cx^n)}{\left(-a+bx^n+cx^{2n}\right)^8}\,dx$

Optimal. Leaf size=25

$$\frac{1}{7n\left(a-bx^n-cx^{2n}\right)^7}$$

```
[Out] 1/(7*n*(a - b*x^n - c*x^(2*n))^7)
```

Rubi [A] time = 0.0290118, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1468, 629}

$$\frac{1}{7n\left(a-bx^n-cx^{2n}\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + n)*(b + 2*c*x^n))/(-a + b*x^n + c*x^(2*n))^8,x]
```

```
[Out] 1/(7*n*(a - b*x^n - c*x^(2*n))^7)
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^{-1+n}(b+2cx^n)}{(-a+bx^n+cx^{2n})^8}\,dx = \frac{\operatorname{Subst}\left(\int \frac{b+2cx}{(-a+bx+cx^2)^8}\,dx, x, x^n\right)}{n}$$
$$= \frac{1}{7n\left(a-bx^n-cx^{2n}\right)^7}$$

Mathematica [A] time = 0.0680033, size = 23, normalized size = 0.92

$$\frac{1}{7n\left(a-x^n\left(b+cx^n\right)\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + n)*(b + 2*c*x^n))/(-a + b*x^n + c*x^(2*n))^8,x]
```

```
[Out] 1/(7*n*(a - x^n*(b + c*x^n))^7)
```

Maple [A] time = 0.059, size = 24, normalized size = 1.

$$\frac{1}{7\,n\left(-c\left(x^n\right)^2-bx^n+a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n)*(b+2*c*x^n)/(-a+b*x^n+c*x^(2*n))^8,x)
```

```
[Out] 1/7/n/(-c*(x^n)^2-b*x^n+a)^7
```

Maxima [B] time = 2.89127, size = 566, normalized size = 22.64

$$-\frac{}{7\left(c^7nx^{14\,n}+7\,bc^6nx^{13\,n}+7\,a^6bnx^n-a^7n+7\left(3\,b^2c^5n-ac^6n\right)x^{12\,n}+7\left(5\,b^3c^4n-6\,abc^5n\right)x^{11\,n}+7\left(5\,b^4c^3n-15\,ab^2c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(-a+b*x^n+c*x^(2*n))^8,x, algorithm="maxima"
)
```

```
[Out] -1/7/(c^7*n*x^(14*n) + 7*b*c^6*n*x^(13*n) + 7*a^6*b*n*x^n - a^7*n + 7*(3*b^
2*c^5*n - a*c^6*n)*x^(12*n) + 7*(5*b^3*c^4*n - 6*a*b*c^5*n)*x^(11*n) + 7*(5
*b^4*c^3*n - 15*a*b^2*c^4*n + 3*a^2*c^5*n)*x^(10*n) + 7*(3*b^5*c^2*n - 20*a
*b^3*c^3*n + 15*a^2*b*c^4*n)*x^(9*n) + 7*(b^6*c*n - 15*a*b^4*c^2*n + 30*a^2
*b^2*c^3*n - 5*a^3*c^4*n)*x^(8*n) + (b^7*n - 42*a*b^5*c*n + 210*a^2*b^3*c^2
*n - 140*a^3*b*c^3*n)*x^(7*n) - 7*(a*b^6*n - 15*a^2*b^4*c*n + 30*a^3*b^2*c^
2*n - 5*a^4*c^3*n)*x^(6*n) + 7*(3*a^2*b^5*n - 20*a^3*b^3*c*n + 15*a^4*b*c^2
*n)*x^(5*n) - 7*(5*a^3*b^4*n - 15*a^4*b^2*c*n + 3*a^5*c^2*n)*x^(4*n) + 7*(5
*a^4*b^3*n - 6*a^5*b*c*n)*x^(3*n) - 7*(3*a^5*b^2*n - a^6*c*n)*x^(2*n))
```

Fricas [B] time = 1.51303, size = 851, normalized size = 34.04

$$-\frac{}{7\left(c^7nx^{14n}+7\,bc^6nx^{13n}+7\,a^6bnx^n-a^7n+7\left(3\,b^2c^5-ac^6\right)nx^{12n}+7\left(5\,b^3c^4-6\,abc^5\right)nx^{11n}+7\left(5\,b^4c^3-15\,ab^2c^4+3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(-a+b*x^n+c*x^(2*n))^8,x, algorithm="fricas"
)
```

```
[Out] -1/7/(c^7*n*x^(14*n) + 7*b*c^6*n*x^(13*n) + 7*a^6*b*n*x^n - a^7*n + 7*(3*b^
2*c^5 - a*c^6)*n*x^(12*n) + 7*(5*b^3*c^4 - 6*a*b*c^5)*n*x^(11*n) + 7*(5*b^4
*c^3 - 15*a*b^2*c^4 + 3*a^2*c^5)*n*x^(10*n) + 7*(3*b^5*c^2 - 20*a*b^3*c^3 +
 15*a^2*b*c^4)*n*x^(9*n) + 7*(b^6*c - 15*a*b^4*c^2 + 30*a^2*b^2*c^3 - 5*a^3
*c^4)*n*x^(8*n) + (b^7 - 42*a*b^5*c + 210*a^2*b^3*c^2 - 140*a^3*b*c^3)*n*x^
(7*n) - 7*(a*b^6 - 15*a^2*b^4*c + 30*a^3*b^2*c^2 - 5*a^4*c^3)*n*x^(6*n) + 7
*(3*a^2*b^5 - 20*a^3*b^3*c + 15*a^4*b*c^2)*n*x^(5*n) - 7*(5*a^3*b^4 - 15*a^
4*b^2*c + 3*a^5*c^2)*n*x^(4*n) + 7*(5*a^4*b^3 - 6*a^5*b*c)*n*x^(3*n) - 7*(3
*a^5*b^2 - a^6*c)*n*x^(2*n))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)/(-a+b*x**n+c*x**(2*n))**8,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23779, size = 31, normalized size = 1.24

$$-\frac{1}{7\left(cx^{2n}+bx^{n}-a\right)^{7}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(-a+b*x^n+c*x^(2*n))^8,x, algorithm="giac")
```

```
[Out] -1/7/((c*x^(2*n) + b*x^n - a)^7*n)
```

3.121 $\int \frac{b+2cx}{bx+cx^2}\,dx$

Optimal. Leaf size=10

$$\log\left(bx+cx^2\right)$$

```
[Out] Log[b*x + c*x^2]
```

Rubi [A] time = 0.0042626, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {628}

$$\log\left(bx+cx^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)/(b*x + c*x^2),x]
```

```
[Out] Log[b*x + c*x^2]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{b+2cx}{bx+cx^2}\,dx=\log\left(bx+cx^2\right)$$

Mathematica [A] time = 0.0038842, size = 9, normalized size = 0.9

$$\log(b+cx)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)/(b*x + c*x^2),x]
```

```
[Out] Log[x] + Log[b + c*x]
```

Maple [A] time = 0.002, size = 9, normalized size = 0.9

$$\ln(x(cx+b))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)/(c*x^2+b*x),x)
```

```
[Out] ln(x*(c*x+b))
```

Maxima [A] time = 1.02648, size = 14, normalized size = 1.4

$$\log\left(cx^2+bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x),x, algorithm="maxima")
```

```
[Out] log(c*x^2 + b*x)
```

Fricas [A] time = 1.09618, size = 24, normalized size = 2.4

$$\log\left(cx^2+bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x),x, algorithm="fricas")
```

```
[Out] log(c*x^2 + b*x)
```

Sympy [A] time = 0.352951, size = 8, normalized size = 0.8

$$\log\left(bx+cx^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x**2+b*x),x)
```

```
[Out] log(b*x + c*x**2)
```

Giac [A] time = 1.09606, size = 15, normalized size = 1.5

$$\log(|cx + b|) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x),x, algorithm="giac")
```

```
[Out] log(abs(c*x + b)) + log(abs(x))
```

3.122 $\int \frac{x(b+2cx^2)}{bx^2+cx^4}\,dx$

Optimal. Leaf size=16

$$\frac{1}{2}\log\left(bx^2+cx^4\right)$$

```
[Out] Log[b*x^2 + c*x^4]/2
```

Rubi [A] time = 0.0239123, antiderivative size = 15, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {1584, 446, 72}

$$\frac{1}{2}\log\left(b+cx^2\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x*(b + 2*c*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] Log[x] + Log[b + c*x^2]/2
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(b+2cx^2\right)}{bx^2+cx^4}\,dx&=\int \frac{b+2cx^2}{x\left(b+cx^2\right)}\,dx\\&=\frac{1}{2}\,\text{Subst}\left(\int \frac{b+2cx}{x(b+cx)}\,dx,x,x^2\right)\\&=\frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{x}+\frac{c}{b+cx}\right)\,dx,x,x^2\right)\\&=\log(x)+\frac{1}{2}\log\left(b+cx^2\right)\end{aligned}$$

Mathematica [A] time = 0.0061463, size = 15, normalized size = 0.94

$$\frac{1}{2}\log\left(b+cx^2\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(b + 2*c*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] Log[x] + Log[b + c*x^2]/2
```

Maple [A] time = 0.004, size = 14, normalized size = 0.9

$$\ln(x)+\frac{\ln\left(cx^2+b\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c*x^2+b)/(c*x^4+b*x^2),x)
```

```
[Out] ln(x)+1/2*ln(c*x^2+b)
```

Maxima [A] time = 0.996828, size = 23, normalized size = 1.44

$$\frac{1}{2}\log\left(cx^2+b\right)+\frac{1}{2}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/2*log(c*x^2 + b) + 1/2*log(x^2)
```

Fricas [A] time = 1.02086, size = 39, normalized size = 2.44

$$\frac{1}{2}\log\left(cx^2+b\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/2*log(c*x^2 + b) + log(x)
```

Sympy [A] time = 0.324457, size = 12, normalized size = 0.75

$$\log\left(x\right)+\frac{\log\left(\frac{b}{c}+x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x**2+b)/(c*x**4+b*x**2),x)
```

```
[Out] log(x) + log(b/c + x**2)/2
```

Giac [A] time = 1.09658, size = 20, normalized size = 1.25

$$\frac{1}{2}\log\left(\left|cx^2+b\right|\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(c*x^2 + b)) + log(abs(x))
```

3.123 $\int \frac{x^2(b+2cx^3)}{bx^3+cx^6}\,dx$

Optimal. Leaf size=16

$$\frac{1}{3}\log\left(bx^3+cx^6\right)$$

```
[Out] Log[b*x^3 + c*x^6]/3
```

Rubi [A] time = 0.0295228, antiderivative size = 15, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {1584, 446, 72}

$$\frac{1}{3}\log\left(b+cx^3\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(b + 2*c*x^3))/(b*x^3 + c*x^6),x]
```

```
[Out] Log[x] + Log[b + c*x^3]/3
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2\left(b+2cx^3\right)}{bx^3+cx^6}\,dx &= \int \frac{b+2cx^3}{x\left(b+cx^3\right)}\,dx\\
&= \frac{1}{3}\operatorname{Subst}\left(\int \frac{b+2cx}{x(b+cx)}\,dx, x, x^3\right)\\
&= \frac{1}{3}\operatorname{Subst}\left(\int \left(\frac{1}{x}+\frac{c}{b+cx}\right)dx, x, x^3\right)\\
&= \log(x)+\frac{1}{3}\log\left(b+cx^3\right)
\end{aligned}
$$

Mathematica [A] time = 0.0070442, size = 15, normalized size = 0.94

$$\frac{1}{3}\log\left(b+cx^3\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(b + 2*c*x^3))/(b*x^3 + c*x^6),x]
```

```
[Out] Log[x] + Log[b + c*x^3]/3
```

Maple [A] time = 0.006, size = 14, normalized size = 0.9

$$\ln(x)+\frac{\ln\left(cx^3+b\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*c*x^3+b)/(c*x^6+b*x^3),x)
```

```
[Out] ln(x)+1/3*ln(c*x^3+b)
```

Maxima [A] time = 1.202, size = 23, normalized size = 1.44

$$\frac{1}{3}\log\left(cx^3+b\right)+\frac{1}{3}\log\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3),x, algorithm="maxima")
```

```
[Out] 1/3*log(c*x^3 + b) + 1/3*log(x^3)
```

Fricas [A] time = 1.16842, size = 39, normalized size = 2.44

$$\frac{1}{3}\log\left(cx^3+b\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3),x, algorithm="fricas")
```

```
[Out] 1/3*log(c*x^3 + b) + log(x)
```

Sympy [A] time = 0.348128, size = 12, normalized size = 0.75

$$\log\left(x\right)+\frac{\log\left(\frac{b}{c}+x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*c*x**3+b)/(c*x**6+b*x**3),x)
```

```
[Out] log(x) + log(b/c + x**3)/3
```

Giac [A] time = 1.12081, size = 20, normalized size = 1.25

$$\frac{1}{3}\log\left(\left|cx^3+b\right|\right)+\log\left(\left|x\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3),x, algorithm="giac")
```

```
[Out] 1/3*log(abs(c*x^3 + b)) + log(abs(x))
```

3.124 $\int \frac{x^{-1+n}(b+2cx^n)}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=15

$$\frac{\log(b+cx^n)}{n}+\log(x)$$

```
[Out] Log[x] + Log[b + c*x^n]/n
```

Rubi [A] time = 0.0350369, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1584, 446, 72}

$$\frac{\log(b+cx^n)}{n}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + n)*(b + 2*c*x^n))/(b*x^n + c*x^(2*n)),x]
```

```
[Out] Log[x] + Log[b + c*x^n]/n
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^{-1+n}(b+2cx^n)}{bx^n+cx^{2n}}\,dx = \int \frac{b+2cx^n}{x(b+cx^n)}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{b+2cx}{x(b+cx)}\,dx, x, x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{x}+\frac{c}{b+cx}\right)dx, x, x^n\right)}{n}$$
$$= \log(x) + \frac{\log(b+cx^n)}{n}$$

Mathematica [A] time = 0.0124065, size = 15, normalized size = 1.

$$\frac{\log(b+cx^n)}{n} + \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + n)*(b + 2*c*x^n))/(b*x^n + c*x^(2*n)),x]
```

```
[Out] Log[x] + Log[b + c*x^n]/n
```

Maple [A] time = 0.018, size = 18, normalized size = 1.2

$$\ln(x) + \frac{\ln\left(ce^{n\ln(x)}+b\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n)*(b+2*c*x^n)/(b*x^n+c*x^(2*n)),x)
```

```
[Out] ln(x)+1/n*ln(c*exp(n*ln(x))+b)
```

Maxima [B] time = 1.0284, size = 63, normalized size = 4.2

$$b\left(\frac{\log(x)}{b} - \frac{\log\left(\frac{cx^n+b}{c}\right)}{bn}\right) + \frac{2\log\left(\frac{cx^n+b}{c}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] b*(log(x)/b - log((c*x^n + b)/c)/(b*n)) + 2*log((c*x^n + b)/c)/n
```

Fricas [A] time = 1.08609, size = 42, normalized size = 2.8

$$\frac{n\log(x)+\log(cx^n+b)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] (n*log(x) + log(c*x^n + b))/n
```

Sympy [A] time = 134.943, size = 48, normalized size = 3.2

$$\begin{cases} \log(x) & \text{for } c=0 \wedge n=0 \\ \frac{(b+2c)\log(x)}{b+c} & \text{for } n=0 \\ \frac{n^2\log(x)}{n^2-n}-\frac{n\log(x)}{n^2-n} & \text{for } c=0 \\ \log(x)+\frac{\log\left(\frac{b}{c}+x^n\right)}{n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] Piecewise((log(x), Eq(c, 0) & Eq(n, 0)), ((b + 2*c)*log(x)/(b + c), Eq(n, 0
)), (n**2*log(x)/(n**2 - n) - n*log(x)/(n**2 - n), Eq(c, 0)), (log(x) + log
(b/c + x**n)/n, True))
```

Giac [A] time = 1.09055, size = 23, normalized size = 1.53

$$\frac{\log(|cx^n+b|)}{n}+\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] log(abs(c*x^n + b))/n + log(abs(x))
```

3.125 $\int \frac{b+2cx}{\left(bx+cx^2\right)^8} dx$

Optimal. Leaf size=15

$$-\frac{1}{7\left(bx+cx^2\right)^7}$$

```
[Out] -1/(7*(b*x + c*x^2)^7)
```

Rubi [A] time = 0.0040906, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {629}

$$-\frac{1}{7\left(bx+cx^2\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)/(b*x + c*x^2)^8,x]
```

```
[Out] -1/(7*(b*x + c*x^2)^7)
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{b+2cx}{\left(bx+cx^2\right)^8} dx = -\frac{1}{7\left(bx+cx^2\right)^7}$$

Mathematica [A] time = 0.0195546, size = 14, normalized size = 0.93

$$-\frac{1}{7x^7(b+cx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)/(b*x + c*x^2)^8,x]
```

```
[Out] -1/(7*x^7*(b + c*x)^7)
```

Maple [B] time = 0.017, size = 177, normalized size = 11.8

$$-\frac{1}{7\,b^7x^7}-132\,\frac{c^6}{b^{13}x}+66\,\frac{c^5}{b^{12}x^2}-30\,\frac{c^4}{b^{11}x^3}+12\,\frac{c^3}{b^{10}x^4}-4\,\frac{c^2}{b^9x^5}+\frac{c}{b^8x^6}+132\,\frac{c^7}{b^{13}\,(cx+b)}+66\,\frac{c^7}{b^{12}\,(cx+b)^2}+30\,\frac{c^7}{b^{11}\,(cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)/(c*x^2+b*x)^8,x)
```

```
[Out] -1/7/b^7/x^7-132/b^13*c^6/x+66/b^12*c^5/x^2-30/b^11*c^4/x^3+12/b^10*c^3/x^4
-4/b^9*c^2/x^5+1/b^8*c/x^6+132/b^13*c^7/(c*x+b)+66/b^12*c^7/(c*x+b)^2+30/b^
11*c^7/(c*x+b)^3+12/b^10*c^7/(c*x+b)^4+4/b^9*c^7/(c*x+b)^5+c^7/b^8/(c*x+b)^
6+1/7*c^7/b^7/(c*x+b)^7
```

Maxima [A] time = 0.981226, size = 18, normalized size = 1.2

$$-\frac{1}{7\left(cx^2+bx\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x)^8,x, algorithm="maxima")
```

```
[Out] -1/7/(c*x^2 + b*x)^7
```

Fricas [B] time = 1.18938, size = 171, normalized size = 11.4

$$-\frac{1}{7\left(c^7x^{14}+7\,bc^6x^{13}+21\,b^2c^5x^{12}+35\,b^3c^4x^{11}+35\,b^4c^3x^{10}+21\,b^5c^2x^9+7\,b^6cx^8+b^7x^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x)^8,x, algorithm="fricas")
```

```
[Out] -1/7/(c^7*x^14 + 7*b*c^6*x^13 + 21*b^2*c^5*x^12 + 35*b^3*c^4*x^11 + 35*b^4*
c^3*x^10 + 21*b^5*c^2*x^9 + 7*b^6*c*x^8 + b^7*x^7)
```

Sympy [B] time = 4.08241, size = 87, normalized size = 5.8

$$-\frac{1}{7b^7x^7 + 49b^6cx^8 + 147b^5c^2x^9 + 245b^4c^3x^{10} + 245b^3c^4x^{11} + 147b^2c^5x^{12} + 49bc^6x^{13} + 7c^7x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x**2+b*x)**8,x)
```

```
[Out] -1/(7*b**7*x**7 + 49*b**6*c*x**8 + 147*b**5*c**2*x**9 + 245*b**4*c**3*x**10
 + 245*b**3*c**4*x**11 + 147*b**2*c**5*x**12 + 49*b*c**6*x**13 + 7*c**7*x**
14)
```

Giac [A] time = 1.11225, size = 18, normalized size = 1.2

$$-\frac{1}{7\left(cx^2 + bx\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x)^8,x, algorithm="giac")
```

```
[Out] -1/7/(c*x^2 + b*x)^7
```

3.126 $\int \frac{x(b+2cx^2)}{(bx^2+cx^4)^8} dx$

Optimal. Leaf size=16

$$-\frac{1}{14x^{14}\left(b+cx^2\right)^7}$$

```
[Out] -1/(14*x^14*(b + c*x^2)^7)
```

Rubi [A] time = 0.0208838, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {1584, 446, 74}

$$-\frac{1}{14x^{14}\left(b+cx^2\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(b + 2*c*x^2))/(b*x^2 + c*x^4)^8,x]
```

```
[Out] -1/(14*x^14*(b + c*x^2)^7)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
```

```
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(b+2cx^2\right)}{\left(bx^2+cx^4\right)^8}\,dx &= \int \frac{b+2cx^2}{x^{15}\left(b+cx^2\right)^8}\,dx\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{b+2cx}{x^8(b+cx)^8}\,dx,x,x^2\right)\\ &= -\frac{1}{14x^{14}\left(b+cx^2\right)^7}\end{aligned}$$

Mathematica [A] time = 0.0277516, size = 16, normalized size = 1.

$$-\frac{1}{14x^{14}\left(b+cx^2\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(b + 2*c*x^2))/(b*x^2 + c*x^4)^8,x]
```

```
[Out] -1/(14*x^14*(b + c*x^2)^7)
```

Maple [B] time = 0.019, size = 197, normalized size = 12.3

$$-\frac{1}{14\,b^7x^{14}}-66\,\frac{c^6}{b^{13}x^2}+33\,\frac{c^5}{b^{12}x^4}-15\,\frac{c^4}{b^{11}x^6}+6\,\frac{c^3}{b^{10}x^8}-2\,\frac{c^2}{b^9x^{10}}+\frac{c}{2\,b^8x^{12}}-\frac{c^8}{2\,b^{13}}\left(-132\,\frac{1}{c\left(cx^2+b\right)}-\frac{b^5}{c\left(cx^2+b\right)^6}-4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c*x^2+b)/(c*x^4+b*x^2)^8,x)
```

```
[Out] -1/14/b^7/x^14-66/b^13*c^6/x^2+33/b^12*c^5/x^4-15/b^11*c^4/x^6+6/b^10*c^3/x
^8-2/b^9*c^2/x^10+1/2/b^8*c/x^12-1/2*c^8/b^13*(-132/c/(c*x^2+b)-1/c*b^5/(c*
x^2+b)^6-4/c*b^4/(c*x^2+b)^5-12/c*b^3/(c*x^2+b)^4-30*b^2/c/(c*x^2+b)^3-1/7/
```

```
c*b^6/(c*x^2+b)^7-66/c*b/(c*x^2+b)^2)
```

Maxima [B] time = 1.03812, size = 109, normalized size = 6.81

$$-\frac{1}{14\left(c^{7}x^{28}+7\,bc^{6}x^{26}+21\,b^{2}c^{5}x^{24}+35\,b^{3}c^{4}x^{22}+35\,b^{4}c^{3}x^{20}+21\,b^{5}c^{2}x^{18}+7\,b^{6}cx^{16}+b^{7}x^{14}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2)^8,x, algorithm="maxima")
```

```
[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 21*b^2*c^5*x^24 + 35*b^3*c^4*x^22 + 35*b^4
*c^3*x^20 + 21*b^5*c^2*x^18 + 7*b^6*c*x^16 + b^7*x^14)
```

Fricas [B] time = 1.05018, size = 177, normalized size = 11.06

$$-\frac{1}{14\left(c^{7}x^{28}+7\,bc^{6}x^{26}+21\,b^{2}c^{5}x^{24}+35\,b^{3}c^{4}x^{22}+35\,b^{4}c^{3}x^{20}+21\,b^{5}c^{2}x^{18}+7\,b^{6}cx^{16}+b^{7}x^{14}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2)^8,x, algorithm="fricas")
```

```
[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 21*b^2*c^5*x^24 + 35*b^3*c^4*x^22 + 35*b^4
*c^3*x^20 + 21*b^5*c^2*x^18 + 7*b^6*c*x^16 + b^7*x^14)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x**2+b)/(c*x**4+b*x**2)**8,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12524, size = 20, normalized size = 1.25

$$-\frac{1}{14\left(cx^{4}+bx^{2}\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2)^8,x, algorithm="giac")
```

```
[Out] -1/14/(c*x^4 + b*x^2)^7
```

3.127 $\int \frac{x^2\left(b+2cx^3\right)}{\left(bx^3+cx^6\right)^8}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{21x^{21}\left(b+cx^3\right)^7}$$

```
[Out] -1/(21*x^21*(b + c*x^3)^7)
```

Rubi [A] time = 0.0253234, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {1584, 446, 74}

$$-\frac{1}{21x^{21}\left(b+cx^3\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(b + 2*c*x^3))/(b*x^3 + c*x^6)^8,x]
```

```
[Out] -1/(21*x^21*(b + c*x^3)^7)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
```

```
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\left(b+2cx^3\right)}{\left(bx^3+cx^6\right)^8}\,dx &= \int \frac{b+2cx^3}{x^{22}\left(b+cx^3\right)^8}\,dx\\ &= \frac{1}{3}\,\text{Subst}\left(\int \frac{b+2cx}{x^8(b+cx)^8}\,dx, x, x^3\right)\\ &= -\frac{1}{21x^{21}\left(b+cx^3\right)^7}\end{aligned}$$

Mathematica [A] time = 0.0348261, size = 16, normalized size = 1.

$$-\frac{1}{21x^{21}\left(b+cx^3\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(b + 2*c*x^3))/(b*x^3 + c*x^6)^8,x]
```

```
[Out] -1/(21*x^21*(b + c*x^3)^7)
```

Maple [B] time = 0.013, size = 197, normalized size = 12.3

$$-\frac{c^8}{3\,b^{13}}\left(-132\,\frac{1}{c\left(cx^3+b\right)}-\frac{b^5}{c\left(cx^3+b\right)^6}-4\,\frac{b^4}{c\left(cx^3+b\right)^5}-12\,\frac{b^3}{c\left(cx^3+b\right)^4}-30\,\frac{b^2}{c\left(cx^3+b\right)^3}-\frac{b^6}{7\,c\left(cx^3+b\right)^7}-66\,\frac{b}{c\left(cx^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*c*x^3+b)/(c*x^6+b*x^3)^8,x)
```

```
[Out] -1/3*c^8/b^13*(-132/c/(c*x^3+b)-1/c*b^5/(c*x^3+b)^6-4/c*b^4/(c*x^3+b)^5-12/
c*b^3/(c*x^3+b)^4-30*b^2/c/(c*x^3+b)^3-1/7/c*b^6/(c*x^3+b)^7-66/c*b/(c*x^3+
b)^2)-1/21/b^7/x^21-44/b^13*c^6/x^3+22/b^12*c^5/x^6-10/b^11*c^4/x^9+4/b^10*
```

```
c^3/x^12-4/3/b^9*c^2/x^15+1/3/b^8*c/x^18
```

Maxima [B] time = 1.08584, size = 109, normalized size = 6.81

$$-\frac{1}{21\left(c^{7}x^{42}+7\,bc^{6}x^{39}+21\,b^{2}c^{5}x^{36}+35\,b^{3}c^{4}x^{33}+35\,b^{4}c^{3}x^{30}+21\,b^{5}c^{2}x^{27}+7\,b^{6}cx^{24}+b^{7}x^{21}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3)^8,x, algorithm="maxima")
```

```
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 21*b^2*c^5*x^36 + 35*b^3*c^4*x^33 + 35*b^4
*c^3*x^30 + 21*b^5*c^2*x^27 + 7*b^6*c*x^24 + b^7*x^21)
```

Fricas [B] time = 1.13802, size = 177, normalized size = 11.06

$$-\frac{1}{21\left(c^{7}x^{42}+7\,bc^{6}x^{39}+21\,b^{2}c^{5}x^{36}+35\,b^{3}c^{4}x^{33}+35\,b^{4}c^{3}x^{30}+21\,b^{5}c^{2}x^{27}+7\,b^{6}cx^{24}+b^{7}x^{21}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3)^8,x, algorithm="fricas")
```

```
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 21*b^2*c^5*x^36 + 35*b^3*c^4*x^33 + 35*b^4
*c^3*x^30 + 21*b^5*c^2*x^27 + 7*b^6*c*x^24 + b^7*x^21)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*c*x**3+b)/(c*x**6+b*x**3)**8,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12139, size = 20, normalized size = 1.25

$$-\frac{1}{21\left(cx^6+bx^3\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3)^8,x, algorithm="giac")
```

```
[Out] -1/21/(c*x^6 + b*x^3)^7
```

3.128 $\int \frac{x^{-1+n}(b+2cx^n)}{\left(bx^n+cx^{2n}\right)^8}\,dx$

Optimal. Leaf size=21

$$-\frac{x^{-7n}}{7n\,(b+cx^n)^7}$$

```
[Out] -1/(7*n*x^(7*n)*(b + c*x^n)^7)
```

Rubi [A] time = 0.0324075, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1584, 446, 74}

$$-\frac{x^{-7n}}{7n\,(b+cx^n)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + n)*(b + 2*c*x^n))/(b*x^n + c*x^(2*n))^8,x]
```

```
[Out] -1/(7*n*x^(7*n)*(b + c*x^n)^7)
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
```

```
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\int \frac{x^{-1+n}(b+2cx^n)}{\left(bx^n+cx^{2n}\right)^8}\,dx = \int \frac{x^{-1-7n}(b+2cx^n)}{(b+cx^n)^8}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{b+2cx}{x^8(b+cx)^8}\,dx, x, x^n\right)}{n}$$
$$= -\frac{x^{-7n}}{7n(b+cx^n)^7}$$

Mathematica [A] time = 0.181327, size = 21, normalized size = 1.

$$-\frac{x^{-7n}}{7n(b+cx^n)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + n)*(b + 2*c*x^n))/(b*x^n + c*x^(2*n))^8,x]
```

```
[Out] -1/(7*n*x^(7*n)*(b + c*x^n)^7)
```

Maple [B] time = 0.05, size = 203, normalized size = 9.7

$$-132\,\frac{c^6}{b^{13}nx^n}+66\,\frac{c^5}{b^{12}n\,(x^n)^2}-30\,\frac{c^4}{b^{11}n\,(x^n)^3}+12\,\frac{c^3}{b^{10}n\,(x^n)^4}-4\,\frac{c^2}{b^9n\,(x^n)^5}+\frac{c}{b^8n\,(x^n)^6}-\frac{1}{7\,b^7n\,(x^n)^7}+\frac{c^7\left(924\,(x^n)^6c^6\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n)*(b+2*c*x^n)/(b*x^n+c*x^(2*n))^8,x)
```

```
[Out] -132/b^13*c^6/n/(x^n)+66/b^12*c^5/n/(x^n)^2-30/b^11*c^4/n/(x^n)^3+12/b^10*c
^3/n/(x^n)^4-4/b^9*c^2/n/(x^n)^5+1/b^8*c/n/(x^n)^6-1/7/b^7/n/(x^n)^7+1/7*c^
7*(924*(x^n)^6*c^6+6006*b*c^5*(x^n)^5+16380*b^2*c^4*(x^n)^4+24024*b^3*c^3*(
x^n)^3+20020*b^4*c^2*(x^n)^2+9009*b^5*c*x^n+1716*b^6)/b^13/n/(b+c*x^n)^7
```

Maxima [B] time = 1.21517, size = 826, normalized size = 39.33

$$-\frac{1}{105}\,b\left(\frac{360360\,c^{13}x^{13\,n}+2342340\,bc^{12}x^{12\,n}+6426420\,b^{2}c^{11}x^{11\,n}+9579570\,b^{3}c^{10}x^{10\,n}+8270262\,b^{4}c^{9}x^{9\,n}+4018014\,b^{5}\cdots}{b^{14}c^{7}nx^{14\,n}+7\,b^{15}c^{6}nx^{13\,n}+21\,b^{16}c^{5}nx^{12\,n}+35\,b^{17}c\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(b*x^n+c*x^(2*n))^8,x, algorithm="maxima")
```

```
[Out] -1/105*b*((360360*c^13*x^(13*n) + 2342340*b*c^12*x^(12*n) + 6426420*b^2*c^1
1*x^(11*n) + 9579570*b^3*c^10*x^(10*n) + 8270262*b^4*c^9*x^(9*n) + 4018014*
b^5*c^8*x^(8*n) + 934362*b^6*c^7*x^(7*n) + 45045*b^7*c^6*x^(6*n) - 5005*b^8
*c^5*x^(5*n) + 1001*b^9*c^4*x^(4*n) - 273*b^10*c^3*x^(3*n) + 91*b^11*c^2*x^
(2*n) - 35*b^12*c*x^n + 15*b^13)/(b^14*c^7*n*x^(14*n) + 7*b^15*c^6*n*x^(13*
n) + 21*b^16*c^5*n*x^(12*n) + 35*b^17*c^4*n*x^(11*n) + 35*b^18*c^3*n*x^(10*
n) + 21*b^19*c^2*n*x^(9*n) + 7*b^20*c*n*x^(8*n) + b^21*n*x^(7*n)) + 360360*
c^7*log(x)/b^15 - 360360*c^7*log((c*x^n + b)/c)/(b^15*n)) + 1/105*c*((36036
0*c^12*x^(12*n) + 2342340*b*c^11*x^(11*n) + 6426420*b^2*c^10*x^(10*n) + 957
9570*b^3*c^9*x^(9*n) + 8270262*b^4*c^8*x^(8*n) + 4018014*b^5*c^7*x^(7*n) +
934362*b^6*c^6*x^(6*n) + 45045*b^7*c^5*x^(5*n) - 5005*b^8*c^4*x^(4*n) + 100
1*b^9*c^3*x^(3*n) - 273*b^10*c^2*x^(2*n) + 91*b^11*c*x^n - 35*b^12)/(b^13*c
^7*n*x^(13*n) + 7*b^14*c^6*n*x^(12*n) + 21*b^15*c^5*n*x^(11*n) + 35*b^16*c^
4*n*x^(10*n) + 35*b^17*c^3*n*x^(9*n) + 21*b^18*c^2*n*x^(8*n) + 7*b^19*c*n*x
^(7*n) + b^20*n*x^(6*n)) + 360360*c^6*log(x)/b^14 - 360360*c^6*log((c*x^n +
 b)/c)/(b^14*n))
```

Fricas [B] time = 1.31501, size = 236, normalized size = 11.24

$$-\frac{1}{7\left(c^{7}nx^{14\,n}+7\,bc^{6}nx^{13\,n}+21\,b^{2}c^{5}nx^{12\,n}+35\,b^{3}c^{4}nx^{11\,n}+35\,b^{4}c^{3}nx^{10\,n}+21\,b^{5}c^{2}nx^{9\,n}+7\,b^{6}cnx^{8\,n}+b^{7}nx^{7\,n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(b*x^n+c*x^(2*n))^8,x, algorithm="fricas")
```

```
[Out] -1/7/(c^7*n*x^(14*n) + 7*b*c^6*n*x^(13*n) + 21*b^2*c^5*n*x^(12*n) + 35*b^3*
c^4*n*x^(11*n) + 35*b^4*c^3*n*x^(10*n) + 21*b^5*c^2*n*x^(9*n) + 7*b^6*c*n*x
^(8*n) + b^7*n*x^(7*n))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)/(b*x**n+c*x**(2*n))**8,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13892, size = 27, normalized size = 1.29

$$-\frac{1}{7\left(cx^{2n}+bx^{n}\right)^{7}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(b*x^n+c*x^(2*n))^8,x, algorithm="giac")
```

```
[Out] -1/7/((c*x^(2*n) + b*x^n)^7*n)
```

3.129 $\int (b+2cx)\left(a+bx+cx^2\right)^p\,dx$

Optimal. Leaf size=20

$$\frac{\left(a+bx+cx^2\right)^{p+1}}{p+1}$$

```
[Out] (a + b*x + c*x^2)^(1 + p)/(1 + p)
```

Rubi [A] time = 0.0054882, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {629}

$$\frac{\left(a+bx+cx^2\right)^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)*(a + b*x + c*x^2)^p,x]
```

```
[Out] (a + b*x + c*x^2)^(1 + p)/(1 + p)
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (b+2cx)\left(a+bx+cx^2\right)^p\,dx=\frac{\left(a+bx+cx^2\right)^{1+p}}{1+p}$$

Mathematica [A] time = 0.0078453, size = 19, normalized size = 0.95

$$\frac{(a+x(b+cx))^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)*(a + b*x + c*x^2)^p,x]
```

```
[Out] (a + x*(b + c*x))^(1 + p)/(1 + p)
```

Maple [A] time = 0.004, size = 21, normalized size = 1.1

$$\frac{\left(cx^2 + bx + a\right)^{1+p}}{1 + p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)*(c*x^2+b*x+a)^p,x)
```

```
[Out] (c*x^2+b*x+a)^(1+p)/(1+p)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x+a)^p,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.07846, size = 63, normalized size = 3.15

$$\frac{\left(cx^2 + bx + a\right)\left(cx^2 + bx + a\right)^p}{p + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x+a)^p,x, algorithm="fricas")
```

```
[Out] (c*x^2 + b*x + a)*(c*x^2 + b*x + a)^p/(p + 1)
```

Sympy [B] time = 51.315, size = 104, normalized size = 5.2

$$\begin{cases} \frac{a(a+bx+cx^2)^p}{p+1} + \frac{bx(a+bx+cx^2)^p}{p+1} + \frac{cx^2(a+bx+cx^2)^p}{p+1} & \text{for } p \neq -1 \\ \log\left(\frac{b}{2c} + x - \frac{\sqrt{-4ac+b^2}}{2c}\right) + \log\left(\frac{b}{2c} + x + \frac{\sqrt{-4ac+b^2}}{2c}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x**2+b*x+a)**p,x)
```

```
[Out] Piecewise((a*(a + b*x + c*x**2)**p/(p + 1) + b*x*(a + b*x + c*x**2)**p/(p +
 1) + c*x**2*(a + b*x + c*x**2)**p/(p + 1), Ne(p, -1)), (log(b/(2*c) + x -
sqrt(-4*a*c + b**2)/(2*c)) + log(b/(2*c) + x + sqrt(-4*a*c + b**2)/(2*c)),
True))
```

Giac [B] time = 1.11774, size = 72, normalized size = 3.6

$$\frac{(cx^2 + bx + a)^p cx^2 + (cx^2 + bx + a)^p bx + (cx^2 + bx + a)^p a}{p + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x+a)^p,x, algorithm="giac")
```

```
[Out] ((c*x^2 + b*x + a)^p*c*x^2 + (c*x^2 + b*x + a)^p*b*x + (c*x^2 + b*x + a)^p*
a)/(p + 1)
```

3.130 $\int x\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^p\,dx$

Optimal. Leaf size=25

$$\frac{\left(a+bx^2+cx^4\right)^{p+1}}{2(p+1)}$$

```
[Out] (a + b*x^2 + c*x^4)^(1 + p)/(2*(1 + p))
```

Rubi [A] time = 0.0191148, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1247, 629}

$$\frac{\left(a+bx^2+cx^4\right)^{p+1}}{2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (a + b*x^2 + c*x^4)^(1 + p)/(2*(1 + p))
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int x\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^p\,dx=\frac{1}{2}\,\text{Subst}\left(\int(b+2cx)\left(a+bx+cx^2\right)^p\,dx,x,x^2\right)$$
$$=\frac{\left(a+bx^2+cx^4\right)^{1+p}}{2(1+p)}$$

Mathematica [A] time = 0.0110264, size = 25, normalized size = 1.

$$\frac{\left(a+bx^2+cx^4\right)^{p+1}}{2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (a + b*x^2 + c*x^4)^(1 + p)/(2*(1 + p))
```

Maple [A] time = 0.003, size = 24, normalized size = 1.

$$\frac{\left(cx^4+bx^2+a\right)^{1+p}}{2+2p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^p,x)
```

```
[Out] 1/2*(c*x^4+b*x^2+a)^(1+p)/(1+p)
```

Maxima [A] time = 1.16291, size = 45, normalized size = 1.8

$$\frac{\left(cx^4+bx^2+a\right)\left(cx^4+bx^2+a\right)^p}{2\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] 1/2*(c*x^4 + b*x^2 + a)*(c*x^4 + b*x^2 + a)^p/(p + 1)
```

Fricas [A] time = 1.18789, size = 74, normalized size = 2.96

$$\frac{\left(cx^4 + bx^2 + a\right)\left(cx^4 + bx^2 + a\right)^p}{2\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] 1/2*(c*x^4 + b*x^2 + a)*(c*x^4 + b*x^2 + a)^p/(p + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x**2+b)*(c*x**4+b*x**2+a)**p,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.14325, size = 84, normalized size = 3.36

$$\frac{\left(cx^4 + bx^2 + a\right)^p cx^4 + \left(cx^4 + bx^2 + a\right)^p bx^2 + \left(cx^4 + bx^2 + a\right)^p a}{2\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] 1/2*((c*x^4 + b*x^2 + a)^p*c*x^4 + (c*x^4 + b*x^2 + a)^p*b*x^2 + (c*x^4 + b
*x^2 + a)^p*a)/(p + 1)
```

3.131 $\int x^2 \left(b+2cx^3\right) \left(a+bx^3+cx^6\right)^p \, dx$

Optimal. Leaf size=25

$$\frac{\left(a+bx^3+cx^6\right)^{p+1}}{3(p+1)}$$

```
[Out] (a + b*x^3 + c*x^6)^(1 + p)/(3*(1 + p))
```

Rubi [A] time = 0.0236525, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1468, 629}

$$\frac{\left(a+bx^3+cx^6\right)^{p+1}}{3(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (a + b*x^3 + c*x^6)^(1 + p)/(3*(1 + p))
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int x^2 \left(b+2cx^3\right) \left(a+bx^3+cx^6\right)^p \, dx = \frac{1}{3}\text{Subst}\left(\int (b+2cx) \left(a+bx+cx^2\right)^p \, dx, x, x^3\right)$$
$$= \frac{\left(a+bx^3+cx^6\right)^{1+p}}{3(1+p)}$$

Mathematica [A] time = 0.0123637, size = 25, normalized size = 1.

$$\frac{\left(a+bx^3+cx^6\right)^{p+1}}{3(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (a + b*x^3 + c*x^6)^(1 + p)/(3*(1 + p))
```

Maple [A] time = 0.006, size = 24, normalized size = 1.

$$\frac{\left(cx^6+bx^3+a\right)^{1+p}}{3+3p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^p,x)
```

```
[Out] 1/3*(c*x^6+b*x^3+a)^(1+p)/(1+p)
```

Maxima [A] time = 1.19067, size = 45, normalized size = 1.8

$$\frac{\left(cx^6+bx^3+a\right)\left(cx^6+bx^3+a\right)^p}{3\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")
```

```
[Out] 1/3*(c*x^6 + b*x^3 + a)*(c*x^6 + b*x^3 + a)^p/(p + 1)
```

Fricas [A] time = 1.08912, size = 74, normalized size = 2.96

$$\frac{\left(cx^6+bx^3+a\right)\left(cx^6+bx^3+a\right)^p}{3\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")
```

```
[Out] 1/3*(c*x^6 + b*x^3 + a)*(c*x^6 + b*x^3 + a)^p/(p + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*c*x**3+b)*(c*x**6+b*x**3+a)**p,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.15765, size = 84, normalized size = 3.36

$$\frac{\left(cx^6+bx^3+a\right)^p cx^6+\left(cx^6+bx^3+a\right)^p bx^3+\left(cx^6+bx^3+a\right)^p a}{3\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^p,x, algorithm="giac")
```

```
[Out] 1/3*((c*x^6 + b*x^3 + a)^p*c*x^6 + (c*x^6 + b*x^3 + a)^p*b*x^3 + (c*x^6 + b
*x^3 + a)^p*a)/(p + 1)
```

3.132 $\int x^{-1+n}(b+2cx^n)\left(a+bx^n+cx^{2n}\right)^p\,dx$

Optimal. Leaf size=27

$$\frac{\left(a+bx^n+cx^{2n}\right)^{p+1}}{n(p+1)}$$

```
[Out] (a + b*x^n + c*x^(2*n))^(1 + p)/(n*(1 + p))
```

Rubi [A] time = 0.0284039, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1468, 629}

$$\frac{\left(a+bx^n+cx^{2n}\right)^{p+1}}{n(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*(b + 2*c*x^n)*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (a + b*x^n + c*x^(2*n))^(1 + p)/(n*(1 + p))
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int x^{-1+n}\left(b+2cx^n\right)\left(a+bx^n+cx^{2n}\right)^p\,dx=\frac{\mathrm{Subst}\left(\int(b+2cx)\left(a+bx+cx^2\right)^p\,dx,x,x^n\right)}{n}$$
$$=\frac{\left(a+bx^n+cx^{2n}\right)^{1+p}}{n(1+p)}$$

Mathematica [A] time = 0.0338035, size = 26, normalized size = 0.96

$$\frac{(a+x^n\,(b+cx^n))^{p+1}}{n(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*(b + 2*c*x^n)*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (a + x^n*(b + c*x^n))^(1 + p)/(n*(1 + p))
```

Maple [A] time = 0.055, size = 40, normalized size = 1.5

$$\frac{\left(a+bx^n+c\,(x^n)^2\right)\left(a+bx^n+c\,(x^n)^2\right)^p}{n\left(1+p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n)*(b+2*c*x^n)*(a+b*x^n+c*x^(2*n))^p,x)
```

```
[Out] (a+b*x^n+c*(x^n)^2)/n/(1+p)*(a+b*x^n+c*(x^n)^2)^p
```

Maxima [A] time = 1.28956, size = 53, normalized size = 1.96

$$\frac{\left(cx^{2\,n}+bx^n+a\right)\left(cx^{2\,n}+bx^n+a\right)^p}{n\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
```

```
[Out] (c*x^(2*n) + b*x^n + a)*(c*x^(2*n) + b*x^n + a)^p/(n*(p + 1))
```

Fricas [A] time = 1.18037, size = 82, normalized size = 3.04

$$\frac{\left(cx^{2n} + bx^n + a\right)\left(cx^{2n} + bx^n + a\right)^p}{np + n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
```

```
[Out] (c*x^(2*n) + b*x^n + a)*(c*x^(2*n) + b*x^n + a)^p/(n*p + n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)*(a+b*x**n+c*x**(2*n))**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15221, size = 36, normalized size = 1.33

$$\frac{\left(cx^{2n} + bx^n + a\right)^{p+1}}{n\left(p + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] (c*x^(2*n) + b*x^n + a)^(p + 1)/(n*(p + 1))
```

3.133 $\int (b+2cx)\left(-a+bx+cx^2\right)^p \, dx$

Optimal. Leaf size=22

$$\frac{\left(-a+bx+cx^2\right)^{p+1}}{p+1}$$

```
[Out] (-a + b*x + c*x^2)^(1 + p)/(1 + p)
```

Rubi [A] time = 0.0050084, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {629}

$$\frac{\left(-a+bx+cx^2\right)^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)*(-a + b*x + c*x^2)^p,x]
```

```
[Out] (-a + b*x + c*x^2)^(1 + p)/(1 + p)
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (b+2cx)\left(-a+bx+cx^2\right)^p \, dx=\frac{\left(-a+bx+cx^2\right)^{1+p}}{1+p}$$

Mathematica [A] time = 0.011004, size = 21, normalized size = 0.95

$$\frac{(x(b+cx)-a)^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)*(-a + b*x + c*x^2)^p,x]
```

```
[Out] (-a + x*(b + c*x))^(1 + p)/(1 + p)
```

Maple [A] time = 0.003, size = 23, normalized size = 1.1

$$\frac{\left(cx^2+bx-a\right)^{1+p}}{1+p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)*(c*x^2+b*x-a)^p,x)
```

```
[Out] (c*x^2+b*x-a)^(1+p)/(1+p)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x-a)^p,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.07991, size = 63, normalized size = 2.86

$$\frac{\left(cx^2+bx-a\right)\left(cx^2+bx-a\right)^p}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x-a)^p,x, algorithm="fricas")
```

```
[Out] (c*x^2 + b*x - a)*(c*x^2 + b*x - a)^p/(p + 1)
```

Sympy [B] time = 51.5311, size = 104, normalized size = 4.73

$$\begin{cases} -\frac{a(-a+bx+cx^2)^p}{p+1} + \frac{bx(-a+bx+cx^2)^p}{p+1} + \frac{cx^2(-a+bx+cx^2)^p}{p+1} & \text{for } p \neq -1 \\ \log\left(\frac{b}{2c} + x - \frac{\sqrt{4ac+b^2}}{2c}\right) + \log\left(\frac{b}{2c} + x + \frac{\sqrt{4ac+b^2}}{2c}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x**2+b*x-a)**p,x)
```

```
[Out] Piecewise((-a*(-a + b*x + c*x**2)**p/(p + 1) + b*x*(-a + b*x + c*x**2)**p/(
p + 1) + c*x**2*(-a + b*x + c*x**2)**p/(p + 1), Ne(p, -1)), (log(b/(2*c) +
x - sqrt(4*a*c + b**2)/(2*c)) + log(b/(2*c) + x + sqrt(4*a*c + b**2)/(2*c))
, True))
```

Giac [B] time = 1.08319, size = 81, normalized size = 3.68

$$\frac{\left(cx^2 + bx - a\right)^p cx^2 + \left(cx^2 + bx - a\right)^p bx - \left(cx^2 + bx - a\right)^p a}{p + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x-a)^p,x, algorithm="giac")
```

```
[Out] ((c*x^2 + b*x - a)^p*c*x^2 + (c*x^2 + b*x - a)^p*b*x - (c*x^2 + b*x - a)^p*
a)/(p + 1)
```

3.134 $\int x\left(b+2cx^2\right)\left(-a+bx^2+cx^4\right)^p\,dx$

Optimal. Leaf size=27

$$\frac{\left(-a+bx^2+cx^4\right)^{p+1}}{2(p+1)}$$

```
[Out] (-a + b*x^2 + c*x^4)^(1 + p)/(2*(1 + p))
```

Rubi [A] time = 0.0195965, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1247, 629}

$$\frac{\left(-a+bx^2+cx^4\right)^{p+1}}{2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(b + 2*c*x^2)*(-a + b*x^2 + c*x^4)^p,x]

[Out] (-a + b*x^2 + c*x^4)^(1 + p)/(2*(1 + p))
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int x\left(b+2cx^2\right)\left(-a+bx^2+cx^4\right)^p\,dx=\frac{1}{2}\operatorname{Subst}\left(\int(b+2cx)\left(-a+bx+cx^2\right)^p\,dx,x,x^2\right)$$
$$=\frac{\left(-a+bx^2+cx^4\right)^{1+p}}{2(1+p)}$$

Mathematica [A] time = 0.0140538, size = 27, normalized size = 1.

$$\frac{\left(-a+bx^2+cx^4\right)^{p+1}}{2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(b + 2*c*x^2)*(-a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (-a + b*x^2 + c*x^4)^(1 + p)/(2*(1 + p))
```

Maple [A] time = 0.003, size = 26, normalized size = 1.

$$\frac{\left(cx^4+bx^2-a\right)^{1+p}}{2+2p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^p,x)
```

```
[Out] 1/2*(c*x^4+b*x^2-a)^(1+p)/(1+p)
```

Maxima [A] time = 1.16771, size = 50, normalized size = 1.85

$$\frac{\left(cx^4+bx^2-a\right)\left(cx^4+bx^2-a\right)^p}{2\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^p,x, algorithm="maxima")
```

```
[Out] 1/2*(c*x^4 + b*x^2 - a)*(c*x^4 + b*x^2 - a)^p/(p + 1)
```

Fricas [A] time = 1.07853, size = 74, normalized size = 2.74

$$\frac{(cx^4+bx^2-a)(cx^4+bx^2-a)^p}{2(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^p,x, algorithm="fricas")
```

```
[Out] 1/2*(c*x^4 + b*x^2 - a)*(c*x^4 + b*x^2 - a)^p/(p + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x**2+b)*(c*x**4+b*x**2-a)**p,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.15072, size = 93, normalized size = 3.44

$$\frac{(cx^4+bx^2-a)^p cx^4+(cx^4+bx^2-a)^p bx^2-(cx^4+bx^2-a)^p a}{2(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^p,x, algorithm="giac")
```

```
[Out] 1/2*((c*x^4 + b*x^2 - a)^p*c*x^4 + (c*x^4 + b*x^2 - a)^p*b*x^2 - (c*x^4 + b
*x^2 - a)^p*a)/(p + 1)
```

3.135 $\int x^2 \left(b+2cx^3\right) \left(-a+bx^3+cx^6\right)^p \, dx$

Optimal. Leaf size=27

$$\frac{\left(-a+bx^3+cx^6\right)^{p+1}}{3(p+1)}$$

```
[Out] (-a + b*x^3 + c*x^6)^(1 + p)/(3*(1 + p))
```

Rubi [A] time = 0.0247507, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1468, 629}

$$\frac{\left(-a+bx^3+cx^6\right)^{p+1}}{3(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(b + 2*c*x^3)*(-a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (-a + b*x^3 + c*x^6)^(1 + p)/(3*(1 + p))
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int x^2 \left(b+2cx^3\right) \left(-a+bx^3+cx^6\right)^p \, dx = \frac{1}{3} \text{Subst}\left(\int (b+2cx) \left(-a+bx+cx^2\right)^p \, dx, x, x^3\right)$$
$$= \frac{\left(-a+bx^3+cx^6\right)^{1+p}}{3(1+p)}$$

Mathematica [A] time = 0.0156965, size = 27, normalized size = 1.

$$\frac{\left(-a+bx^3+cx^6\right)^{p+1}}{3(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(b + 2*c*x^3)*(-a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (-a + b*x^3 + c*x^6)^(1 + p)/(3*(1 + p))
```

Maple [A] time = 0.004, size = 26, normalized size = 1.

$$\frac{\left(cx^6+bx^3-a\right)^{1+p}}{3+3p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^p,x)
```

```
[Out] 1/3*(c*x^6+b*x^3-a)^(1+p)/(1+p)
```

Maxima [A] time = 1.21518, size = 50, normalized size = 1.85

$$\frac{\left(cx^6+bx^3-a\right)\left(cx^6+bx^3-a\right)^p}{3\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^p,x, algorithm="maxima")
```

```
[Out] 1/3*(c*x^6 + b*x^3 - a)*(c*x^6 + b*x^3 - a)^p/(p + 1)
```

Fricas [A] time = 1.06204, size = 74, normalized size = 2.74

$$\frac{\left(cx^6 + bx^3 - a\right)\left(cx^6 + bx^3 - a\right)^p}{3\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^p,x, algorithm="fricas")
```

```
[Out] 1/3*(c*x^6 + b*x^3 - a)*(c*x^6 + b*x^3 - a)^p/(p + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*c*x**3+b)*(c*x**6+b*x**3-a)**p,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.14777, size = 93, normalized size = 3.44

$$\frac{\left(cx^6 + bx^3 - a\right)^p cx^6 + \left(cx^6 + bx^3 - a\right)^p bx^3 - \left(cx^6 + bx^3 - a\right)^p a}{3\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^p,x, algorithm="giac")
```

```
[Out] 1/3*((c*x^6 + b*x^3 - a)^p*c*x^6 + (c*x^6 + b*x^3 - a)^p*b*x^3 - (c*x^6 + b
*x^3 - a)^p*a)/(p + 1)
```

3.136 $\int x^{-1+n}\left(b+2cx^n\right)\left(-a+bx^n+cx^{2n}\right)^p\,dx$

Optimal. Leaf size=29

$$\frac{\left(-a+bx^n+cx^{2n}\right)^{p+1}}{n(p+1)}$$

```
[Out] (-a + b*x^n + c*x^(2*n))^(1 + p)/(n*(1 + p))
```

Rubi [A] time = 0.0282607, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1468, 629}

$$\frac{\left(-a+bx^n+cx^{2n}\right)^{p+1}}{n(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*(b + 2*c*x^n)*(-a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (-a + b*x^n + c*x^(2*n))^(1 + p)/(n*(1 + p))
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int x^{-1+n}(b+2cx^n)\left(-a+bx^n+cx^{2n}\right)^p\,dx=\frac{\text{Subst}\left(\int(b+2cx)\left(-a+bx+cx^2\right)^p\,dx,x,x^n\right)}{n}$$
$$=\frac{\left(-a+bx^n+cx^{2n}\right)^{1+p}}{n(1+p)}$$

<u>Mathematica</u> [A] time = 0.0354757, size = 28, normalized size = 0.97

$$\frac{(x^n(b+cx^n)-a)^{p+1}}{n(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*(b + 2*c*x^n)*(-a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (-a + x^n*(b + c*x^n))^(1 + p)/(n*(1 + p))
```

<u>Maple</u> [A] time = 0.057, size = 45, normalized size = 1.6

$$-\frac{\left(-c(x^n)^2-bx^n+a\right)\left(-a+bx^n+c(x^n)^2\right)^p}{n\left(1+p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n)*(b+2*c*x^n)*(-a+b*x^n+c*x^(2*n))^p,x)
```

```
[Out] -(-c*(x^n)^2-b*x^n+a)/n/(1+p)*(-a+b*x^n+c*(x^n)^2)^p
```

<u>Maxima</u> [A] time = 1.25317, size = 58, normalized size = 2.

$$\frac{\left(cx^{2n}+bx^n-a\right)\left(cx^{2n}+bx^n-a\right)^p}{n\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(-a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima"
)
```

```
[Out] (c*x^(2*n) + b*x^n - a)*(c*x^(2*n) + b*x^n - a)^p/(n*(p + 1))
```

Fricas [A] time = 1.05441, size = 82, normalized size = 2.83

$$\frac{\left(cx^{2\,n}+bx^{n}-a\right)\left(cx^{2\,n}+bx^{n}-a\right)^{p}}{np+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(-a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas"
)
```

```
[Out] (c*x^(2*n) + b*x^n - a)*(c*x^(2*n) + b*x^n - a)^p/(n*p + n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)*(-a+b*x**n+c*x**(2*n))**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18868, size = 39, normalized size = 1.34

$$\frac{\left(cx^{2\,n}+bx^{n}-a\right)^{p+1}}{n\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(-a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] (c*x^(2*n) + b*x^n - a)^(p + 1)/(n*(p + 1))
```

3.137 $\int (b+2cx)\left(bx+cx^2\right)^p\,dx$

Optimal. Leaf size=19

$$\frac{\left(bx+cx^2\right)^{p+1}}{p+1}$$

```
[Out] (b*x + c*x^2)^(1 + p)/(1 + p)
```

Rubi [A] time = 0.004136, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {629}

$$\frac{\left(bx+cx^2\right)^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)*(b*x + c*x^2)^p,x]
```

```
[Out] (b*x + c*x^2)^(1 + p)/(1 + p)
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (b+2cx)\left(bx+cx^2\right)^p\,dx=\frac{\left(bx+cx^2\right)^{1+p}}{1+p}$$

Mathematica [A] time = 0.0098645, size = 17, normalized size = 0.89

$$\frac{(x(b+cx))^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)*(b*x + c*x^2)^p,x]
```

```
[Out] (x*(b + c*x))^(1 + p)/(1 + p)
```

Maple [A] time = 0.004, size = 24, normalized size = 1.3

$$\frac{x(cx+b)\left(cx^2+bx\right)^p}{1+p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)*(c*x^2+b*x)^p,x)
```

```
[Out] x*(c*x+b)/(1+p)*(c*x^2+b*x)^p
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^p,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.0047, size = 53, normalized size = 2.79

$$\frac{\left(cx^2+bx\right)\left(cx^2+bx\right)^p}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^p,x, algorithm="fricas")
```

```
[Out] (c*x^2 + b*x)*(c*x^2 + b*x)^p/(p + 1)
```

Sympy [A] time = 0.576551, size = 46, normalized size = 2.42

$$\begin{cases} \frac{bx(bx+cx^2)^p}{p+1} + \frac{cx^2(bx+cx^2)^p}{p+1} & \text{for } p \neq -1 \\ \log(x) + \log\left(\frac{b}{c} + x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x**2+b*x)**p,x)
```

```
[Out] Piecewise((b*x*(b*x + c*x**2)**p/(p + 1) + c*x**2*(b*x + c*x**2)**p/(p + 1)
, Ne(p, -1)), (log(x) + log(b/c + x), True))
```

Giac [A] time = 1.11184, size = 50, normalized size = 2.63

$$\frac{(cx^2 + bx)^p cx^2 + (cx^2 + bx)^p bx}{p + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^p,x, algorithm="giac")
```

```
[Out] ((c*x^2 + b*x)^p*c*x^2 + (c*x^2 + b*x)^p*b*x)/(p + 1)
```

3.138 $\int x \left(b+2cx^2\right) \left(bx^2+cx^4\right)^p \, dx$

Optimal. Leaf size=24

$$\frac{\left(bx^2+cx^4\right)^{p+1}}{2(p+1)}$$

```
[Out] (b*x^2 + c*x^4)^(1 + p)/(2*(1 + p))
```

Rubi [A] time = 0.0141711, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1588}

$$\frac{\left(bx^2+cx^4\right)^{p+1}}{2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(b + 2*c*x^2)*(b*x^2 + c*x^4)^p,x]
```

```
[Out] (b*x^2 + c*x^4)^(1 + p)/(2*(1 + p))
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int x \left(b+2cx^2\right) \left(bx^2+cx^4\right)^p \, dx = \frac{\left(bx^2+cx^4\right)^{1+p}}{2(1+p)}$$

Mathematica [C] time = 0.0736556, size = 97, normalized size = 4.04

$$\frac{x^2 \left(x^2 \left(b+cx^2\right)\right)^p \left(\frac{cx^2}{b}+1\right)^{-p} \left(2c(p+1)x^2 \, _2F_1\left(-p,p+2;p+3;-\frac{cx^2}{b}\right)+b(p+2) \, _2F_1\left(-p,p+1;p+2;-\frac{cx^2}{b}\right)\right)}{2(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(b + 2*c*x^2)*(b*x^2 + c*x^4)^p,x]
```

```
[Out] (x^2*(x^2*(b + c*x^2))^p*(b*(2 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, -((
c*x^2)/b)] + 2*c*(1 + p)*x^2*Hypergeometric2F1[-p, 2 + p, 3 + p, -((c*x^2)/
b)]))/(2*(1 + p)*(2 + p)*(1 + (c*x^2)/b)^p)
```

Maple [A] time = 0.005, size = 31, normalized size = 1.3

$$\frac{x^2\left(cx^2+b\right)\left(cx^4+bx^2\right)^p}{2+2p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c*x^2+b)*(c*x^4+b*x^2)^p,x)
```

```
[Out] 1/2*(c*x^2+b)*x^2/(1+p)*(c*x^4+b*x^2)^p
```

Maxima [A] time = 1.1558, size = 47, normalized size = 1.96

$$\frac{\left(cx^4+bx^2\right)e^{\left(p\log\left(cx^2+b\right)+2\,p\log\left(x\right)\right)}}{2\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2)^p,x, algorithm="maxima")
```

```
[Out] 1/2*(c*x^4 + b*x^2)*e^(p*log(c*x^2 + b) + 2*p*log(x))/(p + 1)
```

Fricas [A] time = 1.08402, size = 63, normalized size = 2.62

$$\frac{\left(cx^4+bx^2\right)\left(cx^4+bx^2\right)^p}{2\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2)^p,x, algorithm="fricas")
```

```
[Out] 1/2*(c*x^4 + b*x^2)*(c*x^4 + b*x^2)^p/(p + 1)
```

Sympy [B] time = 20.1521, size = 85, normalized size = 3.54

$$\begin{cases} \frac{bx^2(bx^2+cx^4)^p}{2p+2} + \frac{cx^4(bx^2+cx^4)^p}{2p+2} & \text{for } p \neq -1 \\ \log(x) + \frac{\log\left(-i\sqrt{b}\sqrt{\frac{1}{c}}+x\right)}{2} + \frac{\log\left(i\sqrt{b}\sqrt{\frac{1}{c}}+x\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x**2+b)*(c*x**4+b*x**2)**p,x)
```

```
[Out] Piecewise((b*x**2*(b*x**2 + c*x**4)**p/(2*p + 2) + c*x**4*(b*x**2 + c*x**4)
**p/(2*p + 2), Ne(p, -1)), (log(x) + log(-I*sqrt(b)*sqrt(1/c) + x)/2 + log(
I*sqrt(b)*sqrt(1/c) + x)/2, True))
```

Giac [A] time = 1.14638, size = 59, normalized size = 2.46

$$\frac{(cx^4 + bx^2)^p cx^4 + (cx^4 + bx^2)^p bx^2}{2(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2)^p,x, algorithm="giac")
```

```
[Out] 1/2*((c*x^4 + b*x^2)^p*c*x^4 + (c*x^4 + b*x^2)^p*b*x^2)/(p + 1)
```

3.139 $\int x^2 \left(b+2cx^3\right) \left(bx^3+cx^6\right)^p \, dx$

Optimal. Leaf size=24

$$\frac{\left(bx^3+cx^6\right)^{p+1}}{3(p+1)}$$

```
[Out] (b*x^3 + c*x^6)^(1 + p)/(3*(1 + p))
```

Rubi [A] time = 0.0196933, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {1588}

$$\frac{\left(bx^3+cx^6\right)^{p+1}}{3(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(b + 2*c*x^3)*(b*x^3 + c*x^6)^p,x]
```

```
[Out] (b*x^3 + c*x^6)^(1 + p)/(3*(1 + p))
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^2 \left(b+2cx^3\right) \left(bx^3+cx^6\right)^p \, dx = \frac{\left(bx^3+cx^6\right)^{1+p}}{3(1+p)}$$

Mathematica [C] time = 0.0757323, size = 97, normalized size = 4.04

$$\frac{x^3 \left(x^3 \left(b+cx^3\right)\right)^p \left(\frac{cx^3}{b}+1\right)^{-p} \left(2c(p+1)x^3 \, {}_2F_1\left(-p,p+2;p+3;-\frac{cx^3}{b}\right)+b(p+2) \, {}_2F_1\left(-p,p+1;p+2;-\frac{cx^3}{b}\right)\right)}{3(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(b + 2*c*x^3)*(b*x^3 + c*x^6)^p,x]
```

```
[Out] (x^3*(x^3*(b + c*x^3))^p*(b*(2 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, -((
c*x^3)/b)] + 2*c*(1 + p)*x^3*Hypergeometric2F1[-p, 2 + p, 3 + p, -((c*x^3)/
b)]))/(3*(1 + p)*(2 + p)*(1 + (c*x^3)/b)^p)
```

Maple [A] time = 0.004, size = 31, normalized size = 1.3

$$\frac{x^3\left(cx^3+b\right)\left(cx^6+bx^3\right)^p}{3+3p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^p,x)
```

```
[Out] 1/3*(c*x^3+b)*x^3/(1+p)*(c*x^6+b*x^3)^p
```

Maxima [A] time = 1.1813, size = 47, normalized size = 1.96

$$\frac{\left(cx^6+bx^3\right)e^{\left(p\log\left(cx^3+b\right)+3p\log(x)\right)}}{3\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^p,x, algorithm="maxima")
```

```
[Out] 1/3*(c*x^6 + b*x^3)*e^(p*log(c*x^3 + b) + 3*p*log(x))/(p + 1)
```

Fricas [A] time = 1.29856, size = 63, normalized size = 2.62

$$\frac{\left(cx^6+bx^3\right)\left(cx^6+bx^3\right)^p}{3\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^p,x, algorithm="fricas")
```

```
[Out] 1/3*(c*x^6 + b*x^3)*(c*x^6 + b*x^3)^p/(p + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*c*x**3+b)*(c*x**6+b*x**3)**p,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12514, size = 59, normalized size = 2.46

$$\frac{\left(cx^6+bx^3\right)^p cx^6+\left(cx^6+bx^3\right)^p bx^3}{3\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^p,x, algorithm="giac")
```

```
[Out] 1/3*((c*x^6 + b*x^3)^p*c*x^6 + (c*x^6 + b*x^3)^p*b*x^3)/(p + 1)
```

3.140 $\int x^{-1+n}(b+2cx^n)\left(bx^n+cx^{2n}\right)^p\,dx$

Optimal. Leaf size=26

$$\frac{\left(bx^n+cx^{2n}\right)^{p+1}}{n(p+1)}$$

```
[Out] (b*x^n + c*x^(2*n))^(1 + p)/(n*(1 + p))
```

Rubi [A] time = 0.0789861, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {2034, 629}

$$\frac{\left(bx^n+cx^{2n}\right)^{p+1}}{n(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*(b + 2*c*x^n)*(b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (b*x^n + c*x^(2*n))^(1 + p)/(n*(1 + p))
```

Rule 2034

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int x^{-1+n}\left(b+2cx^n\right)\left(bx^n+cx^{2n}\right)^p\,dx=\frac{\text{Subst}\left(\int (b+2cx)\left(bx+cx^2\right)^p\,dx,x,x^n\right)}{n}$$
$$=\frac{\left(bx^n+cx^{2n}\right)^{1+p}}{n(1+p)}$$

Mathematica [C] time = 0.130146, size = 111, normalized size = 4.27

$$\frac{x^{-np}\left(x^n\left(b+cx^n\right)\right)^p\left(\frac{cx^n}{b}+1\right)^{-p}\left(b(p+2)x^{n(p+1)}\,{}_2F_1\left(-p,p+1;p+2;-\frac{cx^n}{b}\right)+2c(p+1)x^{n(p+2)}\,{}_2F_1\left(-p,p+2;p+3;-\frac{cx^n}{b}\right)\right)}{n(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*(b + 2*c*x^n)*(b*x^n + c*x^(2*n))^p,x]
```

```
[Out] ((x^n*(b + c*x^n))^p*(b*(2 + p)*x^(n*(1 + p))*Hypergeometric2F1[-p, 1 + p,
2 + p, -((c*x^n)/b)] + 2*c*(1 + p)*x^(n*(2 + p))*Hypergeometric2F1[-p, 2 +
p, 3 + p, -((c*x^n)/b)]))/(n*(1 + p)*(2 + p)*x^(n*p)*(1 + (c*x^n)/b)^p)
```

Maple [C] time = 0.098, size = 155, normalized size = 6.

$$\frac{x^n\left(b+cx^n\right)}{n\left(1+p\right)}e^{-\frac{p\left(i\pi\left(\text{csgn}\left(ix^n\left(b+cx^n\right)\right)\right)^3-i\pi\left(\text{csgn}\left(ix^n\left(b+cx^n\right)\right)\right)^2\text{csgn}\left(ix^n\right)-i\pi\left(\text{csgn}\left(ix^n\left(b+cx^n\right)\right)\right)^2\text{csgn}\left(i\left(b+cx^n\right)\right)+i\pi\,\text{csgn}\left(ix^n\left(b+cx^n\right)\right)\text{csgn}\left(ix^n\right)\text{csgn}\left(i\left(b+cx^n\right)\right)-2\ln\left(x^n\right)-2\ln\left(b+cx^n\right)\right)}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^p,x)
```

```
[Out] x^n*(b+c*x^n)/n/(1+p)*exp(-1/2*p*(I*Pi*csgn(I*x^n*(b+c*x^n))^3-I*Pi*csgn(I*
x^n*(b+c*x^n))^2*csgn(I*x^n)-I*Pi*csgn(I*x^n*(b+c*x^n))^2*csgn(I*(b+c*x^n))
+I*Pi*csgn(I*x^n*(b+c*x^n))*csgn(I*x^n)*csgn(I*(b+c*x^n))-2*ln(x^n)-2*ln(b+
c*x^n)))
```

Maxima [A] time = 1.32212, size = 54, normalized size = 2.08

$$\frac{\left(cx^{2\,n}+bx^{n}\right)e^{\left(p\log\left(cx^{n}+b\right)+p\log\left(x^{n}\right)\right)}}{n\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^p,x, algorithm="maxima")

[Out] (c*x^(2*n) + b*x^n)*e^(p*log(c*x^n + b) + p*log(x^n))/(n*(p + 1))
```

Fricas [A] time = 1.29047, size = 72, normalized size = 2.77

$$\frac{\left(cx^{2\,n}+bx^{n}\right)\left(cx^{2\,n}+bx^{n}\right)^{p}}{np+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^p,x, algorithm="fricas")

[Out] (c*x^(2*n) + b*x^n)*(c*x^(2*n) + b*x^n)^p/(n*p + n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)*(b*x**n+c*x**(2*n))**p,x)

[Out] Timed out
```

Giac [A] time = 1.14458, size = 35, normalized size = 1.35

$$\frac{\left(cx^{2\,n}+bx^{n}\right)^{p+1}}{n\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] (c*x^(2*n) + b*x^n)^(p + 1)/(n*(p + 1))
```

3.141 $\int \frac{(fx)^m(d+ex^n)}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=196

$$\frac{(fx)^{m+1}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right){}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{f(m+1)\left(b-\sqrt{b^2-4ac}\right)}+\frac{(fx)^{m+1}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right){}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)\left(\sqrt{b^2-4ac}+b\right)}$$

```
[Out] ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*(f*x)^(1 + m)*Hypergeometric2F1[1, (
1 + m)/n, (1 + m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/((b - Sqrt[b^
2 - 4*a*c])*f*(1 + m)) + ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*(f*x)^(1 +
m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2
- 4*a*c])])/((b + Sqrt[b^2 - 4*a*c])*f*(1 + m))
```

Rubi [A] time = 0.288965, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {1560, 364}

$$\frac{(fx)^{m+1}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right){}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{f(m+1)\left(b-\sqrt{b^2-4ac}\right)}+\frac{(fx)^{m+1}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right){}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)\left(\sqrt{b^2-4ac}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n)),x]
```


```
[Out] ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*(f*x)^(1 + m)*Hypergeometric2F1[1, (
1 + m)/n, (1 + m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/((b - Sqrt[b^
2 - 4*a*c])*f*(1 + m)) + ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*(f*x)^(1 +
m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2
- 4*a*c])])/((b + Sqrt[b^2 - 4*a*c])*f*(1 + m))
```


Rule 1560

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ
[q, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(fx)^m (d+ex^n)}{a+bx^n+cx^{2n}}\,dx = \int \left(\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)(fx)^m}{b-\sqrt{b^2-4ac}+2cx^n}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)(fx)^m}{b+\sqrt{b^2-4ac}+2cx^n}\right)dx$$

$$= \left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{(fx)^m}{b+\sqrt{b^2-4ac}+2cx^n}\,dx+\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{(fx)^m}{b-\sqrt{b^2-4ac}+2cx^n}\,dx$$

$$= \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)(fx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\left(b-\sqrt{b^2-4ac}\right)f(1+m)}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)(fx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{n};\frac{1+m+n}{n}\right.}{\left(b+\sqrt{b^2-4ac}\right)f(1+m)}$$

Mathematica [A] time = 0.308321, size = 158, normalized size = 0.81

$$\frac{x(fx)^m\left(\left(d\sqrt{b^2-4ac}-2ae+bd\right){}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)+\left(d\sqrt{b^2-4ac}+2ae-bd\right){}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4}}\right.\right.}{2a(m+1)\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (x*(f*x)^m*((b*d + Sqrt[b^2 - 4*a*c]*d - 2*a*e)*Hypergeometric2F1[1, (1 + m
)/n, (1 + m + n)/n, (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + (-(b*d) + Sqrt[b^
2 - 4*a*c]*d + 2*a*e)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c*
x^n)/(b + Sqrt[b^2 - 4*a*c])]))/(2*a*Sqrt[b^2 - 4*a*c]*(1 + m))
```

Maple [F] time = 0.031, size = 0, normalized size = 0.

$$\int \frac{(fx)^m (d+ex^n)}{a+bx^n+cx^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] int((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^n + d)\left(fx\right)^m}{cx^{2n} + bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate((e*x^n + d)*(f*x)^m/(c*x^(2*n) + b*x^n + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex^n + d)\left(fx\right)^m}{cx^{2n} + bx^n + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] integral((e*x^n + d)*(f*x)^m/(c*x^(2*n) + b*x^n + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(d+e*x**n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^n + d)\left(fx\right)^m}{cx^{2\,n} + bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)*(f*x)^m/(c*x^(2*n) + b*x^n + a), x)
```

3.142 $\int \frac{(fx)^m(d+ex^n)}{\left(a+bx^n+cx^{2n}\right)^2} dx$

Optimal. Leaf size=374

$$-\frac{c(fx)^{m+1}\left((m-n+1)(bd-2ae)-\frac{2aben+4acd(m-2n+1)+b^2(-d)(m-n+1)}{\sqrt{b^2-4ac}}\right){}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{af(m+1)n\left(b^2-4ac\right)\left(b-\sqrt{b^2-4ac}\right)}-\frac{c(fx)^{m+1}\left(\frac{2aben+}{}\right.}{}$$

```
[Out] ((f*x)^(1 + m)*(b^2*d - 2*a*c*d - a*b*e + c*(b*d - 2*a*e)*x^n))/(a*(b^2 - 4
*a*c)*f*n*(a + b*x^n + c*x^(2*n))) - (c*((b*d - 2*a*e)*(1 + m - n) - (4*a*c
*d*(1 + m - 2*n) - b^2*d*(1 + m - n) + 2*a*b*e*n)/Sqrt[b^2 - 4*a*c])*(f*x)^
(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x^n)/(b - Sqrt
[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b - Sqrt[b^2 - 4*a*c])*f*(1 + m)*n) - (c
*((b*d - 2*a*e)*(1 + m - n) + (4*a*c*d*(1 + m - 2*n) - b^2*d*(1 + m - n) +
2*a*b*e*n)/Sqrt[b^2 - 4*a*c])*(f*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n,
 (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b +
Sqrt[b^2 - 4*a*c])*f*(1 + m)*n)
```

Rubi [A] time = 1.38029, antiderivative size = 374, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1558, 1560, 364}

$$-\frac{c(fx)^{m+1}\left((m-n+1)(bd-2ae)-\frac{2aben+4acd(m-2n+1)+b^2(-d)(m-n+1)}{\sqrt{b^2-4ac}}\right){}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{af(m+1)n\left(b^2-4ac\right)\left(b-\sqrt{b^2-4ac}\right)}-\frac{c(fx)^{m+1}\left(\frac{2aben+}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n))^2,x]
```


```
[Out] ((f*x)^(1 + m)*(b^2*d - 2*a*c*d - a*b*e + c*(b*d - 2*a*e)*x^n))/(a*(b^2 - 4
*a*c)*f*n*(a + b*x^n + c*x^(2*n))) - (c*((b*d - 2*a*e)*(1 + m - n) - (4*a*c
*d*(1 + m - 2*n) - b^2*d*(1 + m - n) + 2*a*b*e*n)/Sqrt[b^2 - 4*a*c])*(f*x)^
(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x^n)/(b - Sqrt
[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b - Sqrt[b^2 - 4*a*c])*f*(1 + m)*n) - (c
*((b*d - 2*a*e)*(1 + m - n) + (4*a*c*d*(1 + m - 2*n) - b^2*d*(1 + m - n) +
2*a*b*e*n)/Sqrt[b^2 - 4*a*c])*(f*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n,
 (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b +
Sqrt[b^2 - 4*a*c])*f*(1 + m)*n)
```

Rule 1558

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(a + b*x^n + c*x^(
2*n))^(p + 1)*(d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^n))/(a*f*n*(p +
1)*(b^2 - 4*a*c)), x] + Dist[1/(a*n*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a
+ b*x^n + c*x^(2*n))^(p + 1)*Simp[d*(b^2*(m + n*(p + 1) + 1) - 2*a*c*(m + 2
*n*(p + 1) + 1)) - a*b*e*(m + 1) + (m + n*(2*p + 3) + 1)*(b*d - 2*a*e)*c*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && ILtQ[p + 1, 0]
```

Rule 1560

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ
[q, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(fx)^m (d+ex^n)}{\left(a+bx^n+cx^{2n}\right)^2}\,dx = \frac{(fx)^{1+m}\left(b^2d-2acd-abe+c(bd-2ae)x^n\right)}{a\left(b^2-4ac\right)fn\left(a+bx^n+cx^{2n}\right)} - \frac{\int \frac{(fx)^m\left(-abe(1+m)-2acd(1+m-2n)+b^2d(1+m-n)+c(bd-2ae)\cdots\right)}{a+bx^n+cx^{2n}}}{a\left(b^2-4ac\right)n}$$

$$= \frac{(fx)^{1+m}\left(b^2d-2acd-abe+c(bd-2ae)x^n\right)}{a\left(b^2-4ac\right)fn\left(a+bx^n+cx^{2n}\right)} - \frac{\int \left(\frac{\left(c(bd-2ae)(1+m-n)+\frac{c\left(b^2d-4acd+b^2dm-4acdm-b^2dn+8acdn-2a\cdots\right)}{\sqrt{b^2-4ac}}\right)\cdots}{b-\sqrt{b^2-4ac}+2cx^n}\cdots\right)}{\cdots}$$

$$= \frac{(fx)^{1+m}\left(b^2d-2acd-abe+c(bd-2ae)x^n\right)}{a\left(b^2-4ac\right)fn\left(a+bx^n+cx^{2n}\right)} - \frac{\left(c\left((bd-2ae)(1+m-n)-\frac{4acd(1+m-2n)-b^2d(1+m\cdots}{\sqrt{b^2-4ac}}\cdots\right)\cdots\right)}{a\left(b^2-4ac\right)n}$$

$$= \frac{(fx)^{1+m}\left(b^2d-2acd-abe+c(bd-2ae)x^n\right)}{a\left(b^2-4ac\right)fn\left(a+bx^n+cx^{2n}\right)} - \frac{c\left((bd-2ae)(1+m-n)-\frac{4acd(1+m-2n)-b^2d(1+m-\cdots}{\sqrt{b^2-4ac}}\cdots\right)}{a\left(b^2-4ac\right)\left(b-\sqrt{\cdots}\right)\cdots}$$

Mathematica [B] time = 4.30352, size = 2305, normalized size = 6.16

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n))^2,x]
```

```
[Out] -((x*(f*x)^m*(-((b^2 - 4*a*c)*(-b + Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*
c])*(1 + m)*(1 + m + n)*(b^2*d + b*(-(a*e) + c*d*x^n) - 2*a*c*(d + e*x^n)))
 + 2*b^2*c*Sqrt[b^2 - 4*a*c]*d*(1 + m + n)*(a + x^n*(b + c*x^n))*(-((b + Sq
rt[b^2 - 4*a*c])*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (2*c*x^n)/(
-b + Sqrt[b^2 - 4*a*c])]) + (b - Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, (1
 + m)/n, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]) - 4*a*c^2*Sqrt
[b^2 - 4*a*c]*d*(1 + m + n)*(a + x^n*(b + c*x^n))*(-((b + Sqrt[b^2 - 4*a*c]
)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (2*c*x^n)/(-b + Sqrt[b^2 -
 4*a*c])]) + (b - Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, (1 + m)/n, (1 + m
 + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]) - 2*a*b*c*Sqrt[b^2 - 4*a*c]*e
*(1 + m + n)*(a + x^n*(b + c*x^n))*(-((b + Sqrt[b^2 - 4*a*c])*Hypergeometri
c2F1[1, (1 + m)/n, (1 + m + n)/n, (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]) + (b
 - Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c*
x^n)/(b + Sqrt[b^2 - 4*a*c])]) + 2*b^2*c*Sqrt[b^2 - 4*a*c]*d*m*(1 + m + n)*
(a + x^n*(b + c*x^n))*(-((b + Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, (1 +
m)/n, (1 + m + n)/n, (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]) + (b - Sqrt[b^2 -
 4*a*c])*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x^n)/(b + Sqr
t[b^2 - 4*a*c])]) - 4*a*c^2*Sqrt[b^2 - 4*a*c]*d*m*(1 + m + n)*(a + x^n*(b +
 c*x^n))*(-((b + Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, (1 + m)/n, (1 + m
+ n)/n, (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]) + (b - Sqrt[b^2 - 4*a*c])*Hype
rgeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
])]) - 2*a*b*c*Sqrt[b^2 - 4*a*c]*e*m*(1 + m + n)*(a + x^n*(b + c*x^n))*(-((
b + Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (2*c*
x^n)/(-b + Sqrt[b^2 - 4*a*c])]) + (b - Sqrt[b^2 - 4*a*c])*Hypergeometric2F1
[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]) - 2*b^2*
c*Sqrt[b^2 - 4*a*c]*d*n*(1 + m + n)*(a + x^n*(b + c*x^n))*(-((b + Sqrt[b^2
- 4*a*c])*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (2*c*x^n)/(-b + Sq
rt[b^2 - 4*a*c])]) + (b - Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, (1 + m)/n
, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]) + 8*a*c^2*Sqrt[b^2 -
4*a*c]*d*n*(1 + m + n)*(a + x^n*(b + c*x^n))*(-((b + Sqrt[b^2 - 4*a*c])*Hyp
ergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*
c])]) + (b - Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)
/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]) + 2*b*c^2*Sqrt[b^2 - 4*a*c]*d*(1 +
 m)*x^n*(a + x^n*(b + c*x^n))*(-((b + Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[
1, (1 + m + n)/n, 2 + (1 + m)/n, (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]) + (b
- Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, (1 + m + n)/n, 2 + (1 + m)/n, (-2
```

```
*c*x^n)/(b + Sqrt[b^2 - 4*a*c])]) - 4*a*c^2*Sqrt[b^2 - 4*a*c]*e*(1 + m)*x^n
*(a + x^n*(b + c*x^n))*(-((b + Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, (1 +
 m + n)/n, 2 + (1 + m)/n, (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]) + (b - Sqrt[
b^2 - 4*a*c])*Hypergeometric2F1[1, (1 + m + n)/n, 2 + (1 + m)/n, (-2*c*x^n)
/(b + Sqrt[b^2 - 4*a*c])]) + 2*b*c^2*Sqrt[b^2 - 4*a*c]*d*m*(1 + m)*x^n*(a +
 x^n*(b + c*x^n))*(-((b + Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, (1 + m +
n)/n, 2 + (1 + m)/n, (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]) + (b - Sqrt[b^2 -
 4*a*c])*Hypergeometric2F1[1, (1 + m + n)/n, 2 + (1 + m)/n, (-2*c*x^n)/(b +
 Sqrt[b^2 - 4*a*c])]) - 4*a*c^2*Sqrt[b^2 - 4*a*c]*e*m*(1 + m)*x^n*(a + x^n*
(b + c*x^n))*(-((b + Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, (1 + m + n)/n,
 2 + (1 + m)/n, (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]) + (b - Sqrt[b^2 - 4*a*
c])*Hypergeometric2F1[1, (1 + m + n)/n, 2 + (1 + m)/n, (-2*c*x^n)/(b + Sqrt
[b^2 - 4*a*c])]) - 2*b*c^2*Sqrt[b^2 - 4*a*c]*d*(1 + m)*n*x^n*(a + x^n*(b +
c*x^n))*(-((b + Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, (1 + m + n)/n, 2 +
(1 + m)/n, (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]) + (b - Sqrt[b^2 - 4*a*c])*H
ypergeometric2F1[1, (1 + m + n)/n, 2 + (1 + m)/n, (-2*c*x^n)/(b + Sqrt[b^2
- 4*a*c])]) + 4*a*c^2*Sqrt[b^2 - 4*a*c]*e*(1 + m)*n*x^n*(a + x^n*(b + c*x^n
))*(-((b + Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, (1 + m + n)/n, 2 + (1 +
m)/n, (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])]) + (b - Sqrt[b^2 - 4*a*c])*Hyperg
eometric2F1[1, (1 + m + n)/n, 2 + (1 + m)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c])]))))/(a*(b^2 - 4*a*c)^2*(-b + Sqrt[b^2 - 4*a*c])*(b + Sqrt[b^2 - 4*a*c]
)*(1 + m)*n*(1 + m + n)*(a + x^n*(b + c*x^n))))
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int \frac{\left(fx\right)^{m}\left(d+ex^{n}\right)}{\left(a+bx^{n}+cx^{2\,n}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x)
```

```
[Out] int((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(b^{2}df^{m}-\left(2\,cdf^{m}+bef^{m}\right)a\right)xx^{m}+\left(bcdf^{m}-2\,acef^{m}\right)xe^{(m\log(x)+n\log(x))}}{a^{2}b^{2}n-4\,a^{3}cn+\left(ab^{2}cn-4\,a^{2}c^{2}n\right)x^{2\,n}+\left(ab^{3}n-4\,a^{2}bcn\right)x^{n}}-\int \frac{\left(b^{2}df^{m}(m-n+1)-\left(2\,cdf^{m}(m-2\,n+1)\right.\right.}{a^{2}b^{2}n-4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x, algorithm="maxima")
```

```
[Out] ((b^2*d*f^m - (2*c*d*f^m + b*e*f^m)*a)*x*x^m + (b*c*d*f^m - 2*a*c*e*f^m)*x*
e^(m*log(x) + n*log(x)))/(a^2*b^2*n - 4*a^3*c*n + (a*b^2*c*n - 4*a^2*c^2*n)
*x^(2*n) + (a*b^3*n - 4*a^2*b*c*n)*x^n) - integrate(((b^2*d*f^m*(m - n + 1)
 - (2*c*d*f^m*(m - 2*n + 1) + b*e*f^m*(m + 1))*a)*x^m + (b*c*d*f^m*(m - n +
 1) - 2*a*c*e*f^m*(m - n + 1))*e^(m*log(x) + n*log(x)))/(a^2*b^2*n - 4*a^3*
c*n + (a*b^2*c*n - 4*a^2*c^2*n)*x^(2*n) + (a*b^3*n - 4*a^2*b*c*n)*x^n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex^n+d)\left(fx\right)^m}{c^2x^{4\,n}+b^2x^{2\,n}+2\,abx^n+a^2+2\,(bcx^n+ac)x^{2\,n}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x, algorithm="fricas")
```

```
[Out] integral((e*x^n + d)*(f*x)^m/(c^2*x^(4*n) + b^2*x^(2*n) + 2*a*b*x^n + a^2 +
 2*(b*c*x^n + a*c)*x^(2*n)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(d+e*x**n)/(a+b*x**n+c*x**(2*n))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^n+d)\left(fx\right)^m}{\left(cx^{2\,n}+bx^n+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)*(f*x)^m/(c*x^(2*n) + b*x^n + a)^2, x)
```

3.143 $\int \frac{(fx)^m(d+ex^n)}{\left(a+bx^n+cx^{2n}\right)^3}\,dx$

Optimal. Leaf size=816

$$-\frac{c\left(\left(-d(m-2n+1)b^3+ae(m+1)b^2+2acd(2m-7n+2)b-4a^2ce(m-3n+1)\right)(m-n+1)+\frac{-d\left(m^2+(2-3n)m+2n^2-3n+1\right)}{}\right.}{2a^2\left(b^2-4\right.}$$

```
[Out] ((f*x)^(1 + m)*(b^2*d - 2*a*c*d - a*b*e + c*(b*d - 2*a*e)*x^n))/(2*a*(b^2 -
 4*a*c)*f*n*(a + b*x^n + c*x^(2*n))^2) + ((f*x)^(1 + m)*((b^2 - 2*a*c)*(a*b
*e*(1 + m) + 2*a*c*d*(1 + m - 4*n) - b^2*d*(1 + m - 2*n)) + a*b*c*(b*d - 2*
a*e)*(1 + m - 3*n) + c*(a*b^2*e*(1 + m) + 2*a*b*c*d*(2 + 2*m - 7*n) - 4*a^2
*c*e*(1 + m - 3*n) - b^3*d*(1 + m - 2*n))*x^n))/(2*a^2*(b^2 - 4*a*c)^2*f*n^
2*(a + b*x^n + c*x^(2*n))) - (c*((a*b^2*e*(1 + m) + 2*a*b*c*d*(2 + 2*m - 7*
n) - 4*a^2*c*e*(1 + m - 3*n) - b^3*d*(1 + m - 2*n))*(1 + m - n) + (a*b^3*e*
(1 + m)*(1 + m - n) - 4*a^2*b*c*e*(1 + m^2 + m*(2 - n) - n - 3*n^2) - b^4*d
*(1 + m^2 + m*(2 - 3*n) - 3*n + 2*n^2) + 6*a*b^2*c*d*(1 + m^2 + m*(2 - 4*n)
 - 4*n + 3*n^2) - 8*a^2*c^2*d*(1 + m^2 + m*(2 - 6*n) - 6*n + 8*n^2))/Sqrt[b
^2 - 4*a*c])*(f*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (
-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^2*(b - Sqrt[b^2 -
4*a*c])*f*(1 + m)*n^2) - (c*((a*b^2*e*(1 + m) + 2*a*b*c*d*(2 + 2*m - 7*n) -
 4*a^2*c*e*(1 + m - 3*n) - b^3*d*(1 + m - 2*n))*(1 + m - n) - (a*b^3*e*(1 +
 m)*(1 + m - n) - 4*a^2*b*c*e*(1 + m^2 + m*(2 - n) - n - 3*n^2) - b^4*d*(1
+ m^2 + m*(2 - 3*n) - 3*n + 2*n^2) + 6*a*b^2*c*d*(1 + m^2 + m*(2 - 4*n) - 4
*n + 3*n^2) - 8*a^2*c^2*d*(1 + m^2 + m*(2 - 6*n) - 6*n + 8*n^2))/Sqrt[b^2 -
 4*a*c])*(f*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c
*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^2*(b + Sqrt[b^2 - 4*a*
c])*f*(1 + m)*n^2)
```

Rubi [A] time = 4.55205, antiderivative size = 816, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1558, 1560, 364}

$$-\frac{c\left(\left(-d(m-2n+1)b^3+ae(m+1)b^2+2acd(2m-7n+2)b-4a^2ce(m-3n+1)\right)(m-n+1)+\frac{-d\left(m^2+(2-3n)m+2n^2-3n+1\right)}{}\right.}{2a^2\left(b^2-4\right.}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n))^3,x]
```

```
[Out] ((f*x)^(1 + m)*(b^2*d - 2*a*c*d - a*b*e + c*(b*d - 2*a*e)*x^n))/(2*a*(b^2 -
 4*a*c)*f*n*(a + b*x^n + c*x^(2*n))^2) + ((f*x)^(1 + m)*((b^2 - 2*a*c)*(a*b
*e*(1 + m) + 2*a*c*d*(1 + m - 4*n) - b^2*d*(1 + m - 2*n)) + a*b*c*(b*d - 2*
a*e)*(1 + m - 3*n) + c*(a*b^2*e*(1 + m) + 2*a*b*c*d*(2 + 2*m - 7*n) - 4*a^2
*c*e*(1 + m - 3*n) - b^3*d*(1 + m - 2*n))*x^n))/(2*a^2*(b^2 - 4*a*c)^2*f*n^
2*(a + b*x^n + c*x^(2*n))) - (c*((a*b^2*e*(1 + m) + 2*a*b*c*d*(2 + 2*m - 7*
n) - 4*a^2*c*e*(1 + m - 3*n) - b^3*d*(1 + m - 2*n))*(1 + m - n) + (a*b^3*e*
(1 + m)*(1 + m - n) - 4*a^2*b*c*e*(1 + m^2 + m*(2 - n) - n - 3*n^2) - b^4*d
*(1 + m^2 + m*(2 - 3*n) - 3*n + 2*n^2) + 6*a*b^2*c*d*(1 + m^2 + m*(2 - 4*n)
 - 4*n + 3*n^2) - 8*a^2*c^2*d*(1 + m^2 + m*(2 - 6*n) - 6*n + 8*n^2))/Sqrt[b
^2 - 4*a*c])*(f*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (
-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^2*(b - Sqrt[b^2 -
4*a*c])*f*(1 + m)*n^2) - (c*((a*b^2*e*(1 + m) + 2*a*b*c*d*(2 + 2*m - 7*n) -
 4*a^2*c*e*(1 + m - 3*n) - b^3*d*(1 + m - 2*n))*(1 + m - n) - (a*b^3*e*(1 +
 m)*(1 + m - n) - 4*a^2*b*c*e*(1 + m^2 + m*(2 - n) - n - 3*n^2) - b^4*d*(1
+ m^2 + m*(2 - 3*n) - 3*n + 2*n^2) + 6*a*b^2*c*d*(1 + m^2 + m*(2 - 4*n) - 4
*n + 3*n^2) - 8*a^2*c^2*d*(1 + m^2 + m*(2 - 6*n) - 6*n + 8*n^2))/Sqrt[b^2 -
 4*a*c])*(f*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c
*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^2*(b + Sqrt[b^2 - 4*a*
c])*f*(1 + m)*n^2)
```

Rule 1558

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(a + b*x^n + c*x^(
2*n))^(p + 1)*(d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^n))/(a*f*n*(p +
1)*(b^2 - 4*a*c)), x] + Dist[1/(a*n*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a
+ b*x^n + c*x^(2*n))^(p + 1)*Simp[d*(b^2*(m + n*(p + 1) + 1) - 2*a*c*(m + 2
*n*(p + 1) + 1)) - a*b*e*(m + 1) + (m + n*(2*p + 3) + 1)*(b*d - 2*a*e)*c*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && ILtQ[p + 1, 0]
```

Rule 1560

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ
[q, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{(fx)^m(d+ex^n)}{(a+bx^n+cx^{2n})^3}\,dx = \frac{(fx)^{1+m}\left(b^2d-2acd-abe+c(bd-2ae)x^n\right)}{2a\left(b^2-4ac\right)fn\left(a+bx^n+cx^{2n}\right)^2} - \frac{\int \frac{(fx)^m\left(-abe(1+m)-2acd(1+m-4n)+b^2d(1+m-2n)+c(bd-2\right.}{\left(a+bx^n+cx^{2n}\right)^2}}{2a\left(b^2-4ac\right)n}$$

$$= \frac{(fx)^{1+m}\left(b^2d-2acd-abe+c(bd-2ae)x^n\right)}{2a\left(b^2-4ac\right)fn\left(a+bx^n+cx^{2n}\right)^2} + \frac{(fx)^{1+m}\left(\left(b^2-2ac\right)\left(abe(1+m)+2acd(1+m\right.\right.}{}$$

$$= \frac{(fx)^{1+m}\left(b^2d-2acd-abe+c(bd-2ae)x^n\right)}{2a\left(b^2-4ac\right)fn\left(a+bx^n+cx^{2n}\right)^2} + \frac{(fx)^{1+m}\left(\left(b^2-2ac\right)\left(abe(1+m)+2acd(1+m\right.\right.}{}$$

$$= \frac{(fx)^{1+m}\left(b^2d-2acd-abe+c(bd-2ae)x^n\right)}{2a\left(b^2-4ac\right)fn\left(a+bx^n+cx^{2n}\right)^2} + \frac{(fx)^{1+m}\left(\left(b^2-2ac\right)\left(abe(1+m)+2acd(1+m\right.\right.}{}$$

$$= \frac{(fx)^{1+m}\left(b^2d-2acd-abe+c(bd-2ae)x^n\right)}{2a\left(b^2-4ac\right)fn\left(a+bx^n+cx^{2n}\right)^2} + \frac{(fx)^{1+m}\left(\left(b^2-2ac\right)\left(abe(1+m)+2acd(1+m\right.\right.}{}$$

Mathematica [B] time = 7.35683, size = 13117, normalized size = 16.07

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n))^3,x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.055, size = 0, normalized size = 0.

$$\int \frac{\left(fx\right)^{m} \left(d+e x^{n}\right)}{\left(a+b x^{n}+c x^{2 n}\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x)
```

```
[Out] int((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x, algorithm="maxima")
```

```
[Out] -1/2*((a*b^4*d*f^m*(m - 3*n + 1) + 2*(b*c*e*f^m*(2*m - 5*n + 2) + 2*c^2*d*f
^m*(m - 6*n + 1))*a^3 - (b^2*c*d*f^m*(5*m - 21*n + 5) + b^3*e*f^m*(m - n +
1))*a^2)*x*x^m + (b^3*c^2*d*f^m*(m - 2*n + 1) + 4*a^2*c^3*e*f^m*(m - 3*n +
1) - (2*b*c^3*d*f^m*(2*m - 7*n + 2) + b^2*c^2*e*f^m*(m + 1))*a)*x*e^(m*log(
x) + 3*n*log(x)) + (2*b^4*c*d*f^m*(m - 2*n + 1) + 2*(b*c^2*e*f^m*(4*m - 9*n
 + 4) + 2*c^3*d*f^m*(m - 4*n + 1))*a^2 - (b^2*c^2*d*f^m*(9*m - 29*n + 9) +
2*b^3*c*e*f^m*(m + 1))*a)*x*e^(m*log(x) + 2*n*log(x)) + (b^5*d*f^m*(m - 2*n
 + 1) + 4*a^3*c^2*e*f^m*(m - 5*n + 1) + (b^2*c*e*f^m*(3*m - 4*n + 3) + 2*b*
c^2*d*f^m*n)*a^2 - (4*b^3*c*d*f^m*(m - 3*n + 1) + b^4*e*f^m*(m + 1))*a)*x*e
^(m*log(x) + n*log(x)))/(a^4*b^4*n^2 - 8*a^5*b^2*c*n^2 + 16*a^6*c^2*n^2 + (
a^2*b^4*c^2*n^2 - 8*a^3*b^2*c^3*n^2 + 16*a^4*c^4*n^2)*x^(4*n) + 2*(a^2*b^5*
c*n^2 - 8*a^3*b^3*c^2*n^2 + 16*a^4*b*c^3*n^2)*x^(3*n) + (a^2*b^6*n^2 - 6*a^
3*b^4*c*n^2 + 32*a^5*c^3*n^2)*x^(2*n) + 2*(a^3*b^5*n^2 - 8*a^4*b^3*c*n^2 +
16*a^5*b*c^2*n^2)*x^n) + integrate(1/2*(((m^2 - m*(3*n - 2) + 2*n^2 - 3*n +
 1)*b^4*d*f^m + 2*(2*(m^2 - 2*m*(3*n - 1) + 8*n^2 - 6*n + 1)*c^2*d*f^m + (2
*m^2 - m*(5*n - 4) - 5*n + 2)*b*c*e*f^m)*a^2 - ((5*m^2 - m*(21*n - 10) + 16
*n^2 - 21*n + 5)*b^2*c*d*f^m + (m^2 - m*(n - 2) - n + 1)*b^3*e*f^m)*a)*x^m
+ ((m^2 - m*(3*n - 2) + 2*n^2 - 3*n + 1)*b^3*c*d*f^m + 4*(m^2 - 2*m*(2*n -
1) + 3*n^2 - 4*n + 1)*a^2*c^2*e*f^m - (2*(2*m^2 - m*(9*n - 4) + 7*n^2 - 9*n
 + 2)*b*c^2*d*f^m + (m^2 - m*(n - 2) - n + 1)*b^2*c*e*f^m)*a)*e^(m*log(x) +
 n*log(x)))/(a^3*b^4*n^2 - 8*a^4*b^2*c*n^2 + 16*a^5*c^2*n^2 + (a^2*b^4*c*n^
2 - 8*a^3*b^2*c^2*n^2 + 16*a^4*c^3*n^2)*x^(2*n) + (a^2*b^5*n^2 - 8*a^3*b^3*
```

```
c*n^2 + 16*a^4*b*c^2*n^2)*x^n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex^n+d)\left(fx\right)^m}{c^3x^{6n}+b^3x^{3n}+3\,ab^2x^{2n}+3\,a^2bx^n+a^3+3\left(bc^2x^n+ac^2\right)x^{4n}+3\left(b^2cx^{2n}+2\,abcx^n+a^2c\right)x^{2n}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x, algorithm="fricas")
```

```
[Out] integral((e*x^n + d)*(f*x)^m/(c^3*x^(6*n) + b^3*x^(3*n) + 3*a*b^2*x^(2*n) +
 3*a^2*b*x^n + a^3 + 3*(b*c^2*x^n + a*c^2)*x^(4*n) + 3*(b^2*c*x^(2*n) + 2*a
*b*c*x^n + a^2*c)*x^(2*n)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(d+e*x**n)/(a+b*x**n+c*x**(2*n))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^n+d)\left(fx\right)^m}{\left(cx^{2n}+bx^n+a\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)*(f*x)^m/(c*x^(2*n) + b*x^n + a)^3, x)
```

3.144 $\int \frac{\sqrt[3]{c}-2 \sqrt[3]{d} \sqrt[3]{x}}{c \sqrt[3]{d}x^{2/3}-c^{2/3}d^{2/3}x+\sqrt[3]{c}dx^{4/3}} dx$

Optimal. Leaf size=47

$$-\frac{3 \log \left(c^{2/3}-\sqrt[3]{c} \sqrt[3]{d} \sqrt[3]{x}+d^{2/3} x^{2/3}\right)}{\sqrt[3]{c} d^{2/3}}$$

```
[Out] (-3*Log[c^(2/3) - c^(1/3)*d^(1/3)*x^(1/3) + d^(2/3)*x^(2/3)])/(c^(1/3)*d^(2
/3))
```

Rubi [A] time = 0.062168, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 59, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {1594, 1468, 628}

$$-\frac{3 \log \left(c^{2/3}-\sqrt[3]{c} \sqrt[3]{d} \sqrt[3]{x}+d^{2/3} x^{2/3}\right)}{\sqrt[3]{c} d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c^(1/3) - 2*d^(1/3)*x^(1/3))/(c*d^(1/3)*x^(2/3) - c^(2/3)*d^(2/3)*x +
c^(1/3)*d*x^(4/3)),x]
```

```
[Out] (-3*Log[c^(2/3) - c^(1/3)*d^(1/3)*x^(1/3) + d^(2/3)*x^(2/3)])/(c^(1/3)*d^(2
/3))
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1468

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}\sqrt[3]{x}}{c\sqrt[3]{d}x^{2/3}-c^{2/3}d^{2/3}x+\sqrt[3]{c}dx^{4/3}}\,dx &= \int \frac{\sqrt[3]{c}-2\sqrt[3]{d}\sqrt[3]{x}}{\left(c\sqrt[3]{d}-c^{2/3}d^{2/3}\sqrt[3]{x}+\sqrt[3]{c}dx^{2/3}\right)x^{2/3}}\,dx\\ &= 3\,\text{Subst}\left(\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{c\sqrt[3]{d}-c^{2/3}d^{2/3}x+\sqrt[3]{c}dx^2}\,dx,x,\sqrt[3]{x}\right)\\ &= -\frac{3\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{x}+d^{2/3}x^{2/3}\right)}{\sqrt[3]{c}d^{2/3}}\end{aligned}$$

Mathematica [A] time = 0.0198506, size = 47, normalized size = 1.

$$-\frac{3\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{x}+d^{2/3}x^{2/3}\right)}{\sqrt[3]{c}d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c^(1/3) - 2*d^(1/3)*x^(1/3))/(c*d^(1/3)*x^(2/3) - c^(2/3)*d^(2/3
)*x + c^(1/3)*d*x^(4/3)),x]
```

```
[Out] (-3*Log[c^(2/3) - c^(1/3)*d^(1/3)*x^(1/3) + d^(2/3)*x^(2/3)])/(c^(1/3)*d^(2
/3))
```

Maple [A] time = 0.003, size = 36, normalized size = 0.8

$$-3\,\frac{\ln\left(c^{2/3}d^{2/3}\sqrt[3]{x}-\sqrt[3]{c}x^{2/3}d-c\sqrt[3]{d}\right)}{d^{2/3}\sqrt[3]{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^(1/3)-2*d^(1/3)*x^(1/3))/(c*d^(1/3)*x^(2/3)-c^(2/3)*d^(2/3)*x+c^(1/3
)*d*x^(4/3)),x)
```

```
[Out] -3/d^(2/3)/c^(1/3)*ln(c^(2/3)*d^(2/3)*x^(1/3)-c^(1/3)*x^(2/3)*d-c*d^(1/3))
```

Maxima [A] time = 1.06346, size = 46, normalized size = 0.98

$$-\frac{3\log\left(c^{\frac{1}{3}}dx^{\frac{2}{3}}-c^{\frac{2}{3}}d^{\frac{2}{3}}x^{\frac{1}{3}}+cd^{\frac{1}{3}}\right)}{c^{\frac{1}{3}}d^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^(1/3)-2*d^(1/3)*x^(1/3))/(c*d^(1/3)*x^(2/3)-c^(2/3)*d^(2/3)*x+
c^(1/3)*d*x^(4/3)),x, algorithm="maxima")
```

```
[Out] -3*log(c^(1/3)*d*x^(2/3) - c^(2/3)*d^(2/3)*x^(1/3) + c*d^(1/3))/(c^(1/3)*d^
(2/3))
```

Fricas [A] time = 1.38533, size = 109, normalized size = 2.32

$$-\frac{3\log\left(dx^{\frac{2}{3}}-c^{\frac{1}{3}}d^{\frac{2}{3}}x^{\frac{1}{3}}+c^{\frac{2}{3}}d^{\frac{1}{3}}\right)}{c^{\frac{1}{3}}d^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^(1/3)-2*d^(1/3)*x^(1/3))/(c*d^(1/3)*x^(2/3)-c^(2/3)*d^(2/3)*x+
c^(1/3)*d*x^(4/3)),x, algorithm="fricas")
```

```
[Out] -3*log(d*x^(2/3) - c^(1/3)*d^(2/3)*x^(1/3) + c^(2/3)*d^(1/3))/(c^(1/3)*d^(2
/3))
```

Sympy [C] time = 7.14819, size = 126, normalized size = 2.68

$$-\frac{3\log\left(-\frac{\sqrt[3]{c}}{2\sqrt[3]{d}}+\sqrt[3]{x}-\frac{\sqrt{3}i\sqrt{c^{\frac{4}{3}}}\sqrt{d^{\frac{4}{3}}}}{2\sqrt[3]{cd}}\right)}{\sqrt[3]{c}d^{\frac{2}{3}}}-\frac{3\log\left(-\frac{\sqrt[3]{c}}{2\sqrt[3]{d}}+\sqrt[3]{x}+\frac{\sqrt{3}i\sqrt{c^{\frac{4}{3}}}\sqrt{d^{\frac{4}{3}}}}{2\sqrt[3]{cd}}\right)}{\sqrt[3]{c}d^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**(1/3)-2*d**(1/3)*x**(1/3))/(c*d**(1/3)*x**(2/3)-c**(2/3)*d**(
2/3)*x+c**(1/3)*d*x**(4/3)),x)
```

```
[Out] -3*log(-c**(1/3)/(2*d**(1/3)) + x**(1/3) - sqrt(3)*I*sqrt(c**(4/3))*sqrt(d*
*(4/3))/(2*c**(1/3)*d))/(c**(1/3)*d**(2/3)) - 3*log(-c**(1/3)/(2*d**(1/3))
+ x**(1/3) + sqrt(3)*I*sqrt(c**(4/3))*sqrt(d**(4/3))/(2*c**(1/3)*d))/(c**(1
/3)*d**(2/3))
```

Giac [A] time = 1.16123, size = 46, normalized size = 0.98

$$-\frac{3 \log\left(c^{\frac{1}{3}}dx^{\frac{2}{3}} - c^{\frac{2}{3}}d^{\frac{2}{3}}x^{\frac{1}{3}} + cd^{\frac{1}{3}}\right)}{c^{\frac{1}{3}}d^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^(1/3)-2*d^(1/3)*x^(1/3))/(c*d^(1/3)*x^(2/3)-c^(2/3)*d^(2/3)*x+
c^(1/3)*d*x^(4/3)),x, algorithm="giac")
```

```
[Out] -3*log(c^(1/3)*d*x^(2/3) - c^(2/3)*d^(2/3)*x^(1/3) + c*d^(1/3))/(c^(1/3)*d^
(2/3))
```

3.145 $\int \frac{(fx)^m(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=245

$$\frac{2c(fx)^{m+1}\left(d+ex^n\right)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(\frac{m+1}{n};1,-q;\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{f(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c(fx)^{m+1}\left(d+ex^n\right)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(\frac{m+1}{n};1,-\right.}{f(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-\right.}}$$

```
[Out] (2*c*(f*x)^(1 + m)*(d + e*x^n)^q*AppellF1[(1 + m)/n, 1, -q, (1 + m + n)/n,
(-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^n)/d)])/(Sqrt[b^2 - 4*a*c]*(b -
Sqrt[b^2 - 4*a*c])*f*(1 + m)*(1 + (e*x^n)/d)^q) - (2*c*(f*x)^(1 + m)*(d + e
*x^n)^q*AppellF1[(1 + m)/n, 1, -q, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2
- 4*a*c]), -((e*x^n)/d)])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*f*(1 +
 m)*(1 + (e*x^n)/d)^q)
```

Rubi [A] time = 0.539941, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1556, 511, 510}

$$\frac{2c(fx)^{m+1}\left(d+ex^n\right)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(\frac{m+1}{n};1,-q;\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{f(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c(fx)^{m+1}\left(d+ex^n\right)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(\frac{m+1}{n};1,-\right.}{f(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (2*c*(f*x)^(1 + m)*(d + e*x^n)^q*AppellF1[(1 + m)/n, 1, -q, (1 + m + n)/n,
(-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^n)/d)])/(Sqrt[b^2 - 4*a*c]*(b -
Sqrt[b^2 - 4*a*c])*f*(1 + m)*(1 + (e*x^n)/d)^q) - (2*c*(f*x)^(1 + m)*(d + e
*x^n)^q*AppellF1[(1 + m)/n, 1, -q, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2
- 4*a*c]), -((e*x^n)/d)])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*f*(1 +
 m)*(1 + (e*x^n)/d)^q)
```

Rule 1556

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_))/((a_) + (c_.)*(x_)^(
n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[
(2*c)/r, Int[((f*x)^m*(d + e*x^n)^q)/(b - r + 2*c*x^n), x], x] - Dist[(2*c)
/r, Int[((f*x)^m*(d + e*x^n)^q)/(b + r + 2*c*x^n), x], x]] /; FreeQ[{a, b,
```

```
c, d, e, f, m, n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(fx)^m(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx = \frac{(2c)\int \frac{(fx)^m(d+ex^n)^q}{b-\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{(fx)^m(d+ex^n)^q}{b+\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{\left(2c(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\right)\int \frac{(fx)^m\left(1+\frac{ex^n}{d}\right)^q}{b-\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}} - \frac{\left(2c(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\right)\int \frac{(fx)^m\left(1+\frac{ex^n}{d}\right)^q}{b+\sqrt{b^2-4ac}+2cx^n}\,d}{\sqrt{b^2-4ac}}$$

$$= \frac{2c(fx)^{1+m}(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}F_1\left(\frac{1+m}{n};1,-q;\frac{1+m+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)f(1+m)} - \frac{2c(fx)^{1+m}(d+ex}{}$$

Mathematica [F] time = 0.174706, size = 0, normalized size = 0.

$$\int \frac{(fx)^m(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] Integrate[((f*x)^m*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)), x]
```

Maple [F] time = 0.076, size = 0, normalized size = 0.

$$\int \frac{\left(fx\right)^{m} \left(d+e x^{n}\right)^{q}}{a+b x^{n}+c x^{2 n}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x)

[Out] int((f*x)^m*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{{\left(e x^{n} + d\right)}^{q} \left(f x\right)^{m}}{c x^{2 \, n} + b x^{n} + a}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")

[Out] integrate((e*x^n + d)^q*(f*x)^m/(c*x^(2*n) + b*x^n + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left(\frac{{\left(e x^{n} + d\right)}^{q} \left(f x\right)^{m}}{c x^{2 \, n} + b x^{n} + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")

[Out] integral((e*x^n + d)^q*(f*x)^m/(c*x^(2*n) + b*x^n + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(d+e*x**n)**q/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^n + d)^q (fx)^m}{cx^{2n} + bx^n + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)^q*(f*x)^m/(c*x^(2*n) + b*x^n + a), x)
```

3.146 $\int \frac{x^2(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=210

$$-\frac{2cx^3\left(d+ex^n\right)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(\frac{3}{n};1,-q;\frac{n+3}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{3\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)}-\frac{2cx^3\left(d+ex^n\right)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(\frac{3}{n};1,-q;\frac{n+3}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right.}{3\left(b\sqrt{b^2-4ac}-4ac+b^2\right)}$$

```
[Out] (-2*c*x^3*(d + e*x^n)^q*AppellF1[3/n, 1, -q, (3 + n)/n, (-2*c*x^n)/(b - Sqr
t[b^2 - 4*a*c]), -((e*x^n)/d)])/(3*(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(1 +
 (e*x^n)/d)^q) - (2*c*x^3*(d + e*x^n)^q*AppellF1[3/n, 1, -q, (3 + n)/n, (-2
*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), -((e*x^n)/d)])/(3*(b^2 - 4*a*c + b*Sqrt[b^
2 - 4*a*c])*(1 + (e*x^n)/d)^q)
```

Rubi [A] time = 0.501204, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1556, 511, 510}

$$-\frac{2cx^3\left(d+ex^n\right)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(\frac{3}{n};1,-q;\frac{n+3}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{3\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)}-\frac{2cx^3\left(d+ex^n\right)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(\frac{3}{n};1,-q;\frac{n+3}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right.}{3\left(b\sqrt{b^2-4ac}-4ac+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (-2*c*x^3*(d + e*x^n)^q*AppellF1[3/n, 1, -q, (3 + n)/n, (-2*c*x^n)/(b - Sqr
t[b^2 - 4*a*c]), -((e*x^n)/d)])/(3*(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(1 +
 (e*x^n)/d)^q) - (2*c*x^3*(d + e*x^n)^q*AppellF1[3/n, 1, -q, (3 + n)/n, (-2
*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), -((e*x^n)/d)])/(3*(b^2 - 4*a*c + b*Sqrt[b^
2 - 4*a*c])*(1 + (e*x^n)/d)^q)
```

Rule 1556

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_))/((a_) + (c_.)*(x_)^(
n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[
(2*c)/r, Int[((f*x)^m*(d + e*x^n)^q)/(b - r + 2*c*x^n), x], x] - Dist[(2*c)
/r, Int[((f*x)^m*(d + e*x^n)^q)/(b + r + 2*c*x^n), x], x]] /; FreeQ[{a, b,
c, d, e, f, m, n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x^2 (d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx = \frac{(2c)\int \frac{x^2(d+ex^n)^q}{b-\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{x^2(d+ex^n)^q}{b+\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{\left(2c(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\right)\int \frac{x^2\left(1+\frac{ex^n}{d}\right)^q}{b-\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}} - \frac{\left(2c(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\right)\int \frac{x^2\left(1+\frac{ex^n}{d}\right)^q}{b+\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}}$$

$$= -\frac{2cx^3(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q} F_1\left(\frac{3}{n};1,-q;\frac{3+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{3\left(b^2-4ac-b\sqrt{b^2-4ac}\right)} - \frac{2cx^3(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q} F_1}{3\left(b^2-4ac\right.}$$

Mathematica [F] time = 0.187065, size = 0, normalized size = 0.

$$\int \frac{x^2 (d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^2*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] Integrate[(x^2*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)), x]
```

Maple [F] time = 0.054, size = 0, normalized size = 0.

$$\int \frac{x^{2} \left(d+e x^{n}\right)^{q}}{a+b x^{n}+c x^{2 n}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] int(x^2*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(e x^{n}+d\right)^{q} x^{2}}{c x^{2 n}+b x^{n}+a}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate((e*x^n + d)^q*x^2/(c*x^(2*n) + b*x^n + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(e x^{n}+d\right)^{q} x^{2}}{c x^{2 n}+b x^{n}+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] integral((e*x^n + d)^q*x^2/(c*x^(2*n) + b*x^n + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d+e*x**n)**q/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^n + d)^q x^2}{cx^{2n} + bx^n + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)^q*x^2/(c*x^(2*n) + b*x^n + a), x)
```

3.147 $\int \frac{x(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=206

$$-\frac{cx^2\left(d+ex^n\right)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(\frac{2}{n};1,-q;\frac{n+2}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}-\frac{cx^2\left(d+ex^n\right)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(\frac{2}{n};1,-q;\frac{n+2}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},-\right.}{b\sqrt{b^2-4ac}-4ac+b^2}$$

```
[Out] -((c*x^2*(d + e*x^n)^q*AppellF1[2/n, 1, -q, (2 + n)/n, (-2*c*x^n)/(b - Sqrt
[b^2 - 4*a*c]), -((e*x^n)/d)])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(1 + (e
*x^n)/d)^q)) - (c*x^2*(d + e*x^n)^q*AppellF1[2/n, 1, -q, (2 + n)/n, (-2*c*x
^n)/(b + Sqrt[b^2 - 4*a*c]), -((e*x^n)/d)])/((b^2 - 4*a*c + b*Sqrt[b^2 - 4*
a*c])*(1 + (e*x^n)/d)^q)
```

Rubi [A] time = 0.375739, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1556, 511, 510}

$$-\frac{cx^2\left(d+ex^n\right)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(\frac{2}{n};1,-q;\frac{n+2}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}-\frac{cx^2\left(d+ex^n\right)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(\frac{2}{n};1,-q;\frac{n+2}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},-\right.}{b\sqrt{b^2-4ac}-4ac+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -((c*x^2*(d + e*x^n)^q*AppellF1[2/n, 1, -q, (2 + n)/n, (-2*c*x^n)/(b - Sqrt
[b^2 - 4*a*c]), -((e*x^n)/d)])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(1 + (e
*x^n)/d)^q)) - (c*x^2*(d + e*x^n)^q*AppellF1[2/n, 1, -q, (2 + n)/n, (-2*c*x
^n)/(b + Sqrt[b^2 - 4*a*c]), -((e*x^n)/d)])/((b^2 - 4*a*c + b*Sqrt[b^2 - 4*
a*c])*(1 + (e*x^n)/d)^q)
```

Rule 1556

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_))/((a_) + (c_.)*(x_)^(
n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[
(2*c)/r, Int[((f*x)^m*(d + e*x^n)^q)/(b - r + 2*c*x^n), x], x] - Dist[(2*c)
/r, Int[((f*x)^m*(d + e*x^n)^q)/(b + r + 2*c*x^n), x], x]] /; FreeQ[{a, b,
c, d, e, f, m, n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{x(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx = \frac{(2c)\int \frac{x(d+ex^n)^q}{b-\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{x(d+ex^n)^q}{b+\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{\left(2c\,(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\right)\int \frac{x\left(1+\frac{ex^n}{d}\right)^q}{b-\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}} - \frac{\left(2c\,(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\right)\int \frac{x\left(1+\frac{ex^n}{d}\right)^q}{b+\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}}$$

$$= -\frac{cx^2\,(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}F_1\left(\frac{2}{n};1,-q;\frac{2+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{b^2-4ac-b\sqrt{b^2-4ac}} - \frac{cx^2\,(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}F_1\left(\frac{2}{n}\right.}{b^2-4ac+b}$$

Mathematica [F] time = 0.137504, size = 0, normalized size = 0.

$$\int \frac{x(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] Integrate[(x*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)), x]
```

Maple [F] time = 0.055, size = 0, normalized size = 0.

$$\int \frac{x(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] int(x*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^n + d)^q x}{cx^{2n} + bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate((e*x^n + d)^q*x/(c*x^(2*n) + b*x^n + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex^n + d)^q x}{cx^{2n} + bx^n + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] integral((e*x^n + d)^q*x/(c*x^(2*n) + b*x^n + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+e*x**n)**q/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^n + d)^q x}{cx^{2n} + bx^n + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)^q*x/(c*x^(2*n) + b*x^n + a), x)
```

3.148 $\int \frac{(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=194

$$-\frac{2cx\,(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(\frac{1}{n};1,-q;1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}-\frac{2cx\,(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(\frac{1}{n};1,-q;1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4a}}\right.}{b\sqrt{b^2-4ac}-4ac+b^2}$$

```
[Out] (-2*c*x*(d + e*x^n)^q*AppellF1[n^(-1), 1, -q, 1 + n^(-1), (-2*c*x^n)/(b - S
qrt[b^2 - 4*a*c]), -((e*x^n)/d)])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(1 +
 (e*x^n)/d)^q) - (2*c*x*(d + e*x^n)^q*AppellF1[n^(-1), 1, -q, 1 + n^(-1), (
-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), -((e*x^n)/d)])/((b^2 - 4*a*c + b*Sqrt[b^
2 - 4*a*c])*(1 + (e*x^n)/d)^q)
```

Rubi [A] time = 0.299677, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1428, 430, 429}

$$-\frac{2cx\,(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(\frac{1}{n};1,-q;1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}-\frac{2cx\,(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(\frac{1}{n};1,-q;1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4a}}\right.}{b\sqrt{b^2-4ac}-4ac+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^n)^q/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (-2*c*x*(d + e*x^n)^q*AppellF1[n^(-1), 1, -q, 1 + n^(-1), (-2*c*x^n)/(b - S
qrt[b^2 - 4*a*c]), -((e*x^n)/d)])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(1 +
 (e*x^n)/d)^q) - (2*c*x*(d + e*x^n)^q*AppellF1[n^(-1), 1, -q, 1 + n^(-1), (
-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), -((e*x^n)/d)])/((b^2 - 4*a*c + b*Sqrt[b^
2 - 4*a*c])*(1 + (e*x^n)/d)^q)
```

Rule 1428

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/r, Int[(d + e*x^
n)^q/(b - r + 2*c*x^n), x], x] - Dist[(2*c)/r, Int[(d + e*x^n)^q/(b + r + 2
*c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n, q}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[q]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx &= \frac{(2c)\int \frac{(d+ex^n)^q}{b-\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{(d+ex^n)^q}{b+\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}}\\ &= \frac{\left(2c\,(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\right)\int \frac{\left(1+\frac{ex^n}{d}\right)^q}{b-\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}} - \frac{\left(2c\,(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\right)\int \frac{\left(1+\frac{ex^n}{d}\right)^q}{b+\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}}\\ &= -\frac{2cx\,(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}F_1\left(\frac{1}{n};1,-q;1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{b^2-4ac-b\sqrt{b^2-4ac}} - \frac{2cx\,(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}F_1}{b^2-4ac}\end{aligned}$$

Mathematica [F] time = 0.0837886, size = 0, normalized size = 0.

$$\int \frac{(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x^n)^q/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] Integrate[(d + e*x^n)^q/(a + b*x^n + c*x^(2*n)), x]
```

Maple [F] time = 0.052, size = 0, normalized size = 0.

$$\int \frac{(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] int((d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^n + d)^q}{cx^{2n} + bx^n + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate((e*x^n + d)^q/(c*x^(2*n) + b*x^n + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex^n + d)^q}{cx^{2n} + bx^n + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] integral((e*x^n + d)^q/(c*x^(2*n) + b*x^n + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)**q/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^n + d)^q}{cx^{2n} + bx^n + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)^q/(c*x^(2*n) + b*x^n + a), x)
```

3.149 $\int \frac{(d+ex^n)^q}{x\left(a+bx^n+cx^{2n}\right)}\,dx$

Optimal. Leaf size=263

$$\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)(d+ex^n)^{q+1}\ {}_2F_1\left(1,q+1;q+2;\frac{2c(ex^n+d)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{an(q+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}+\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^n)^{q+1}\ {}_2F_1\left(1,q+1;q+2;\frac{2c(e}{2cd-(b+}\right.}{an(q+1)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}$$

```
[Out] (c*(1 + b/Sqrt[b^2 - 4*a*c])*(d + e*x^n)^(1 + q)*Hypergeometric2F1[1, 1 + q
, 2 + q, (2*c*(d + e*x^n))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)])/(a*(2*c*d
- (b - Sqrt[b^2 - 4*a*c])*e)*n*(1 + q)) + (c*(1 - b/Sqrt[b^2 - 4*a*c])*(d +
 e*x^n)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^n))/(2*c*d
 - (b + Sqrt[b^2 - 4*a*c])*e)])/(a*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*n*(1
 + q)) - ((d + e*x^n)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, 1 + (e*x^n
)/d])/(a*d*n*(1 + q))
```

Rubi [A] time = 0.733565, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1474, 960, 65, 830, 68}

$$\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)(d+ex^n)^{q+1}\ {}_2F_1\left(1,q+1;q+2;\frac{2c(ex^n+d)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{an(q+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}+\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^n)^{q+1}\ {}_2F_1\left(1,q+1;q+2;\frac{2c(e}{2cd-(b+}\right.}{an(q+1)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^n)^q/(x*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] (c*(1 + b/Sqrt[b^2 - 4*a*c])*(d + e*x^n)^(1 + q)*Hypergeometric2F1[1, 1 + q
, 2 + q, (2*c*(d + e*x^n))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)])/(a*(2*c*d
- (b - Sqrt[b^2 - 4*a*c])*e)*n*(1 + q)) + (c*(1 - b/Sqrt[b^2 - 4*a*c])*(d +
 e*x^n)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^n))/(2*c*d
 - (b + Sqrt[b^2 - 4*a*c])*e)])/(a*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*n*(1
 + q)) - ((d + e*x^n)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, 1 + (e*x^n
)/d])/(a*d*n*(1 + q))
```

Rule 1474

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 960

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 830

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \frac{(d+ex^n)^q}{x\left(a+bx^n+cx^{2n}\right)}\,dx = \frac{\text{Subst}\left(\int \frac{(d+ex)^q}{x\left(a+bx+cx^2\right)}\,dx, x, x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{(d+ex)^q}{ax} + \frac{(-b-cx)(d+ex)^q}{a\left(a+bx+cx^2\right)}\right)dx, x, x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \frac{(d+ex)^q}{x}\,dx, x, x^n\right)}{an} + \frac{\text{Subst}\left(\int \frac{(-b-cx)(d+ex)^q}{a+bx+cx^2}\,dx, x, x^n\right)}{an}$$

$$= -\frac{(d+ex^n)^{1+q}\ {}_2F_1\left(1,1+q;2+q;1+\frac{ex^n}{d}\right)}{adn(1+q)} + \frac{\text{Subst}\left(\int \left(\frac{\left(-c-\frac{bc}{\sqrt{b^2-4ac}}\right)(d+ex)^q}{b-\sqrt{b^2-4ac}+2cx} + \frac{\left(-c+\frac{bc}{\sqrt{b^2-4ac}}\right)(d+ex)^q}{b+\sqrt{b^2-4ac}+2cx}\right.\right.}{an}$$

$$= -\frac{(d+ex^n)^{1+q}\ {}_2F_1\left(1,1+q;2+q;1+\frac{ex^n}{d}\right)}{adn(1+q)} - \frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{(d+ex)^q}{b+\sqrt{b^2-4ac}+2cx}\,dx, x, x\right.}{an}$$

$$= \frac{c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^n)^{1+q}\ {}_2F_1\left(1,1+q;2+q;\frac{2c(d+ex^n)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{a\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)n(1+q)} + \frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^n)^{1+}}{a\left(2cd-\left(b+\right.\right.}$$

Mathematica [A] time = 0.727833, size = 218, normalized size = 0.83

$$\frac{(d+ex^n)^{q+1}\left(\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right){}_2F_1\left(1,q+1;q+2;\frac{2c\left(ex^n+d\right)}{2cd+\left(\sqrt{b^2-4ac}-b\right)e}\right)}{e\left(\sqrt{b^2-4ac}-b\right)+2cd} + \frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right){}_2F_1\left(1,q+1;q+2;\frac{2c\left(ex^n+d\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2cd-e\left(\sqrt{b^2-4ac}+b\right)} - \frac{{}_2F_1\left(1,q+1;q+2;\frac{ex^n}{d}+1\right)}{d}\right)}{an(q+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^n)^q/(x*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] ((d + e*x^n)^(1 + q)*((c*(1 + b/Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, 1 +
 q, 2 + q, (2*c*(d + e*x^n))/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)])/(2*c*d
+ (-b + Sqrt[b^2 - 4*a*c])*e) + (c*(1 - b/Sqrt[b^2 - 4*a*c])*Hypergeometric
2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^n))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]
)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e) - Hypergeometric2F1[1, 1 + q, 2 + q,
1 + (e*x^n)/d]/d))/(a*n*(1 + q))
```

Maple [F] time = 0.079, size = 0, normalized size = 0.

$$\int \frac{(d+ex^n)^q}{x(a+bx^n+cx^{2n})}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x^n)^q/x/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] int((d+e*x^n)^q/x/(a+b*x^n+c*x^(2*n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^n+d)^q}{(cx^{2n}+bx^n+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^q/x/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate((e*x^n + d)^q/((c*x^(2*n) + b*x^n + a)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex^n+d)^q}{cxx^{2n}+bxx^n+ax}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^q/x/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] integral((e*x^n + d)^q/(c*x*x^(2*n) + b*x*x^n + a*x), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)**q/x/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^n + d)^q}{\left(cx^{2n} + bx^n + a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^q/x/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)^q/((c*x^(2*n) + b*x^n + a)*x), x)
```

3.150 $\int \frac{(d+ex^n)^q}{x^2\left(a+bx^n+cx^{2n}\right)}\,dx$

Optimal. Leaf size=212

$$\frac{2c\,(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(-\frac{1}{n};1,-q;-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{x\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)}+\frac{2c\,(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(-\frac{1}{n};1,-q;-\frac{1-n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4a}}\right)}{x\left(b\sqrt{b^2-4ac}-4ac+b^2\right)}$$

```
[Out] (2*c*(d + e*x^n)^q*AppellF1[-n^(-1), 1, -q, -((1 - n)/n), (-2*c*x^n)/(b - S
qrt[b^2 - 4*a*c]), -((e*x^n)/d)])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*x*(1
 + (e*x^n)/d)^q) + (2*c*(d + e*x^n)^q*AppellF1[-n^(-1), 1, -q, -((1 - n)/n)
, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), -((e*x^n)/d)])/((b^2 - 4*a*c + b*Sqrt
[b^2 - 4*a*c])*x*(1 + (e*x^n)/d)^q)
```

Rubi [A] time = 0.488911, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1556, 511, 510}

$$\frac{2c\,(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(-\frac{1}{n};1,-q;-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{x\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)}+\frac{2c\,(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(-\frac{1}{n};1,-q;-\frac{1-n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4a}}\right)}{x\left(b\sqrt{b^2-4ac}-4ac+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^n)^q/(x^2*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] (2*c*(d + e*x^n)^q*AppellF1[-n^(-1), 1, -q, -((1 - n)/n), (-2*c*x^n)/(b - S
qrt[b^2 - 4*a*c]), -((e*x^n)/d)])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*x*(1
 + (e*x^n)/d)^q) + (2*c*(d + e*x^n)^q*AppellF1[-n^(-1), 1, -q, -((1 - n)/n)
, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), -((e*x^n)/d)])/((b^2 - 4*a*c + b*Sqrt
[b^2 - 4*a*c])*x*(1 + (e*x^n)/d)^q)
```

Rule 1556

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_))/((a_) + (c_.)*(x_)^(
n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[
(2*c)/r, Int[((f*x)^m*(d + e*x^n)^q)/(b - r + 2*c*x^n), x], x] - Dist[(2*c)
/r, Int[((f*x)^m*(d + e*x^n)^q)/(b + r + 2*c*x^n), x], x]] /; FreeQ[{a, b,
c, d, e, f, m, n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(d+ex^n)^q}{x^2\left(a+bx^n+cx^{2n}\right)}\,dx = \frac{(2c)\int \frac{(d+ex^n)^q}{x^2\left(b-\sqrt{b^2-4ac}+2cx^n\right)}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{(d+ex^n)^q}{x^2\left(b+\sqrt{b^2-4ac}+2cx^n\right)}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{\left(2c\,(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\right)\int \frac{\left(1+\frac{ex^n}{d}\right)^q}{x^2\left(b-\sqrt{b^2-4ac}+2cx^n\right)}\,dx}{\sqrt{b^2-4ac}} - \frac{\left(2c\,(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\right)\int \frac{\left(1+\frac{ex^n}{d}\right)}{x^2\left(b+\sqrt{b^2-4ac}\right.}}{\sqrt{b^2-4ac}}$$

$$= \frac{2c\,(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q} F_1\left(-\frac{1}{n};1,-q;-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{\left(b^2-4ac-b\sqrt{b^2-4ac}\right)x} + \frac{2c\,(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q} F_1}{\left(b^2-4ac\right.}$$

Mathematica [F] time = 0.142108, size = 0, normalized size = 0.

$$\int \frac{(d+ex^n)^q}{x^2\left(a+bx^n+cx^{2n}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x^n)^q/(x^2*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] Integrate[(d + e*x^n)^q/(x^2*(a + b*x^n + c*x^(2*n))), x]
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int \frac{(d+e x^n)^q}{x^2 \left(a+b x^n+c x^{2 n}\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x^n)^q/x^2/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] int((d+e*x^n)^q/x^2/(a+b*x^n+c*x^(2*n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e x^n+d)^q}{\left(c x^{2 n}+b x^n+a\right) x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^q/x^2/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate((e*x^n + d)^q/((c*x^(2*n) + b*x^n + a)*x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(e x^n+d)^q}{c x^2 x^{2 n}+b x^2 x^n+a x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^q/x^2/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] integral((e*x^n + d)^q/(c*x^2*x^(2*n) + b*x^2*x^n + a*x^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)**q/x**2/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^n + d)^q}{\left(cx^{2\,n} + bx^n + a\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^q/x^2/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)^q/((c*x^(2*n) + b*x^n + a)*x^2), x)
```

3.151 $\int \frac{(d+ex^n)^q}{x^3\left(a+bx^n+cx^{2n}\right)}\,dx$

Optimal. Leaf size=210

$$\frac{c\,(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(-\frac{2}{n};1,-q;-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{x^2\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)}+\frac{c\,(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(-\frac{2}{n};1,-q;-\frac{2-n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\right.}{x^2\left(b\sqrt{b^2-4ac}-4ac+b^2\right)}$$

```
[Out] (c*(d + e*x^n)^q*AppellF1[-2/n, 1, -q, -((2 - n)/n), (-2*c*x^n)/(b - Sqrt[b
^2 - 4*a*c]), -((e*x^n)/d)])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*x^2*(1 +
(e*x^n)/d)^q) + (c*(d + e*x^n)^q*AppellF1[-2/n, 1, -q, -((2 - n)/n), (-2*c*
x^n)/(b + Sqrt[b^2 - 4*a*c]), -((e*x^n)/d)])/((b^2 - 4*a*c + b*Sqrt[b^2 - 4
*a*c])*x^2*(1 + (e*x^n)/d)^q)
```

Rubi [A] time = 0.475619, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1556, 511, 510}

$$\frac{c\,(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(-\frac{2}{n};1,-q;-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{x^2\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)}+\frac{c\,(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(-\frac{2}{n};1,-q;-\frac{2-n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\right.}{x^2\left(b\sqrt{b^2-4ac}-4ac+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^n)^q/(x^3*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] (c*(d + e*x^n)^q*AppellF1[-2/n, 1, -q, -((2 - n)/n), (-2*c*x^n)/(b - Sqrt[b
^2 - 4*a*c]), -((e*x^n)/d)])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*x^2*(1 +
(e*x^n)/d)^q) + (c*(d + e*x^n)^q*AppellF1[-2/n, 1, -q, -((2 - n)/n), (-2*c*
x^n)/(b + Sqrt[b^2 - 4*a*c]), -((e*x^n)/d)])/((b^2 - 4*a*c + b*Sqrt[b^2 - 4
*a*c])*x^2*(1 + (e*x^n)/d)^q)
```

Rule 1556

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_))/((a_) + (c_.)*(x_)^(
n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[
(2*c)/r, Int[((f*x)^m*(d + e*x^n)^q)/(b - r + 2*c*x^n), x], x] - Dist[(2*c)
/r, Int[((f*x)^m*(d + e*x^n)^q)/(b + r + 2*c*x^n), x], x]] /; FreeQ[{a, b,
c, d, e, f, m, n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(d+ex^n)^q}{x^3\left(a+bx^n+cx^{2n}\right)}\,dx = \frac{(2c)\int \frac{(d+ex^n)^q}{x^3\left(b-\sqrt{b^2-4ac}+2cx^n\right)}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{(d+ex^n)^q}{x^3\left(b+\sqrt{b^2-4ac}+2cx^n\right)}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{\left(2c\,(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\right)\int \frac{\left(1+\frac{ex^n}{d}\right)^q}{x^3\left(b-\sqrt{b^2-4ac}+2cx^n\right)}\,dx}{\sqrt{b^2-4ac}} - \frac{\left(2c\,(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\right)\int \frac{\left(1+\frac{ex^n}{d}\right.}{x^3\left(b+\sqrt{b^2-4ac}\right.}}{\sqrt{b^2-4ac}}$$

$$= \frac{c\,(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q} F_1\left(-\frac{2}{n};1,-q;-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{\left(b^2-4ac-b\sqrt{b^2-4ac}\right)x^2} + \frac{c\,(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q} F_1\left(-\right.}{\left(b^2-4ac+\right.}$$

Mathematica [F] time = 0.128474, size = 0, normalized size = 0.

$$\int \frac{(d+ex^n)^q}{x^3\left(a+bx^n+cx^{2n}\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d + e*x^n)^q/(x^3*(a + b*x^n + c*x^(2*n))),x]

[Out] Integrate[(d + e*x^n)^q/(x^3*(a + b*x^n + c*x^(2*n))), x]
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int \frac{(d+ex^n)^q}{x^3\left(a+bx^n+cx^{2n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x^n)^q/x^3/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] int((d+e*x^n)^q/x^3/(a+b*x^n+c*x^(2*n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^n+d)^q}{\left(cx^{2n}+bx^n+a\right)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^q/x^3/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate((e*x^n + d)^q/((c*x^(2*n) + b*x^n + a)*x^3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(ex^n+d)^q}{cx^3x^{2n}+bx^3x^n+ax^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^q/x^3/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] integral((e*x^n + d)^q/(c*x^3*x^(2*n) + b*x^3*x^n + a*x^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)**q/x**3/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^n+d)^q}{(cx^{2n}+bx^n+a)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^q/x^3/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)^q/((c*x^(2*n) + b*x^n + a)*x^3), x)
```

3.152 $\int (fx)^m \left(d+ex^n\right)^2 \left(a+bx^n+cx^{2n}\right)^p \, dx$

Optimal. Leaf size=498

$$\frac{d^2(fx)^{m+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{m+1}{n};-p,-p;\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)}+2dex$$

```
[Out] (d^2*(f*x)^(1 + m)*(a + b*x^n + c*x^(2*n))^p*AppellF1[(1 + m)/n, -p, -p, (1
 + m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 -
 4*a*c])])/(f*(1 + m)*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x
^n)/(b + Sqrt[b^2 - 4*a*c]))^p) + (2*d*e*x^(1 + n)*(f*x)^m*(a + b*x^n + c*x
^(2*n))^p*AppellF1[(1 + m + n)/n, -p, -p, (1 + m + 2*n)/n, (-2*c*x^n)/(b -
Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((1 + m + n)*(1 +
(2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])
)^p) + (e^2*x^(1 + 2*n)*(f*x)^m*(a + b*x^n + c*x^(2*n))^p*AppellF1[(1 + m +
 2*n)/n, -p, -p, (1 + m + 3*n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c
*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((1 + m + 2*n)*(1 + (2*c*x^n)/(b - Sqrt[b^2
 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] time = 0.609488, antiderivative size = 498, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1560, 1385, 510, 20}

$$\frac{d^2(fx)^{m+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{m+1}{n};-p,-p;\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)}+2dex$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (d^2*(f*x)^(1 + m)*(a + b*x^n + c*x^(2*n))^p*AppellF1[(1 + m)/n, -p, -p, (1
 + m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 -
 4*a*c])])/(f*(1 + m)*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x
^n)/(b + Sqrt[b^2 - 4*a*c]))^p) + (2*d*e*x^(1 + n)*(f*x)^m*(a + b*x^n + c*x
^(2*n))^p*AppellF1[(1 + m + n)/n, -p, -p, (1 + m + 2*n)/n, (-2*c*x^n)/(b -
Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((1 + m + n)*(1 +
(2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])
)^p) + (e^2*x^(1 + 2*n)*(f*x)^m*(a + b*x^n + c*x^(2*n))^p*AppellF1[(1 + m +
 2*n)/n, -p, -p, (1 + m + 3*n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c
*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((1 + m + 2*n)*(1 + (2*c*x^n)/(b - Sqrt[b^2
```

```
 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 1560

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ
[q, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rubi steps

$$\int (fx)^m (d+ex^n)^2 \left(a+bx^n+cx^{2n}\right)^p dx = \int \left(d^2(fx)^m \left(a+bx^n+cx^{2n}\right)^p + 2dex^n(fx)^m \left(a+bx^n+cx^{2n}\right)^p + e^2x^{2n}(fx)^m\right.$$
$$= d^2 \int (fx)^m \left(a+bx^n+cx^{2n}\right)^p dx + (2de) \int x^n(fx)^m \left(a+bx^n+cx^{2n}\right)^p dx + e$$
$$= \left(2dex^{-m}(fx)^m\right) \int x^{m+n} \left(a+bx^n+cx^{2n}\right)^p dx + \left(e^2x^{-m}(fx)^m\right) \int x^{m+2n} \left(a+\right.$$
$$= \frac{d^2(fx)^{1+m}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{1+m}{n};-p\right.}{f(1+m)}$$
$$= \frac{d^2(fx)^{1+m}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{1+m}{n};-p\right.}{f(1+m)}$$

Mathematica [A] time = 1.16659, size = 391, normalized size = 0.79

$$\frac{x(fx)^m\left(\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^n}{\sqrt{b^2-4ac}+b}\right)^{-p}(a+x^n(b+cx^n))^p\left(d^2\left(m^2+m(3n+2)+2n^2+3n+1\right)F_1\left(\frac{m+1}{n};-p,-p;\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f*x)^m*(d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (x*(f*x)^m*(a + x^n*(b + c*x^n))^p*(d^2*(1 + m^2 + 3*n + 2*n^2 + m*(2 + 3*n
))*AppellF1[(1 + m)/n, -p, -p, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + e*(1 + m)*x^n*(2*d*(1 + m + 2*
n)*AppellF1[(1 + m + n)/n, -p, -p, (1 + m + 2*n)/n, (-2*c*x^n)/(b + Sqrt[b^
2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + e*(1 + m + n)*x^n*Appell
F1[(1 + m + 2*n)/n, -p, -p, (1 + m + 3*n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])))/((1 + m)*(1 + m + n)*(1 + m +
2*n)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sq
rt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F] time = 0.054, size = 0, normalized size = 0.

$$\int \left(fx\right)^{m}(d+ex^{n})^2\left(a+bx^{n}+cx^{2\,n}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d+e*x^n)^2*(a+b*x^n+c*x^(2*n))^p,x)
```

```
[Out] int((f*x)^m*(d+e*x^n)^2*(a+b*x^n+c*x^(2*n))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex^n + d)^2 \left(cx^{2n} + bx^n + a\right)^p \left(fx\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)^2*(a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x^n + d)^2*(c*x^(2*n) + b*x^n + a)^p*(f*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(e^2x^{2n} + 2\,dex^n + d^2\right)\left(cx^{2n} + bx^n + a\right)^p \left(fx\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)^2*(a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^(2*n) + 2*d*e*x^n + d^2)*(c*x^(2*n) + b*x^n + a)^p*(f*x)^m,
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(d+e*x**n)**2*(a+b*x**n+c*x**(2*n))**p,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)^2*(a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.153 $\int (fx)^m (d + ex^n) \left(a + bx^n + cx^{2n}\right)^p \, dx$

Optimal. Leaf size=323

$$\frac{d(fx)^{m+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{m+1}{n};-p,-p;\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)}+\frac{ex^{n+1}(\ldots}{}$$

```
[Out] (d*(f*x)^(1 + m)*(a + b*x^n + c*x^(2*n))^p*AppellF1[(1 + m)/n, -p, -p, (1 +
 m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4
*a*c])])/(f*(1 + m)*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n
)/(b + Sqrt[b^2 - 4*a*c]))^p) + (e*x^(1 + n)*(f*x)^m*(a + b*x^n + c*x^(2*n)
)^p*AppellF1[(1 + m + n)/n, -p, -p, (1 + m + 2*n)/n, (-2*c*x^n)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((1 + m + n)*(1 + (2*c*x
^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] time = 0.37063, antiderivative size = 323, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1560, 1385, 510, 20}

$$\frac{d(fx)^{m+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{m+1}{n};-p,-p;\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)}+\frac{ex^{n+1}(\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (d*(f*x)^(1 + m)*(a + b*x^n + c*x^(2*n))^p*AppellF1[(1 + m)/n, -p, -p, (1 +
 m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4
*a*c])])/(f*(1 + m)*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n
)/(b + Sqrt[b^2 - 4*a*c]))^p) + (e*x^(1 + n)*(f*x)^m*(a + b*x^n + c*x^(2*n)
)^p*AppellF1[(1 + m + n)/n, -p, -p, (1 + m + 2*n)/n, (-2*c*x^n)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((1 + m + n)*(1 + (2*c*x
^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 1560

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ
```

```
[q, 0])
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rubi steps

$$\int (fx)^m (d+ex^n)\left(a+bx^n+cx^{2n}\right)^p\,dx = \int \left(d(fx)^m\left(a+bx^n+cx^{2n}\right)^p + ex^n(fx)^m\left(a+bx^n+cx^{2n}\right)^p\right)dx$$

$$= d\int (fx)^m\left(a+bx^n+cx^{2n}\right)^p\,dx + e\int x^n(fx)^m\left(a+bx^n+cx^{2n}\right)^p\,dx$$

$$= \left(ex^{-m}(fx)^m\right)\int x^{m+n}\left(a+bx^n+cx^{2n}\right)^p\,dx + \left(d\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{}{b}\right.\right.$$

$$= \frac{d(fx)^{1+m}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{1+m}{n};-p,-\right.}{f(1+m)}$$

$$= \frac{d(fx)^{1+m}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{1+m}{n};-p,-\right.}{f(1+m)}$$

Mathematica [A] time = 0.633243, size = 273, normalized size = 0.85

$$\frac{x(fx)^m\left(\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^n}{\sqrt{b^2-4ac}+b}\right)^{-p}(a+x^n(b+cx^n))^p\left(d(m+n+1)F_1\left(\frac{m+1}{n};-p,-p;\frac{m+n+1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4a}}\right.\right.}{(m+1)(m+n+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f*x)^m*(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (x*(f*x)^m*(a + x^n*(b + c*x^n))^p*(d*(1 + m + n)*AppellF1[(1 + m)/n, -p, -
p, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[
b^2 - 4*a*c])] + e*(1 + m)*x^n*AppellF1[(1 + m + n)/n, -p, -p, (1 + m + 2*n
)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])
]))/((1 + m)*(1 + m + n)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 -
 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int (fx)^m (d+ex^n)\left(a+bx^n+cx^{2n}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d+e*x^n)*(a+b*x^n+c*x^(2*n))^p,x)
```

```
[Out] int((f*x)^m*(d+e*x^n)*(a+b*x^n+c*x^(2*n))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (ex^n+d)\left(cx^{2n}+bx^n+a\right)^p (fx)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)*(a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
```

```
[Out] integrate((e*x^n + d)*(c*x^(2*n) + b*x^n + a)^p*(f*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex^n + d)\left(cx^{2n} + bx^n + a\right)^p \left(fx\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)*(a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
```

```
[Out] integral((e*x^n + d)*(c*x^(2*n) + b*x^n + a)^p*(f*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(d+e*x**n)*(a+b*x**n+c*x**(2*n))**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex^n + d)\left(cx^{2n} + bx^n + a\right)^p \left(fx\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)*(a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] integrate((e*x^n + d)*(c*x^(2*n) + b*x^n + a)^p*(f*x)^m, x)
```

3.154 $\int (fx)^m \left(a+bx^n+cx^{2n}\right)^p \, dx$

Optimal. Leaf size=158

$$\frac{(fx)^{m+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{m+1}{n};-p,-p;\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)}$$

```
[Out] ((f*x)^(1 + m)*(a + b*x^n + c*x^(2*n))^p*AppellF1[(1 + m)/n, -p, -p, (1 + m
 + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c])])/(f*(1 + m)*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/
(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rubi [A] time = 0.123837, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1385, 510}

$$\frac{(fx)^{m+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{m+1}{n};-p,-p;\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] ((f*x)^(1 + m)*(a + b*x^n + c*x^(2*n))^p*AppellF1[(1 + m)/n, -p, -p, (1 + m
 + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c])])/(f*(1 + m)*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/
(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 1385

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n + c*x^(2*n))^FracPart[p])/((1 + (
2*c*x^n)/(b + Rt[b^2 - 4*a*c, 2]))^FracPart[p]*(1 + (2*c*x^n)/(b - Rt[b^2 -
 4*a*c, 2]))^FracPart[p]), Int[(d*x)^m*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c
]))^p*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
```

```
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int (fx)^m \left(a+bx^n+cx^{2n}\right)^p dx = \left(\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p\right)\int (fx)^m \left(1+\frac{2c}{b-\sqrt{b}}\right.$$

$$= \frac{(fx)^{1+m}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{1+m}{n};-p,-p;\frac{1+m+n}{n};\right.}{f(1+m)}$$

Mathematica [A] time = 0.330698, size = 181, normalized size = 1.15

$$\frac{x(fx)^m\left(\frac{-\sqrt{b^2-4ac}+b+2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{\sqrt{b^2-4ac}+b+2cx^n}{\sqrt{b^2-4ac}+b}\right)^{-p}\left(a+x^n\left(b+cx^n\right)\right)^p F_1\left(\frac{m+1}{n};-p,-p;\frac{m+n+1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{\sqrt{b^2-4ac}-b}\right)}{m+1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f*x)^m*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (x*(f*x)^m*(a + x^n*(b + c*x^n))^p*AppellF1[(1 + m)/n, -p, -p, (1 + m + n)/
n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])
/((1 + m)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*((b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F] time = 0.05, size = 0, normalized size = 0.

$$\int (fx)^m \left(a+bx^n+cx^{2n}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*x^n+c*x^(2*n))^p,x)
```

```
[Out] int((f*x)^m*(a+b*x^n+c*x^(2*n))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^{2n} + bx^n + a\right)^p \left(fx\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^p*(f*x)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^{2n} + bx^n + a\right)^p \left(fx\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^(2*n) + b*x^n + a)^p*(f*x)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*x**n+c*x**(2*n))**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^{2n} + bx^n + a\right)^p \left(fx\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^p*(f*x)^m, x)
```

3.155 $\int \frac{(fx)^m(a+bx^n+cx^{2n})^p}{d+ex^n}\,dx$

Optimal. Leaf size=33

$$\text{Unintegrable}\left(\frac{(fx)^m\left(a+bx^n+cx^{2n}\right)^p}{d+ex^n},x\right)$$

```
[Out] Defer[Int][((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n), x]
```

Rubi [A] time = 0.0255954, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{(fx)^m\left(a+bx^n+cx^{2n}\right)^p}{d+ex^n}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n),x]

[Out] Defer[Int][((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n), x]
```

Rubi steps

$$\int \frac{(fx)^m\left(a+bx^n+cx^{2n}\right)^p}{d+ex^n}\,dx=\int \frac{(fx)^m\left(a+bx^n+cx^{2n}\right)^p}{d+ex^n}\,dx$$

Mathematica [A] time = 0.208149, size = 0, normalized size = 0.

$$\int \frac{(fx)^m\left(a+bx^n+cx^{2n}\right)^p}{d+ex^n}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n),x]
```

```
[Out] Integrate[((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n), x]
```

Maple [A] time = 0.073, size = 0, normalized size = 0.

$$\int \frac{(fx)^m (a + bx^n + cx^{2n})^p}{d + ex^n} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n),x)
```

```
[Out] int((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^{2n} + bx^n + a)^p (fx)^m}{ex^n + d} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n),x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^p*(f*x)^m/(e*x^n + d), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx^{2n} + bx^n + a)^p (fx)^m}{ex^n + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n),x, algorithm="fricas")
```

```
[Out] integral((c*x^(2*n) + b*x^n + a)^p*(f*x)^m/(e*x^n + d), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*x**n+c*x**(2*n))**p/(d+e*x**n),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^{2n} + bx^n + a\right)^p \left(fx\right)^m}{ex^n + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n),x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^p*(f*x)^m/(e*x^n + d), x)
```

3.156 $\int \frac{(fx)^m\left(a+bx^n+cx^{2n}\right)^p}{(d+ex^n)^2}\,dx$

Optimal. Leaf size=33

$$\text{Unintegrable}\left(\frac{(fx)^m\left(a+bx^n+cx^{2n}\right)^p}{(d+ex^n)^2},x\right)$$

```
[Out] Defer[Int][((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n)^2, x]
```

Rubi [A] time = 0.0251436, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(fx)^m\left(a+bx^n+cx^{2n}\right)^p}{(d+ex^n)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n)^2,x]
```

```
[Out] Defer[Int][((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n)^2, x]
```

Rubi steps

$$\int \frac{(fx)^m\left(a+bx^n+cx^{2n}\right)^p}{(d+ex^n)^2}\,dx=\int \frac{(fx)^m\left(a+bx^n+cx^{2n}\right)^p}{(d+ex^n)^2}\,dx$$

Mathematica [A] time = 0.284787, size = 0, normalized size = 0.

$$\int \frac{(fx)^m\left(a+bx^n+cx^{2n}\right)^p}{(d+ex^n)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n)^2,x]
```

```
[Out] Integrate[((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n)^2, x]
```

Maple [A] time = 0.074, size = 0, normalized size = 0.

$$\int \frac{(fx)^m (a + bx^n + cx^{2n})^p}{(d + ex^n)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^2,x)
```

```
[Out] int((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^{2n} + bx^n + a)^p (fx)^m}{(ex^n + d)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^2,x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^p*(f*x)^m/(e*x^n + d)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx^{2n} + bx^n + a)^p (fx)^m}{e^2x^{2n} + 2\,dex^n + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral((c*x^(2*n) + b*x^n + a)^p*(f*x)^m/(e^2*x^(2*n) + 2*d*e*x^n + d^2),
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*x**n+c*x**(2*n))**p/(d+e*x**n)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^{2n}+bx^n+a\right)^p\left(fx\right)^m}{\left(ex^n+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^p*(f*x)^m/(e*x^n + d)^2, x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
```

```

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal, debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=", ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
```

```
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
   as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
```

```
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
```

```
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
```

```
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
```

```
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
```

```
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(
    flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```
def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
```

```
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0], Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1], Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0], Rational)
                return 1
```

```
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
 elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
     m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
     m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
     return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
 elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
     return max(3,expnType(expn.operands()[0]))
 elif is_special_function(expn.operator()): #is_special_function(expn.func)
     m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
     return max(4,m1)   #max(4,m1)
 elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
     m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
     return max(5,m1)   #max(5,m1)
 elif is_appell_function(expn.operator()):
     m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
     return max(6,m1)   #max(6,m1)
 elif str(expn).find("Integral") != -1: #this will never happen, since it
             #is checked before calling the grading function that is passed.
             #but kept it here.
     m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
     return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
 else:
     return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)
```

```


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```